Experiment # 18

Optics Three
The Prism

Principles

Definition

A prism is a three dimensional triangular block of glass or clear acrylic. The triangle is either isosceles or equilateral or a right angled triangle. It has two parameters: (i) the refractive index n, and (ii) the apex angle θ_a. The refractive index belongs to the material of which the prism is made. The apex angle is usually understood to be the angle opposite the base. Scientifically, however, apex angle is the angle between those two sides of the prism that are used as interfaces. Any of the three angles may serve as the apex angle.,

Function

Prisms have two important functions: (i) Dispersion (ii) Reflection.

(i) Dispersion

In free space, electromagnetic waves of *all* frequencies travel at 3×10^8 m/s. But in a medium other than the free space, waves of no two frequencies have the same speed! The difference in speed for two neighboring frequencies may not be appreciable but for waves whose frequencies are far apart, the effect is certainly noticeable. In the visible spectrum, the frequencies of the seven rainbow colors are sufficiently far apart for the effect to be noticeable. The speed of a wave in a medium is popularly described in terms of the refractive index. Thus when we say that waves (of different frequencies) have different speeds, we really mean to tell you that each has a different refractive index in the *same* medium! The difference shows up in the second and third places of decimal. In flint glass, for example, the refractive index of red light (one end of the visible spectrum) is 1.618 while that of violet light (the other end) is 1.665.

According to Snell's law, waves of different frequencies that become incident on a material medium at the same point and at the same angle, will be transmitted into that medium at different angles. Each ray will then travel in a different direction inside the medium. When exiting the medium, these rays will be incident on the second interface at different points and at different angles. The positions of emergence and the angles of emergence will also be distinctly different. As each ray (or each bunch of rays) emerges at a different point and at a different angle, each is viewed by us as a distinct and independent ray. If white light be incident on a medium, in the manner described above, then upon emergence, one should be able to see the seven colors.

The effect is called *Dispersion*.

A prism is perhaps the simplest dispersive device and is reasonably effective. As such it is extensively used in spectrometers, spectrophotometers and other such instruments that require monochromatic beams of light for their operation.

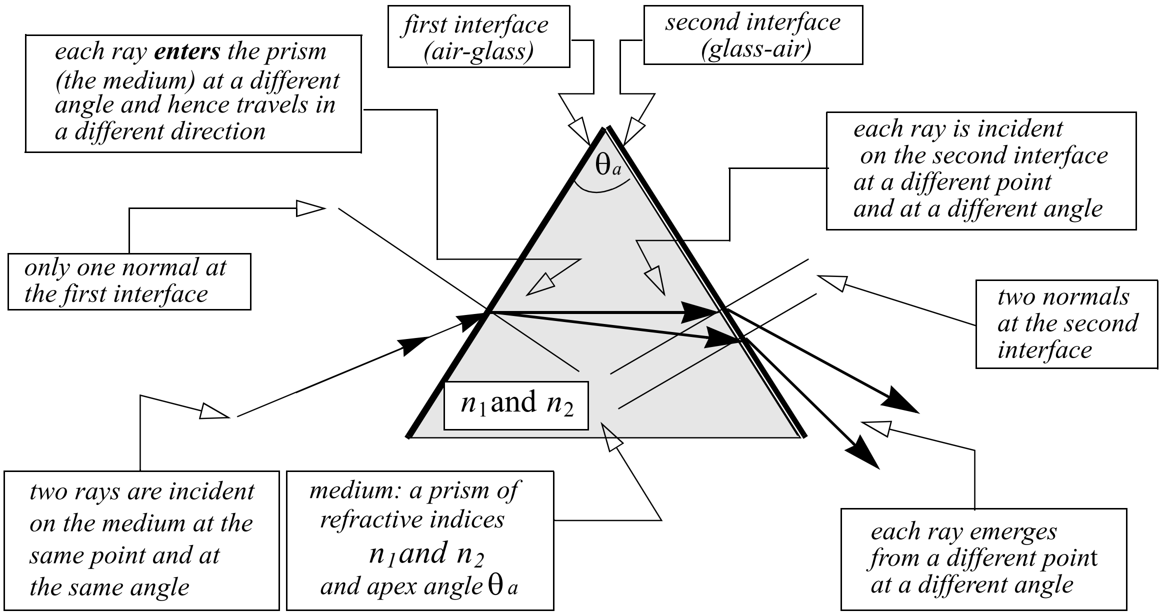

Fig (1) Anatomy of Dispersion of Two Rays of Different Frequencies, Effected by a Prism

(ii) Reflection

Even though prisms are made from excellent transmitting materials, they may be required to reflect light rays! This is done by using the principle of total internal reflection. The phenomenon was studied in the experiment *Optics One* and we need not repeat the theory here. The anatomy of total internal reflection as it occurs in a prism, is shown in Fig (2).

The ray (to be reflected) enters the prism at the first interface (air-glass) at *normal* incidence. There is no change of direction for this ray *inside* the prism. As it approaches the second interface (glass-air), it finds that the angle of incidence is greater than the critical angle. The ray gets totally internally reflected and stays within the glass. It travels toward the second glass-air interface (the third side of the prism). It so happens that the angle of incidence at this interface is also greater than the critical angle. The ray is again unable to get out of the prism because of the second occurrence of the total internal reflection. By cleverly arranging the geometry, it is possible to have the twice-totally-internally-reflected ray travel back to the first interface (still glass-air) and be incident on it *normally.* The ray finally emerges from the prism on the same side from which it had entered Because of normal incidence, the ray suffered no change of direction at the last leg of its journey. Thus it got reflected from the prism just as it would have, from a simple plane mirror!

One would tend to remark: “why bother”? If it is just the reflection you want, why not use a plane mirror! One justification for the *botheration* is that the reflected ray does not retrace its path. The reflected ray is significantly laterally displaced with respect to the incident ray. For the other, the prism causes the path-length of the ray to increase, which allows us to condense telescope into binoculars. Then there is the *free gift:* the quality of reflection here, is significantly superior to that of a plane mirror.

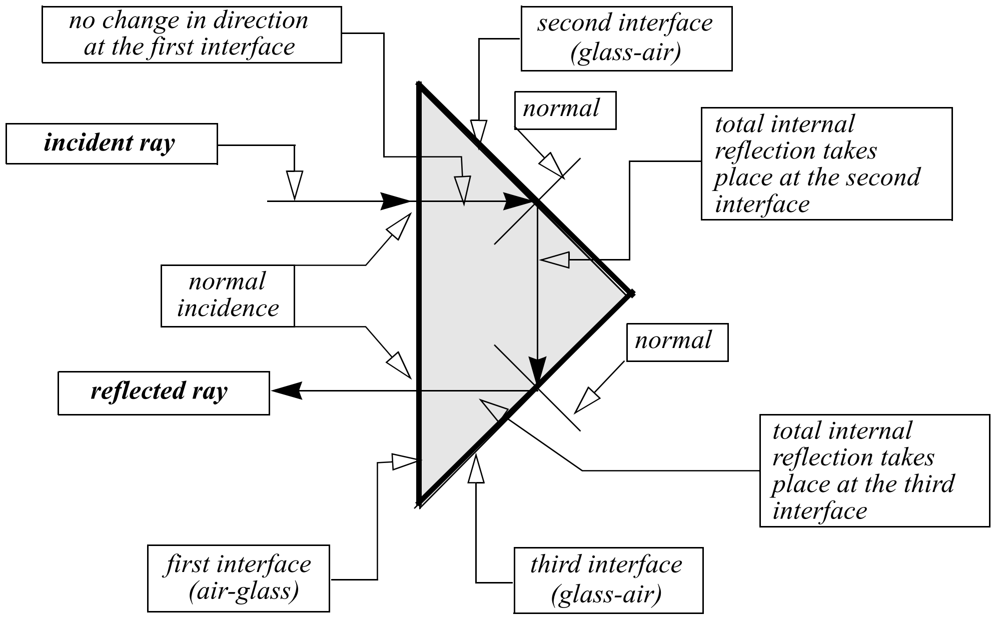

Fig (2) The Anatomy of Reflection by a Prism

Mathematical Equations

Like all (well, almost all) material media, a prism also has two interfaces. These two interfaces are at an angle θ_a with respect to each other. We shall apply Snell's Law to each of the two interface. Let the angles at the entrance interface be θ_1 and θ_2, while those at the exit interface be θ_3 and θ_4. This is shown in Fig (3).

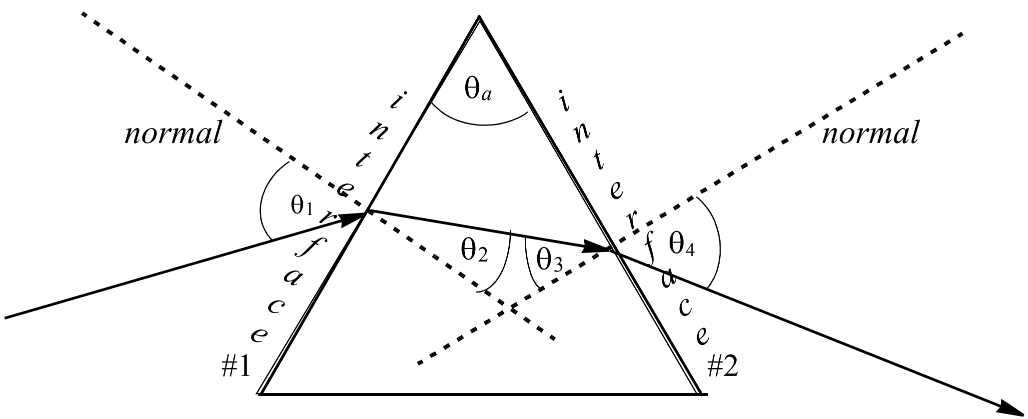

Fig (3) The Two Interfaces of a Prism

The two equations are:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \text{..........(1)}$$

$$n_2 \sin\theta_3 = n_1 \sin\theta_4 \qquad \text{..........(2)}$$

The two angles inside the prism are related to each other by:

$$\theta_3 = \theta_a - \theta_2 \qquad \text{..........(3)}$$

Objectives of the Experiment

To determine:

(1) the apex angle θ_a of the given prism, and

(2) the refractive index n, of the material of which it is made.

Setting Up

We can let a beam of light be incident on the prism and study the emergent ray. The angle of emergence depends on *both* characteristics of the prism. Hence, in principle, it should be possible to meet the objectives of the experiment.

We shall carry out some mathematical work to eliminate θ_2 and θ_3. The rationale for this is two-fold: (i) to eliminate measurement of angles inside the medium (which cannot be done anyway), and (ii) to have only two parameters in the *processed* equation, which is a necessary condition for plotting a graph.

From Eqn (1):

$$\sin\theta_2 = (n_1/n_2)\sin\theta_1 \qquad \text{..........(4)}$$

From Eqn (2):

$$\sin\theta_3 = (n_1/n_2)\sin\theta_4 \qquad \text{..........(5)}$$

Using Eqn (3):

$$\sin\theta_3 = \sin(\theta_a - \theta_2) \qquad \text{..........(6)}$$

From Eqns (5) & (6), we get:

$$(n_1/n_2)\sin\theta_4 = \sin(\theta_a - \theta_2) \qquad \text{..........(7)}$$

From Trigonometry:

$$\sin(\theta_a - \theta_2) = \sin\theta_a \cos\theta_2 - \cos\theta_a \sin\theta_2 \qquad \text{..........(8)}$$

Plug in the value of $\sin(\theta_a - \theta_2)$ from Eqn (8) in Eqn (7), to get:

$$(n_1/n_2)\sin\theta_4 = \sin\theta_a \cos\theta_2 - \cos\theta_a \sin\theta_2 \qquad \text{..........(9)}$$

Plug in the value of $\sin\theta_2$ from Eqn (4) in Eqn (9) to get:

$$(n_1/n_2)\sin\theta_4 = \sin\theta_a \cos\theta_2 - \cos\theta_a \left\{(n_1/n_2)\sin\theta_1\right\} \qquad \text{..........(10)}$$

Again, from Trigonometry:

$$\cos\theta_2 = \sqrt{(1-\sin^2\theta_2)} = (1-\sin^2\theta_2)^{1/2} \qquad \text{..........(11)}$$

Applying the Binomial Theorem to $(1-\sin^2\theta_2)^{1/2}$, we get:

$$(1-\sin^2\theta_2)^{1/2} = \left(1 + \tfrac{1}{2}\sin^2\theta_2 - \tfrac{1}{8}\sin^4\theta_2\right) \qquad \text{..........(12)}$$

where we have retained only three terms of the infinite series.
From Eqns (11) & (12) we get:

$$\cos\theta_2 = \left(1 + \tfrac{1}{2}\sin^2\theta_2 - \tfrac{1}{8}\sin^4\theta_2\right) \quad \text{..........(13)}$$

Now, from Eqn (4), squaring both sides, we get:

$$\sin^2\theta_2 = (n_1/n_2)^2 \sin^2\theta_1$$

and squaring a second time:

$$\sin^4\theta_2 = (n_1/n_2)^4 \sin^4\theta_1$$

Plug in these values in Eqn (13) to get:

$$\cos\theta_2 = \left(1 + (\tfrac{1}{2})(n_1/n_2)^2\sin^2\theta_1 - (\tfrac{1}{8})(n_1/n_2)^4\sin^4\theta_1\right)$$

Plug in this value of $\cos\theta_2$ in Eqn (10). We get:

$$(n_1/n_2)\sin\theta_4 =$$
$$\sin\theta_a\left(1 + (\tfrac{1}{2})(n_1/n_2)^2\sin^2\theta_1 - (\tfrac{1}{8})(n_1/n_2)^4\sin^4\theta_1\right) - \cos\theta_a\left\{(n_1/n_2)\sin\theta_1\right\}$$

Letting $n_1 = 1$ (for air) and multiplying throughout by n_2, we get:

$$\sin\theta_4 = n_2\sin\theta_a - \cos\theta_a\sin\theta_1 + \left\{\frac{\sin\theta_a}{2n_2}\right\}\sin^2\theta_1 - \left\{\frac{\sin\theta_a}{8n_2^3}\right\}\sin^4\theta_1 \quad \text{..........(14)}$$

This is our final result! Eqn (14) expresses the sine of the angle of emergence in terms of the sine of the angle of incidence. This is really **exotic** because we have been able to eliminate everything that happens inside the transmitting medium (the prism). The two angles lie in air and are easily measurable.

Our final result, Eqn (14), is in the form of a power series. Introducing constants:

$$A = n_2\sin\theta_a \quad \text{..........(15)}$$

$$B = \cos\theta_a \quad \text{..........(16)}$$

$$C = \frac{\sin\theta_a}{2n_2} \quad \text{..........(17)}$$

$$D = \frac{\sin\theta_a}{8n_2^3} \quad \text{..........(18)}$$

we can re-write Eqn (14) as:

$$\sin\theta_4 = A - B\sin\theta_1 + C\sin^2\theta_1 - D\sin^4\theta_1 \quad \text{..........(19)}$$

If plotted as such, the constants will enable us to determine both, the refractive index n and the apex angle θ_a .We have, however, one snag. The computer program *Cricket Graph*, does not plot a convergent power series!

A little deliberation shows that our convergent power series is indeed a polynomial of order n . All powers of the independent variable ($\sin\theta_1$) are positive integers. None is negative or fractional.

Luckily, we only need the first two coefficients! Hence a polynomial of second order will serve our purpose and we shall be able to get the values of n and θ_a from coefficients A & B.

<u>Procedure</u>

You ned a well sharpened, type #2 pencil for the experiment.

(1) Set up the circular table on the optical bench as was done in Expt: *Optics One*.

(2) Set up a plane wave front (as per Expt: *Optics Two*.

(3) Place a sheet of white paper on the circular table such that the edge of the paper along its length, coincides with the *component* line. Fix the paper with two short pieces of cello-tape.

(4) Place the given prism on the *component* line with one of the two long (and equal) sides coinciding with the line.

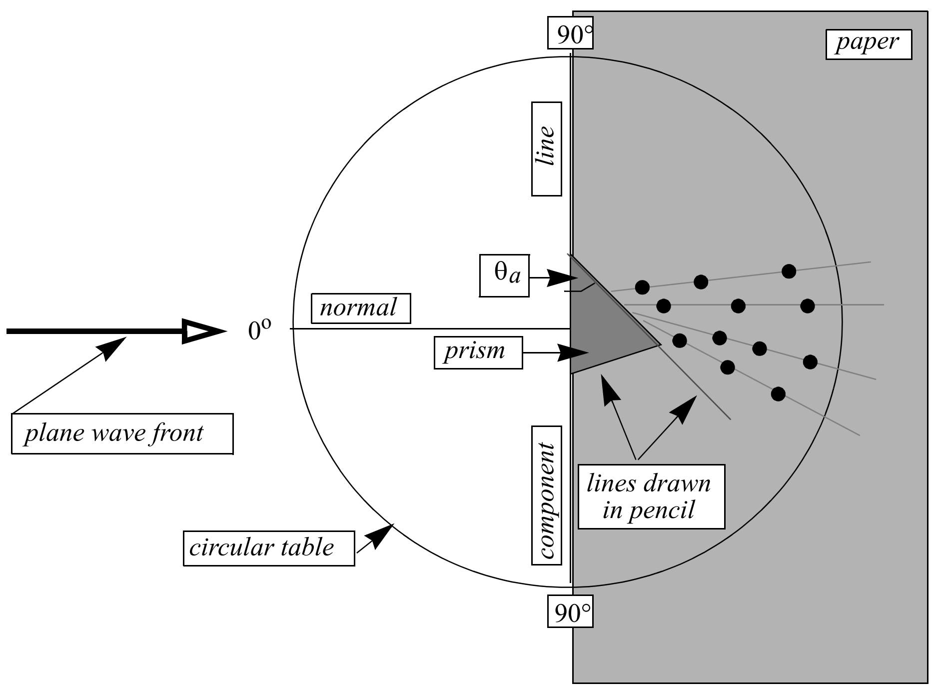

Fig (4) The Set up for the Experiment

(5) Holding the prism firmly, draw the boundary of the prism in pencil, as shown in Fig (4). Extreme care should be taken to see that the prism does not move from its position.

(6) Align the reflected ray such that $\theta_r = \theta_i$ for $\theta = 60^o$. The reflected light is rather feeble but can be seen clearly along the translucent surface of the circular table. This will ensure that the interface is perpendicular to the normal. Select following angles of incidence θ_1: $0°$, $5°$, $10°$...... $65°$. Set these angles one by one, by gently rotating the circular table. Make sure that the paper or the prism does not get disturbed.

(7) For each angle of incidence, you will find an emergent beam on the other side of the prism. The beam will be about a millimeter wide. We need mark the position of this beam but we cannot use a ruler to draw the line. We shall, therefore, mark several dots

along the beam (only 3 are shown in the above diagram, 5 are recommended). This is shown by black dots in the diagram. The dots should be as fine (but still visible) as possible. When the tip of the pencil is placed on the beam, a distinct shadow is formed. It will be very easy to make the marks, in the center of the beam. The three marks need not be at equal intervals. One mark should be near the prism, the second near the end of the table, and the third anywhere in between.

(8) Repeat for all other angles of incidence.

(9) The experiment ends. Remove the prism and then the paper. Arrange all apparatus and components neatly on the table.

Calculations & Graph

(1) The line drawn in pencil for the glass-air interface should now be made permanent. Draw a line with a ball pen of *fine* tip over this line. Call this line the *interface*.

(2) Connect the first set of three (or more) points corresponding to angle of incidence $\theta_1 = 0°$ by a long and thin straight line, using the sharp pencil and the given ruler. This line represents the out-coming ray. Extend it to meet the interface and call it the *emergent beam*. ***Draw only one line at a time.***

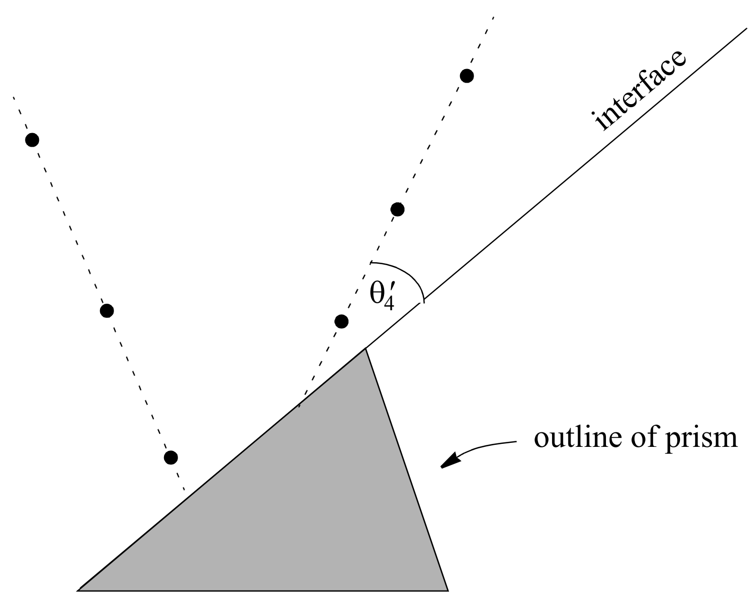

Fig (5) Measuring Emergent Angles

(3) Using the large, transparent protractor, measure the angle of the *emergent beam*, θ_4', with respect to the interface (and not with respect to the normal, that we did not draw). This is why the angle is named θ_4' and not θ_4. This measurement should be made as accurately as possible. Try to estimate the angle up to half of a degree or better. For best results, you should look at the protractor and the lines from the *top* and not from the side. Record the value in your data sheet. Once the angle is measured and recorded, you should erase that part of the *emergent beam* which is near the interface. This will allow you to see the next *emergent beam* clearly.

(4) Connect the next set of three (or more) points for the second *emergent beam*, corresponding to the second angle of incidence. Repeat step (2) and step (3).

(5) Repeat the above for all other trials. Be warned that the lines drawn for different emergent beams will meet the interface at different points. These points will get closer and closer as the angles of incidence become larger and larger.

(6) Calculate *sine* of all angles of incidence, θ_1, and *cosines* of all angles of emergence, θ_4'. Plot $\cos\theta_4'$ on y-axis and $\sin\theta_1$ on the x-axis. Ask the computer to fit a second order polynomial to the data with r^2.

(7) The coefficient of x in the equation printed out by the computer is $B = \cos\theta_a$ (Eqn 16). Take *cosine*$^{-1}$ of this coefficient. This is the apex angle θ_a of the prism.

(8) Now that we know θ_a, we can plug in the value of $\sin\theta_a$ on the right hand side of Eqn (15), where A is the *intercept* in the equation printed out by the computer. Solve for n_2. This is the refractive index of the material of which the prism is made.

(9) Compare the experimentally obtained values of θ_a (step 7) and n_2 (steps 8), above, with their expected values: 40° for θ_a and 1.50 for n_2. Find percent errors.

(10) Compile the "Results" of the experiment.

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Note (1)*: We measured angles θ_4' instead of θ_4 not for laziness but for getting improved accuracy. By measuring angles θ_4', we eliminated drawing normals for every emergent beam. As you know, the more lines we draw, the more will be the error. Hence by reducing the number of lines we draw, we hope to get better results.*

Note (2)*: According to Eqn (14), we should have plotted $\sin\theta_4$. Now θ_4 is $(90 - \theta_4')$. From Trigonometry, however, we find that:*

$$\sin\theta_4 = \sin(90 - \theta_4') = \cos\theta_4'$$

This is why we are plotting $\cos\theta_4'$.

Note (3)*: The accuracy of the result depends on how carefully and patiently you carry out the geometrical construction and how accurately you read the angles.*

Data & Data Tables

Name.................................... **Date**..............................
Instructor.............................. **Lab Section**....................
Partner.......................... **Table #**..........................

Expected value of the refractive index of the material of the given prism:1.5

Expected value of the apex angle of the given prism:40^o

Table 1: The Incident and the Emergent Angles

Trial #	Angle of Incidence θ_1	Angle of Emergence θ_4'	Trial #	Angle of Incidence θ_1	Angle of Emergence θ_4'
1	0^o		8	35^o	
2	5^o		9	40^o	
3	10^o		10	45^o	
4	15^o		11	50^o	
5	20^o		12	55^o	
6	25^o		13	60^o	
7	30^o		14	65^o	

Additional Information or Data (if any):

Additional data table (just in case....)

Table 2: The Incident and the Emergent Angles

Trial #	Angle of Incidence θ_1	Angle of Emergence θ_4'	Trial #	Angle of Incidence θ_1	Angle of Emergence θ_4'
1	0^o		8	35^o	
2	5^o		9	40^o	
3	10^o		10	45^o	
4	15^o		11	50^o	
5	20^o		12	55^o	
6	25^o		13	60^o	
7	30^o		14	65^o	