Experiment # 17

Optics Two
Plane Wavefronts & Curved Interfaces

More "Principles"

"Simple" Curved Surfaces and Devices

If an optical medium is of irregular shape and size, then its boundaries will also be correspondingly irregular. An irregular boundary is, in general, a mixture of regular and irregular curvatures. The interface at any point on such a surface, will be a tangent to the surface at that point. The normal will then be perpendicular to the tangent. For a totally irregular interface, the tangents and the normals will all be criss-crossing and overlapping one another. There may not even be a definable *image*. Obviously, we are not interested in studying the optical behavior of such objects. We, however, will be interested in studying the optical behavior of media that have regularity of some sort, *regularly curved boundaries,* for example. Some media of this type are cylinders, spheres, ellipsoids, paraboloids and so on. Spheres are extensively used for making *spherical mirrors* and *lenses*. These are, in turn, used to make a variety of optical devices; from make-up and shaving mirrors to parabolic mirrors, cameras, telescopes microscopes, binoculars etc. Adequate mathematical tools are available for dealing with them.

Spheres remain the simplest and the most regularly-curved media. In the ensuing discussions, therefore, we shall confine ourselves to spheres or parts thereof. Thus our *curved* interface will be a *slice* of the *surface* of a sphere. The *slice* will have a *radius of curvatur*e which it would have inherited from the sphere of which it is a slice. Thus the radius of curvature of the *slice* is the radius of the sphere itself. One such slice is shown in Fig (1) and is called a *curved interface*.

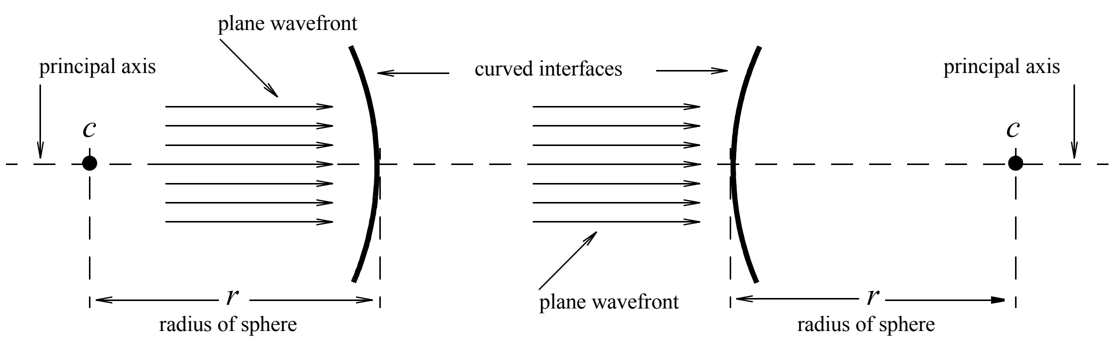


Fig (1) Plane Wavefronts & Curved Interfaces

A line drawn from the center of the sphere to the geometrical center of the *curved interface* (and extended in both directions) is called *principal axis*. Such an interface can face a plane wavefront in two different ways, as shown in Fig (1). Reflection type media have one curved interface while the transmission type media have two curved interfaces. Some mirrors and lenses are shown in Fig (2).

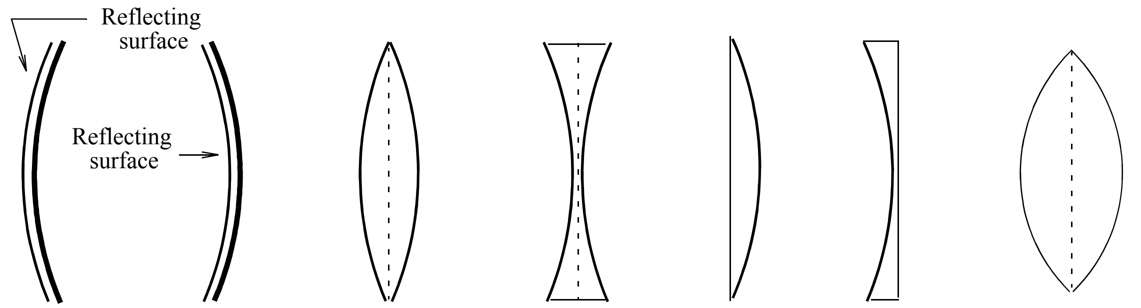


Fig (2) Some Devices made of Simple Curved Surfaces

Concept of Interface for Curved Surfaces

For a plane wavefront, incident on a plane interface, the wavefront can be represented by a single ray and this is what we did for the experiment: *Optics One*. For a curved interface, however, such a simple representation is not possible because different parts of the wavefront (different rays of the same beam) are incident on different parts of the curvature. For each of these rays, the angle of incidence is different. In fact, for all practical purposes, we can say that each ray is really incident on a *different* interface. These interfaces are tangents to the curvature at the point of incidence. Since radii are perpendicular to tangents, the radii serve as normals to the interfaces and all angles of incidence *and* reflection are constructed with respect to the radii. Fig (3) shows a plane wavefront incident on a reflecting type curved surface. It is seen that all reflected rays meet at a single point, which lies on the principal axis.

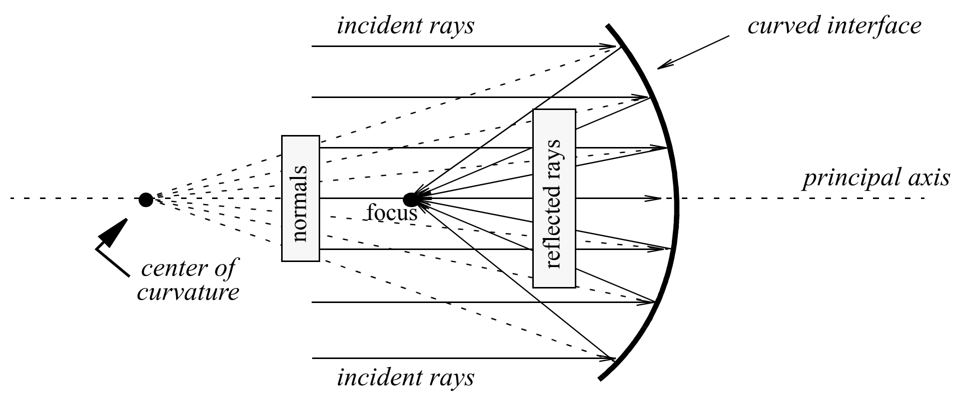
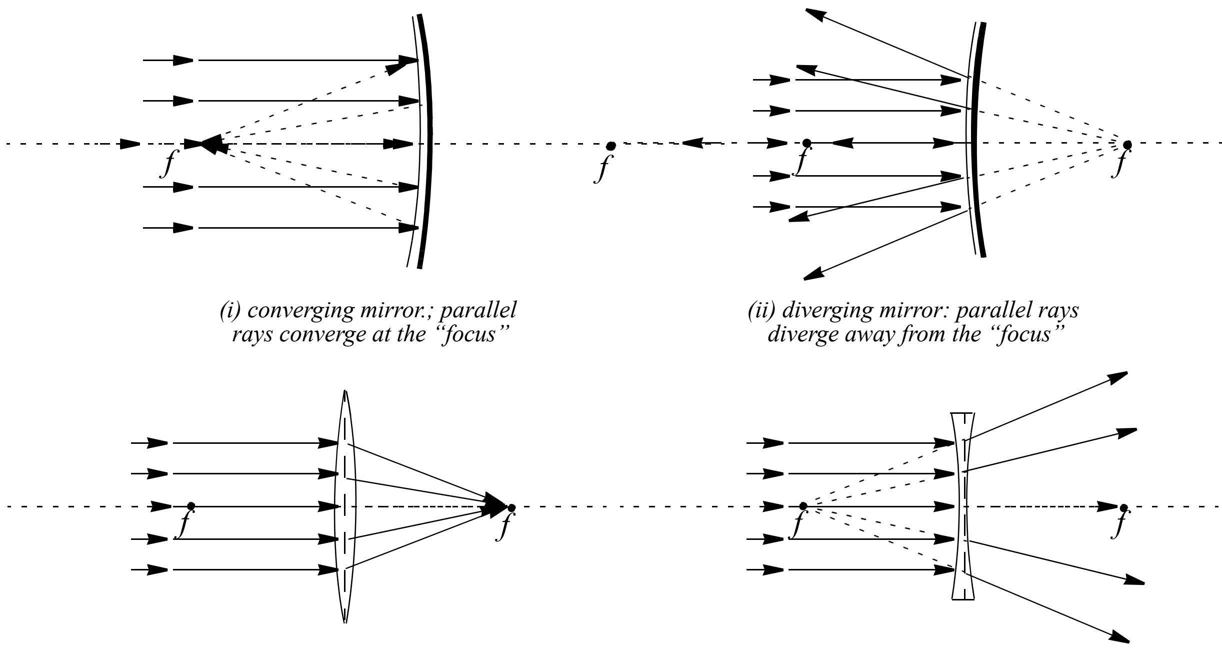


Fig (3) Incident & Reflected Rays on a Curved Interface

The reflection and transmission of a plane wavefront (parallel rays of light) is shown in Fig (4), for four different types of devices. These are respectively: (i) converging mirror, (ii) diverging mirror, (iii) converging lens, and (iv) diverging lens. The construction is based on the laws of reflection and transmission. It is interesting to see that, in every case, all rays of light meet at one point. This point is called the *focal point* or the *focus* of the device. The distance of the focal point from the center of the lens or mirror is one-half the radius of curvature; or one-quarter of the diameter of the sphere of which the curved surface is a slice.

$$f = r/2 = d/4, \quad r = \text{radius}, \ d = \text{diameter} \qquad \text{..........(1)}$$



(i) converging mirror.; parallel rays converge at the "focus"

(ii) diverging mirror: parallel rays diverge away from the "focus"

(iii) converging lens: parallel rays converge at the "focus"

(iv) Diverging lens: parallel rays diverge away from the "focus"

Fig (4) Thin lenses & Mirrors

It will be noticed, in general, that a converging device converges the beam on the focal point or at the focus. All rays meet at this point. A diverging device diverges the rays of the beam away from the focal point. The rays move away from one another and never meet, but they appear to have come from the *other* side of the device.

If the plane wavefront is not parallel to the principal axis, the rays do not get focussed at the focal point. They get focussed, however, on a point in the *focal plane* of the device. This is shown in Fig (5).

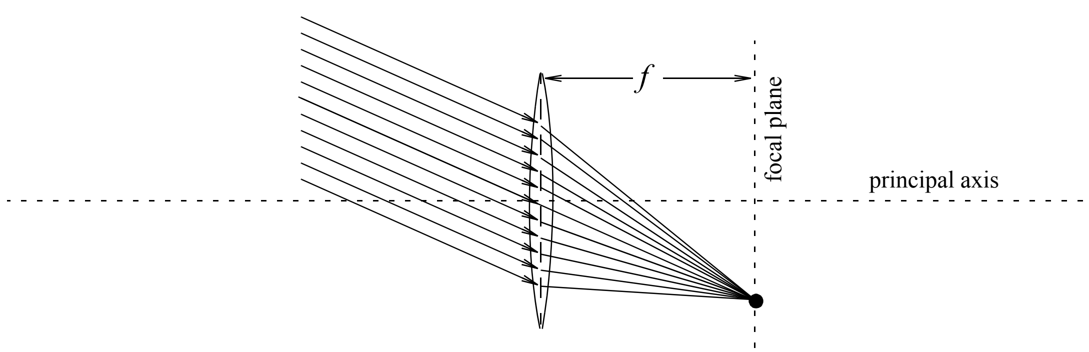


Fig (5) Thin Lens: Parallel rays are brought to focus in the focal plane

Again, in case of thick lenses (and mirrors), it is found that *all* parallel rays of a beam of light do not get focussed at one point. The rays are focussed in a small region of space on the principal axis. This is shown in Fig (6). Such lenses do not make useful devices. They may, however, be used as *condensing lenses* (and mirrors) to condense light from an extended source, in a small region of space.

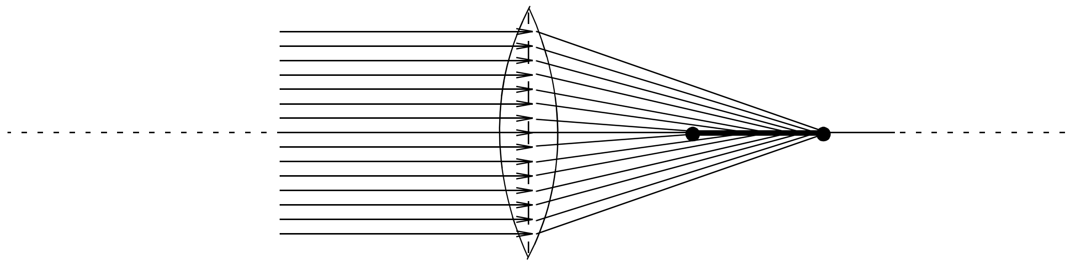


Fig (6) Thick Lens: Rays are brought to focus at different points

Image: Definition

So far we talked of wavefronts being incident *directly* on the interfaces. This discussion is mainly of academic interest. For practical applications of the principles, we consider *objects, illuminated* by plane wavefronts. When a plane wavefront is incident on the object, each point of the object serves as a new source of light. Light is re-propagated in accordance with the Huyghen's principle, in *all* directions in the three dimensional space. We say that the plane wavefront *scattered off* the object. When these (scattered) rays are incident on a device, they interact with the interface as per the rules of reflection and transmission. The resulting configuration of the rays (if any) is called *image*.

Image: Characteristics

There are four characteristics of optical images that one would like to find or determine:

(a) Position: in front of the device or behind it.

(b) Size: magnified or diminished as compared to the actual size.

(c) Orientation: upright or inverted.

(d) Nature: real image or virtual.

Image: Formation

The four characteristics of an *image* depend on (a) the type of lens or mirror used, and (b) the position (location) of *object* with respect to the lens or mirror. To determine these characteristics, we shall presently introduce some parameters. For what follows, we shall assume that the rays that scattered off an object, are incident on the device from the left hand side. We shall call it the *light side*. The right hand side will then be called the *opposite side*.

Principal Axis: A horizontal line, passing through the geometrical center of the lens or mirror. Points *F* and *C* lie on this line.

c = *center of curvature* of the lens or mirror

f = *focal length* of a lens or mirror; *positive* for *converging* lens or mirror, *negative* for the *diverging* ones.

d_o = *distance of the object* from the lens or mirror; positive if on the light side and negative if on the opposite side.

d_i = *distance of the image* from the lens or mirror; positive if on the opposite side and negative if on the light side.

h_o = *height of the object*; positive if above the principal axis and negative if below it.

h_i = *height of the image*; positive if above the principal axis and negative if below it.

m = *orientation*; positive *m* implies an upright image while a negative *m* implies an inverted image.

m = *magnification*; $m > 1$ implies a magnified image while $m < 1$ implies a diminished image.

It is of interest to know that the afore-mentioned characteristics of images can be determined either geometrically, using *ray diagrams*; or analytically, by developing mathematical formulae. We shall presently consider the ray diagrams.

Ray Diagrams

Of the multitude of rays that scatter off the object *and* become incident on the device, we shall select four rays for reasons of their strategic positions and use them to make a *ray diagram*; the purpose of which is to locate the image. The selected rays follow simple geometrical rules for reflection and transmission. It is, however, not necessary to use all four rays for a ray diagram. One can use as few as two and as many as all four rays for this purpose. The rules are:

(a) a ray travelling parallel to the principal axis will pass through the focal point f, after reflection or transmission from the device.

(b) a ray incident on the device at its geometrical center will travel undeflected through a transmission type interface. In case of reflection type interface, it will be reflected at an angle of reflection equal to the angle of incidence.

(c) a ray passing through the focal point f will become parallel to the principal axis, after reflection or transmission from the device.

(d) a ray passing through the center of curvature $\boldsymbol{c}$, will travel undeflected through the device for transmission type device. For a reflection type device, the ray will retrace its path. This is a case of *normal incidence* as the ray actually travels along the radius of the sphere.

The object is placed on the *light side* of the lens or mirror and we let a plane wavefront be incident upon it. Rays scatter off every part of the object. These scattered rays travel in all directions. Some of them travel towards the lens or mirror and are reflected or transmitted at the interfaces, at the point of contact (with the device).

To study the image formation, it is customary to stand the object on the principal axis, on the light side of the lens or mirror, and follow the path of rays that scatter from the upper-most part of the object. One would select two or three rays from the list of four rays that have been described above. A ray scattering off the lowest part of the object (which lies on the principal axis), travels radially and keeps moving along the principal axis (Rule b). The image of the bottom of the object will thus be formed on the principal axis itself. We, therefore, need locate the position of the image of the *top* of the object only.

Converging Devices.

Ray diagrams leading to the formation of images are shown for a converging lens, for three different positions of the object (with respect to the lens). Converging devices **usually** produces real images, as can be seen in Fig (7a) and (7b). In Fig (7c), however, the image is virtual. In case of real image, transmitted rays actually meet at one point. Such an image can always be placed on a screen.

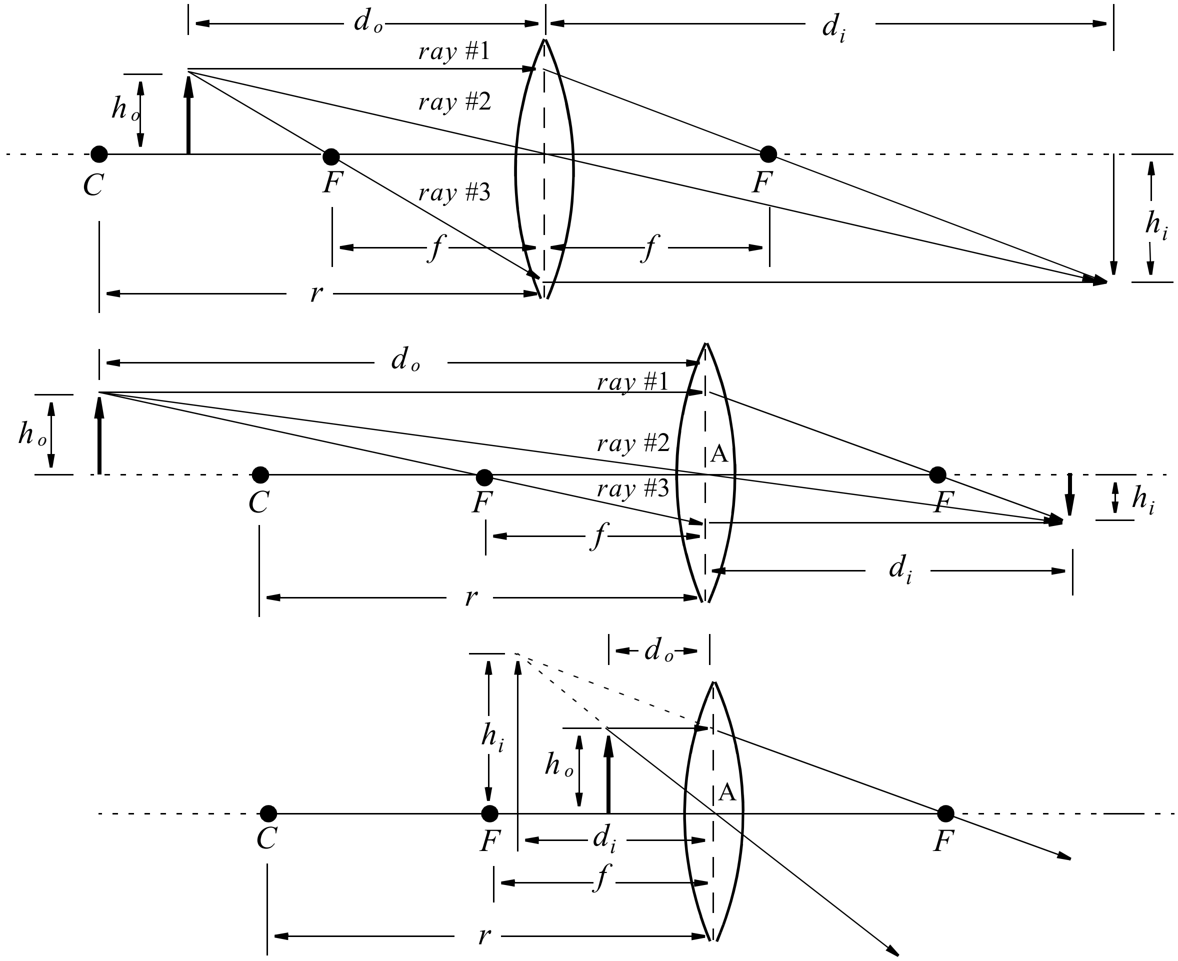


Fig (7) Image Formation by Tracing Rays: Converging lenses

Diverging Devices

Diverging devices *do not* produce real images. After interacting with the device, the rays diverge away in all directions. To our eyes, it *appears* as if they are coming from some point *behind* the diverging device (which may or may not be transparent). Such an image is (obviously) not real and, as such, cannot be placed on a screen. We call it a *virtual* image. Even though the image is not real and cannot be placed on a screen, our eyes have the extraordinary capability of *projecting* these rays backward and actually seeing an image. See Figs (8a) & (8b).

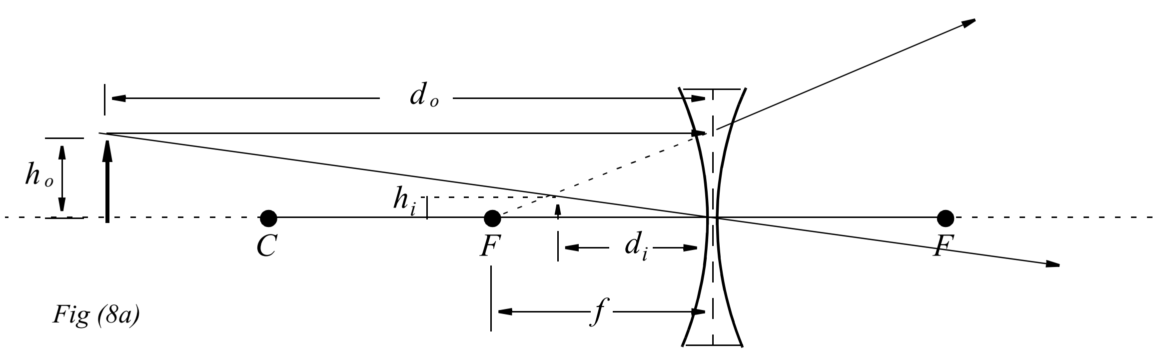


Fig (8a)

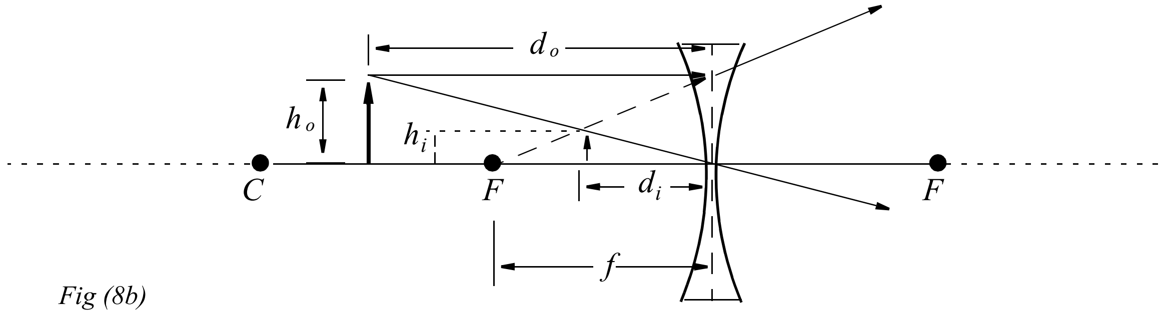


Fig (8b)

Fig (8) (a) & (b): Image Formation: Tracing Rays: Diverging Lenses

Combinations of converging and diverging devices

For some applications, one may use two or more devices (converging, diverging or their combinations) together, by placing them in such a way that they have a common principal axis. The devices may either be in contact with one another (separation $s = 0$) or some distance s apart (separation $s > 0$). In such cases, the image formed by the device facing the object serves as the object for the device behind it (away from the object).

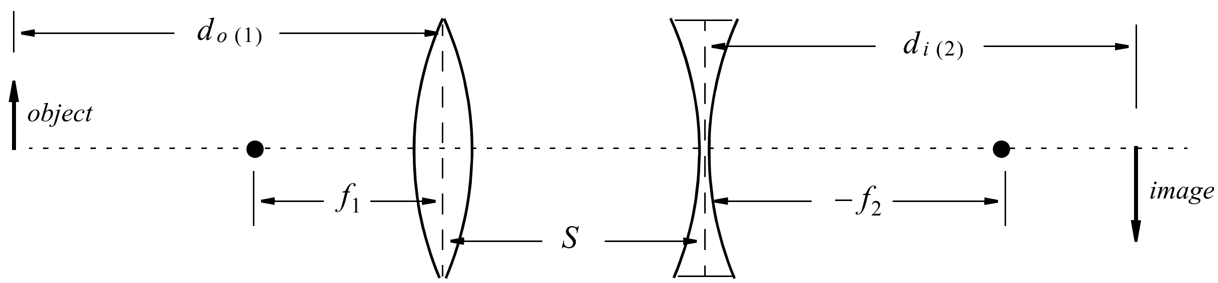


Fig (9) Using two lenses

Mathematical Analysis

The purpose of mathematical analysis is to be able to predict the four characteristics of the image: position, size, orientation and nature, for a given object and a given device (lens or mirror). Following are the two equations that interconnect the above listed parameters.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad ....(2)$$

$$m = -\frac{d_i}{d_o} = \frac{h_i}{h_o} \qquad ....(3)$$

A negative d_i implies that the image is virtual while a negative m implies that the image is inverted (upside down).

Combination of Lenses

When using two lenses in combination (placed a distance s apart, where $s \geq 0$,), we do not have to start all over again. We do not need any new formulae either. All we do is to remember that the image formed by the lens in front (the first lens), serves as the object for the second lens. We use the general lens formula twice (once for each lens) and use:

$$d_{o(2)} = s - d_{i(1)} \qquad \text{..........(4)}$$

Objectives of the Experiment

To determine the focal length of:
(a) a converging lens, and
(b) a diverging lens.

Setting Up

Focal Length of the Converging Lens

The objective can be accomplished easily by using Eqn (2). One would set up a plane wavefront and let it be incident on an object. The rays that will scatter off the object, will be received on the curved interface of the converging lens. An image will be formed (hopefully). To receive this image, one would adjust the position of the screen for a best-focus image. Knowing the object distance (from the lens) and the image distance (also from the lens), the focal length can be determined. To determine the focal length scientifically, however, one would need plot a graph. The relationship, as given in Eqn (2), does not match the equation of a straight line. This difficulty can be overcome by restructuring Eqn (2), as shown below:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Adding the fractions on the left hand side:

$$\frac{d_o + d_i}{d_o\, d_i} = \frac{1}{f}$$

Cross multiplying and rearranging, we get:

$$d_o\, d_i = f(d_o + d_i) \qquad \text{..........(5)}$$

This is the restructured equation which is comparable to the equation of a straight line. We are *guided* by this comparison, to plot the product of the two distances against their sum, to obtain a straight line. The slope will directly be the desired focal length of the converging lens.

Focal Length of the Diverging Lens

As for the focal length of the diverging lens, the above procedure cannot be adopted because diverging lenses do not make real images. There will be nothing on the screen and there will be no way of finding the *image distance.* This can also be shown mathematically. The focal length f is negative for diverging lenses. If we take $(1/d_o)$ to the right hand side, it would also become negative. The two negative terms will force d_i to be negative as well. As we know, negative d_i stands for a virtual image. Since a virtual image cannot be placed on the screen, d_i remains undetermined.

One way out is to force the diverging lens to produce a real image by feeding it a negative object distance. If d_o itself is negative then it would appear as a positive quantity on the right hand side of Eqn (2); and, further, if $(1/d_o)$ happens to be greater than $(1/f)$, then $(1/d_o - 1/f)$ will be a positive quantity. This will cause d_i to be positive, place-able on a screen and measurable!

Where do we find a negative object distance? On the face of it, it appears to be an illogical proposition. A deep knowledge of the principles, however, tells us that the real image of an object is as good an object as the object itself. If we place a real image on the *opposite* side of the diverging lens, then it will indeed form a negative object distance for the diverging lens. A real image of the object can easily be produced by a converging lens. The converging lens, in turn, can be positioned in such a way as to produce the real image *behind* the diverging lens.

We find that forcing the diverging lens to produce a real image wasn't all that far-fetched after all! Let there be a converging lens of focal length f_1. Let the object be placed at $d_{o\,(1)}$ to make a real image at $d_{i\,(1)}$. Let the diverging lens of focal length $-f_2$ be placed some distance s on the *dark* side (*opposite side*) of the converging lens. The object distance for the second lens: $d_{o\,(2)}$ is found from Eqn (4). Hopefully it will be negative.

Once the two lenses are in position and the object has been placed at a distance $d_{o\,(1)}$ to the left of the converging lens of known focal length f_1, the next step will be to determine $d_{i\,(1)}$. We, however, will not be able to determine $d_{i\,(1)}$ because of the presence of the diverging lens. But we may calculate it using Eqn (2). We shall then calculate $d_{o\,(2)}$ from Eqn (4). We shall next determine experimentally the image distance $d_{i\,(2)}$, of the real image as formed by the diverging lens. Inserting the values of both, $d_{o\,(2)}$ and $d_{i\,(2)}$ into Eqn (2) a second time, we shall (at last) be able to calculate f_2. Let's rewrite the concerned equations over again:

$$\frac{1}{d_{o\,(1)}} + \frac{1}{d_{i\,(1)}} = \frac{1}{f_1} \qquad \text{..........(6)}$$

$$d_{o\,(2)} = s - d_{i\,(1)} \qquad \text{..........(7)}$$

$$\frac{1}{d_{o\,(2)}} + \frac{1}{d_{i\,(2)}} = \frac{1}{f_2} \qquad \text{..........(8)}$$

To do the work scientifically, one needs to obtain the required focal length from a graph. This is possible by reshaping and restructuring the three equations (mathematically) in such a way as to yield a single equation, that should be comparable to the equation of a straight line. When this is done, one would need select a value of lens separation, s (to be kept constant throughout the experiment) and select several values of $d_{o\,(1)}$. For each $d_{o\,(1)}$, one would determine $d_{i\,(2)}$ experimentally. Following are the mathematical steps that lead to the final equation.

Step (1): Solve Eqn (6) for $d_{i\,(1)}$. Rearranging and adding fractions, we get:

$$d_{i\,(1)} = \frac{f_1\, d_{o\,(1)}}{d_{o\,(1)} - f_1}$$

Step (2): Plug in this value of $d_{i\,(1)}$ in Eqn (7) to get $d_{o\,(2)}$:

$$d_{o\,(2)} = \left[\frac{s}{1} - \frac{f_1\, d_{o\,(1)}}{d_{o\,(1)} - f_1}\right]$$

$$d_{o(2)} = \frac{s\, d_{o(1)} - sf_1 - f_1\, d_{o(1)}}{d_{o(1)} - f_1}$$

Step (3): Now plug in this value of $d_{o(2)}$ in Eqn (8):

$$\frac{d_{o(1)} - f_1}{[s\, d_{o(1)} - sf_1 - f_1\, d_{o(1)}]} + \frac{1}{d_{i(2)}} = \frac{1}{f_2}$$

Adding fractions on the left hand side, we get:

$$\frac{(d_{i(2)})(d_{o(1)} - f_1) + [s\, d_{o(1)} - sf_1 - f_1\, d_{o(1)}]}{(d_{i(2)})[s\, d_{o(1)} - sf_1 - f_1\, d_{o(1)}]} = \frac{1}{f_2}$$

Cross multiplying and rearranging:

$$(d_{i(2)})[s\, d_{o(1)} - sf_1 - f_1\, d_{o(1)}] = (f_2)\{(d_{i(2)})(d_{o(1)} - f_1) + [s\, d_{o(1)} - sf_1 - f_1\, d_{o(1)}]\} \quad \text{..........(9)}$$

This is the final result. As can be seen, if we plot:

$(d_{i(2)})[s\, d_{o(1)} - sf_1 - f_1\, d_{o(1)}]$ ....on y-axis ....*(10)*

and

$\{(d_{i(2)})(d_{o(1)} - f_1) + [s\, d_{o(1)} - sf_1 - f_1\, d_{o(1)}]\}$ ....on x-axis ....*(11)*

we should be able to get the focal length of the diverging lens f_2, as the slope of the straight line!

Procedure

(a) Focal length of the converging lens

(1) Read and record the value of the focal length of the given converging lens, as written on the lens (in mm).

(2) Obtain a plane wavefront, using the procedure used in the previous experiment. You may try placing the light source at 80 mm, on the optical bench. Place the *parallel-rays len*s at 140 mm, on a component-holder, facing the light source. Place the *crossed-arrow target* on the opposite side of the *parallel-rays lens*, on the same component-holder. The position of the *crossed-arrow target* (the object) will be 150 mm.

(3) Select the following 10 values for the object distance d_o, (the *crossed-arrow target*,): 250 mm, 240 mm, 230 mm,......160 mm. You should locate the position of the lens (on the optical bench) by adding the values of d_o to that of the *crossed-arrow target*. As the *crossed-arrow target* cannot be moved, then to select different d_o, the lens must be moved. Record the position of the lens (as read on the optical bench), in the data table.

(4) For each d_o selected, find the image by moving the screen back and forth on the optical bench. When a sharp and well focussed image has been found, read the position of the screen on the optical bench and enter in the data table. The distance of the image from the lens: d_i, is found by subtracting the position of the lens (in mm, on the optical bench) from that of the screen (in mm, also on the optical bench). Also enter in the data table, The instructor will give you some hint for locating the best image.

(b) Focal length of the diverging lens

(5) Read and record the value of the focal length of the given diverging lens, as written on the lens (in mm). Please note that it comes with a negative sign.

(6) Place the diverging lens 60 mm behind the converging lens (the opposite side). Thus s is 60 mm.

(7) Select the following 7 values of $d_{o(1)}$: 200 mm, 190 mm,.......140 mm. Remember that the *crossed-arrow target* cannot be moved. Hence when you move the converging lens to set a value of $d_{o(1)}$, the diverging lens must also be moved to maintain an inter-lens distance of 60 mm. Record the positions of both lenses (as read on the optical bench), in the data table.

(8) For each $d_{o(1)}$, locate the image $d_{i(2)}$, at its sharpest and brightest, on the screen. Read and record the position of the screen in the data sheet. Find the value of $d_{i(2)}$ by subtracting the position of the diverging lens (on the optical bench) from the position of the screen (also on the optical bench). Also record in the data table.

(9) The experiment is over. Arrange all parts neatly on the table.

Calculations and Graphs.

(a) Focal length of the converging lens

(1) For all values of d_o and d_i, calculate $(d_o \, d_i)$ and $(d_o + d_i)$.

(2) Plot $(d_o \, d_i)$ on y-axis and $(d_o + d_i)$ on the x-axis. Draw the best fit straight line and find its slope. This is the focal length of the given converging lens. Compare with the value recorded in the data sheet and find percent error.

(b) Focal length of the diverging lens

(3) Using the values of $d_{o(1)}$, $d_{i(2)}$, f_1 and s, calculate the expressions given in Eqns (10) and (11), for all 7 trials.

(4) Plot values from Eqn (10) on y-axis and the values from Eqn (11) on the x-axis. Draw a best fit straight line and find its slope. You will get a straight line with a negative slope! This is f_2, the focal length of the given diverging lens. Compare with the value recorded in the data sheet and find percent error.

(5) Compile the "Results".

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Helpful Notes

(1) Remember you are finding f_2 . You do not know its value at this time. Do not (repeat do not) use its value ($-150\ mm$) in any calculation.

(2) As the diverging lens has a negative focal length, the straight line in your graph must have a negative slope.

Data & Data Tables

Name.................................... **Date**..............................

Instructor.............................. **Lab Section**....................

Partner.......................... **Table #**..........................

Focal length of the given converging lens: 75 mm

Position of the edge of the light box: 80 mm OR ....(mm)

Position of the Parallel Rays Lens: 140 mm OR ....(mm)

Position of the crossed-arrow target, **to be called "Pcat"** ...150...(mm)

(a) Focal length of the converging lens

Table 1: Data for the Focal Length of the Converging Lens

Trial # (1)	Distance of the Crossed-Arrow Target from the Lens d_o (mm) (2)	Position of Lens "PL" = (Pcat + d_o); (mm) (3)	Position of Screen "PS"; (mm) (4)	Image Distance d_i = (PS - PL) (mm) (5)
1	250			
2	240			
3	230			
4	220			
5	210			
6	200			
7	190			
8	180			
9	170			
10	160			

(b) Focal length of the diverging lens

Focal length of the given diverging lens: -150 mm

Position of the edge of the light box: 80 mm OR ....(mm)

Position of the Parallel Rays Lens: 140 mm OR ....(mm)

Position of the Crossed -Arrow Target, **to be called "Pcat"** ....150......(mm)

Separation s of the two lenses: 60 mm

Table 2: Data for the Focal Length of the Diverging Lens

Trial # (1)	Distance of the Crossed-Arrow Target from the Converging Lens $d_{o\ (1)}$ (mm) (2)	Position of the converging lens: "PCL" = (Pcat + $d_{o\ (1)}$); (mm) (3)	Position of the diverging lens: "PDL" = (PCL + 60); (mm) (4)	Position of the screen "PS" (mm) (5)	Image distance $d_{i\ (2)}$ = (PS - PDL) (mm) (6)
1	200				
2	190				
3	180				
4	170				
5	160				
6	150				
7	140				

Additional information or data (if any):