Experiment # 12

RC Circuits

Principles

Definition

An *RC* circuit is a series combination of a resistor and a capacitor, fed by DC electricity.

Capacitors & DC Sources of Electricity

Capacitors are basically rectangular blocks of insulating materials, placed in between a pair of parallel metallic sheets, called "plates". When a battery (source of DC electricity) is connected to the metallic plates and the switch is closed, electricity travels over the connecting wires to the plates but no further. The insulator block stops the flow of electrons. Electrons stay on the plate and form the charge $-Q$ (in coulombs). The opposite metallic plate acquires an equal and opposite charge $+Q$. We say that the capacitor is *charged* by the battery (or that charges have been *deposited* on the plates) in accordance with the law:

$$Q_C = C\,V_C \qquad \text{..........(1)}$$

where C is the capacitance of the capacitor and V_C is the voltage across the capacitor. According to Kirchhoff's loop rule, capacitor voltage V_C is equal to the source voltage ε . Thus we may write:

$$V_C = \varepsilon$$

A circuit diagram of a capacitor connected to a battery is shown below:

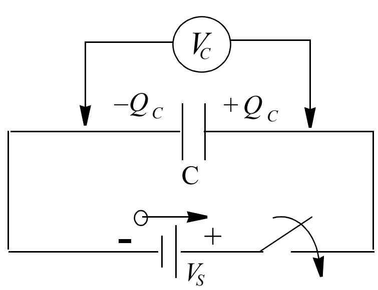

Fig (1) A Capacitor and a DC Source

When the battery is disconnected the charges $\pm Q_C$ stay on the plates and so does the electrical potential difference V_C (which equals ε). For ideal capacitors, the charges and the voltages will maintain their respective values indefinitely. But in case of real life capacitors, charges slowly leak to the surroundings (usually moist air) and after some time, all charges disappear, leaving the capacitor uncharged. Again, if the two terminals of the capacitor be connected together (shorted), then charges will flow violently from one plate to another and get neutralized. No charge or voltage will be left on the capacitor in this case.

The Charging Time

Even though most of the above information was given in a previous experiment, we did not talk about the time needed to charge or discharge a capacitor. It is interesting to note that a capacitor gets charged (or discharged) instantaneously. Upon closing the switch, a very large current flows from the battery to the capacitor. The charging current is limited only by the finite resistance of the connecting wires. A 20 ampere or a 30 ampere *fuse* between the battery and the capacitor has little chance of surviving. The discharging current is equally strong. If not handled carefully, such a discharge may cause a painful shock. Usually the discharging of the capacitor is accompanied with sparks and some crackling sound.

Controlling the Charging Time

One can slow down the motion of electrons (that travel to the capacitor plates and get deposited on it) by placing a resistor in their path. Not only that it is logical and is acceptable by *common sense*, it is also substantiated by the unit of R . The unit of R is *ohm* which represents (V/I) or $(JS)/(C^2)$. This shows that a resistor *does* involve a *time* component. The greater the resistance, the greater will be the time of travel of electrons through the resistor because of the increased random motion in their path. And the larger the time of travel of electrons, the less will be the number of electrons reaching the capacitor plates, *per unit time*. And as less electrons will be reaching the plates of the capacitor, *per unit time*, the capacitor will take longer to get charged. Such an arrangement will permit us to *control* the charging time of the capacitor. A suitable circuit is shown in Fig (2).

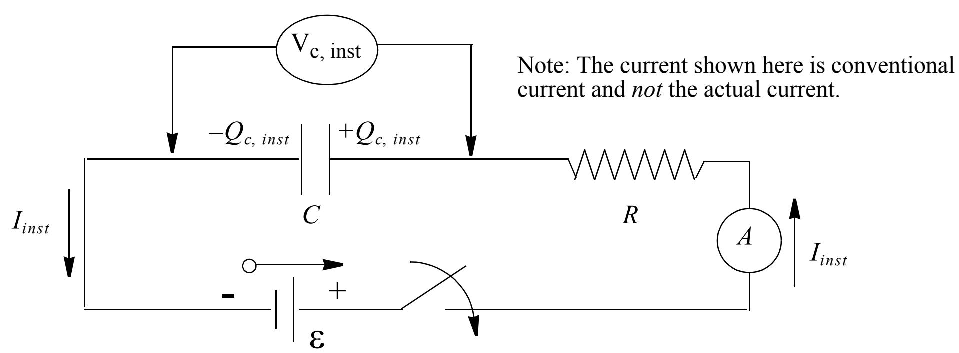

Fig (2) A Circuit to Control Charging Time

Strange as it may seem, but it turns out that the composite unit of the capacitor-resistor pair is just *time*. The respective units of resistance R and capacitance C, are $(JS)/(C^2)$ and C^2/J. (Refer to the experiments on insulators and conductors.) Consider:

$$RC = \left(\frac{JS}{C^2}\right)\left(\frac{C^2}{J}\right) = S$$

The product is called the *capacitive time constant* (or just *time constant*) and we write τ for it. The unit of τ is, of course, *time (*in seconds). The magnitude of τ varies from one pair of RC to another, and is found by multiplying their values in ohms and farads.

Mathematical Analysis

(a) Charging

To study mathematically the manner in which the charging time is affected by the resistance, one would write down a loop equation for the circuit of Fig (2), using Kirchhoff's loop rule. There are two *sinks* and one *source.* We get the following loop equation:

$$\varepsilon = V_C + V_R \qquad \text{..........(2)}$$

Applying Ohm's law to the resistor, we find that:

$$V_R = IR \qquad \text{..........(3)}$$

From the capacitor equation: $Q_C = CV_C$, we find that:

$$V_C = \frac{Q_C}{C} \qquad \text{..........(4)}$$

Inserting these values in Eqn (2), we get:

$$\varepsilon = \frac{Q_C}{C} + IR$$

Here I is the charging current. Writing I as $\Delta Q_C / \Delta t$, we get:

$$\varepsilon = \frac{Q_C}{C} + R\,\frac{\Delta Q_C}{\Delta t}$$

Multiplying by C throughout, we get:

$$C\,\varepsilon = Q_C + RC\,\frac{\Delta Q_C}{\Delta t} \qquad \text{..........(5)}$$

When the capacitor voltage V_C reaches its maximum value ε, (the source voltage), the charge upon it will also reach its maximum value $Q_{c,\,max}$. Letting $Q_C = Q_{c,\,max}$ and $V_C = \varepsilon$ in Eqn (1), we get $Q_{c,\,max} = C\varepsilon$. Replacing $C\varepsilon$ in Eqn (5) by $Q_{c,\,max}$, we get:

$$Q_{c,\,max} = Q_C + RC\,\frac{\Delta Q_C}{\Delta t} \qquad \text{..........(6)}$$

Q_C, as it appears in above equations, is in fact the instantaneous value of charge as it is being deposited on the capacitor. Writing $Q_{C,\,inst}$ for Q_C we get:

$$Q_{c,\,max} = Q_{C,\,inst} + RC\,\frac{\Delta Q_C}{\Delta t} \qquad \text{..........(7)}$$

Equation (7) is a differential equation. It can be solved only with the help of calculus. We, therefore, proceed no further and borrow the result from the mathematicians for the variation of capacitor charge $Q_{C,\,inst}$ with time:

$$Q_{C,\,inst} = Q_{c,\,max}(1 - e^{-t/RC}) \qquad \text{..........(8)}$$

Next, we use this result to find $V_{C,\,inst}$ for the capacitor. Dividing both sides by C and making use of the appropriate capacitor equations, we get:

$$V_{C,\,inst} = \varepsilon\,(1 - e^{-t/RC}) \qquad \text{..........(9)}$$

A similar equation representing the instantaneous current $I_{C,\,inst}$ for the capacitor, is found from a continued use of calculus, and is as follows:

$$I_{C,inst} = I_{max}\,(e^{-t/RC}) \qquad \text{..........(10)}$$

where I_{max} is given by Ohm's law as:

$$I_{max} = \varepsilon/R \qquad \text{..........(11)}$$

It is to be noted that the charging of the capacitor is not linear. It is exponential. The rate of charging is large to begin with and slows down as the time progresses. This is also predicted, qualitatively, by Eqn (6). Since the right hand side is necessarily constant, and so is RC, therefore as $Q_{C,inst}$ increases, $\Delta Q_C/\Delta t$ must decrease. Mathematically speaking, the time needed for the capacitor to reach full charge is infinitely long; but it acquires about 99% of Q_{max} in about five time constants. A table of $Q_{C,inst}$, $V_{C,inst}$ for different time constants τ, is given in table (1); while that for $I_{C,inst}$ is given in Table (2). A graph showing the rise of $Q_{C,inst}$, $V_{C,inst}$ and the fall of $I_{C,inst}$ is shown in Fig (3).

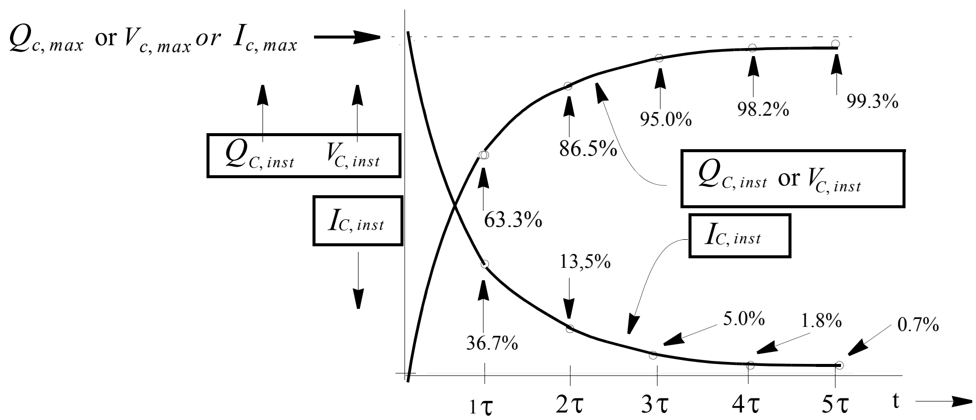

Fig (3) Anatomy of the Exponential Charging of a Capacitor

(b) Discharging

When the capacitor is fully charged and it is to be slowly discharged, one may use a suitable resistor in series with the capacitor. By joining together the two ends of the series *RC* circuit, the capacitor will begin to discharge slowly. Fig (4) shows the necessary circuit for a controlled discharge of the capacitor

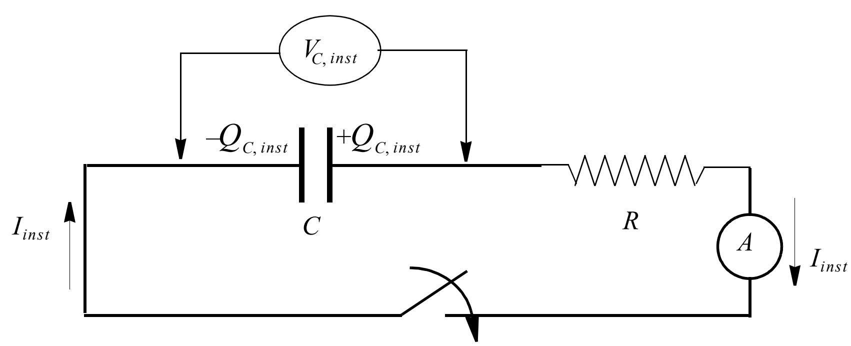

Fig (4) A Circuit to Control the Discharging Time

Kirchhoff's loop equation for the circuit is:

$$0 = V_C + V_R \qquad \text{..........(12)}$$

Proceeding on the same lines as for the charging of the capacitor, we obtain:

$$0 = Q_{C,\,inst} + RC\,\frac{\Delta Q_C}{\Delta t} \quad \text{..........(13)}$$

When solved using calculus, we find the following three equations for the variation of (i) capacitor charge $Q_{C,\,inst}$ (ii) the capacitor voltage $V_{C,\,inst}$, and (iii) the capacitor current $I_{C,\,inst}$, with time:

$$Q_{C,\,inst} = Q_{max}(e^{-t/RC}) \quad \text{..........(14)}$$

$$V_{C,\,inst} = \varepsilon(e^{-t/RC}) \quad \text{..........(15)}$$

$$I_{C,\,inst} = -I_{max}(e^{-t/RC}) \quad \text{..........(16)}$$

Please note that: (1) all three parameters now decrease from their 100% values to zero, (2) the direction of current flow is now opposite to that of the charging circuit. The current *always* decreases in an RC circuit!

A table of values of $Q_{C,\,inst}$ and $V_{C,\,inst}$ for different time constants τ, is given in table (1); while that for $I_{C,\,inst}$ is given in Table (2). A graph showing the fall of $Q_{C,\,inst}$, $V_{C,\,inst}$, and $I_{C,\,inst}$ is shown in Fig (5).

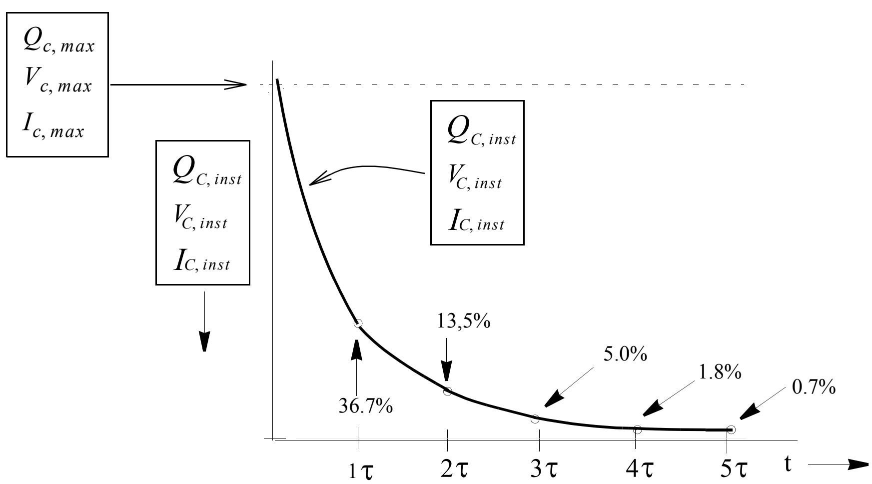

Fig (5) Anatomy of the Exponential Discharging of a Capacitor

Table 1: Voltages: Percent Charge & Discharge in Different Time Constants

mode	$t = 1\tau$	$t = 2\tau$	$t = 3\tau$	$t = 4\tau$	$t = 5\tau$
Instantaneous Charges & Voltages	$Q_{C,\,inst}$ or $V_{C,\,inst}$	$Q_{C,\,inst}$ or $V_{C,\,inst}$	$Q_{C,\,inst}$ or $V_{C,\,inst}$	$Q_{C,\,inst}$ or $V_{C,\,inst}$	$Q_{C,\,inst}$ or $V_{C,\,inst}$
When Charging	Rises to: 63.212%	Rises to: 86.466%	Rises to: 95.021%	Rises to: 98.168%	Rises to: 99.326%
When Discharging	Falls to: 36.788%	Falls to: 13.534%	Falls to: 4.979%	Falls to: 1.832%	Falls to: 0.674%

Table 2: Currents: Percent Charge & Discharge in Different Time Constants

mode	$t = 1\tau$	$t = 2\tau$	$t = 3\tau$	$t = 4\tau$	$t = 5\tau$
Instantaneous Currents	$I_{C,\,inst}$	$I_{C,\,inst}$	$I_{C,\,inst}$	$I_{C,\,inst}$	$I_{C,\,inst}$
When Charging & Discharging	Falls to: 36.788%	Falls to: 13.534%	Falls to: 4.979%	Falls to: 1.832%	Falls to: 0.674%

In Table (3), all necessary formulae have been collected for easy reference.

Table 3: Charge & Discharge of a Capacitor in an RC Circuit

Charging $Q_{C,\,inst}$	Charging $V_{C,\,inst}$	Charging $I_{C,\,inst}$	Discharging $Q_{C,\,inst}$	Discharging $V_{C,\,inst}$	Discharging $I_{C,\,inst}$
$Q_{max}\left(1-e^{-\frac{t}{RC}}\right)$	$V_{max}\left(1-e^{-\frac{t}{RC}}\right)$	$I_{max}e^{-\frac{t}{RC}}$	$Q_{max}e^{-\frac{t}{RC}}$	$V_{max}e^{-\frac{t}{RC}}$	$-I_{max}e^{-\frac{t}{RC}}$
$Q_{max} = C\,\varepsilon$	$V_{max} = \varepsilon$	$I_{max} = \varepsilon/R$	$Q_{max} = C\,\varepsilon$	$V_{max} = \varepsilon$	$I_{max} = -\varepsilon/R$

Objectives of the Experiment

To study the exponential charge and discharge of a capacitor in an RC circuit, find a value of its time constant τ, experimentally and estimate the magnitude of resistance r , representing the leakage effect.

Setting Up

Selection of Circuit Components

The circuit is simple and the experiment is straight forward. No major setting-up problems are encountered. One should select a large capacitor and a large resistor to obtain a large time constant. A large time constant is necessary for being able to record several values of voltages and currents within a time constant. A time constant of the order of 100 seconds should be adequate. One can record 5 sets of values of voltages and currents (at 20-second intervals) for each time constant. Large resistors will cause current to be too small to be adequately measured. In particular, resistors in M¾ range. will produce currents in fractions of microamperes; which is not really welcome. Resistors in the range of hundreds of kilo-ohms may be acceptable. This requires the capacitors to be in the range of hundreds of microfarads. Such large capacitors are usually *electrolytic* and have polarity. Electrolytic capacitors have larger than normal *leakage*. *Leakage* is the loss of charge due to the inability of the capacitor to hold-on to large quantities of charge. This results in longer charging time and one gets larger time constant than the expected one.

Leakage should accelerate the discharging process. The discharge is very fast in the early stages and the effect of leakage may be less pronounced here. In the later part of the discharge process, when a relatively small quantity of charge is left on the plates, leakage will again be less pronounced. Thus, when using large (electrolytic) capacitors, it will be safer to study the time constant during the discharge mode of the capacitor. A study of the time constant during the charging mode, on the other hand, may give us some idea of the leakage, present in the capacitor. Such a leakage is best expressed in terms of an *internal* resistance of the capacitor. Resistors always dissipate energy and thus will simulate the leakage present in the capacitor. It is possible to estimate the magnitude of this resistance.

Let the charging time constant, found experimentally be τ', which we expect to be greater than τ. Then:

$$\tau' = R'C \qquad \text{..........(17)}$$

The resistance R' includes the leakage effect and we may write:

$$R' = R + r \qquad \text{..........(18)}$$

where r is the additional resistance and represents leakage; R is the actual resistance used. Subtracting, one can determine the leakage resistance r.

Restructuring the Formulae

The formulae, as developed in *Principles* (Eqns 9, 10, 15 & 16) are not in the form of the equation of a straight line. They must be restructured for use in the laboratory. We shall do this for one equation and leave the rest as an exercise for the students. Recall Eqn (8):

$$V_{C,\,inst} = \varepsilon\,(1 - e^{-t/RC})$$

Divide both sides by ε to get:

$$\frac{V_{C,\,inst}}{\varepsilon} = (1 - e^{-t/RC})$$

Subtract both sides from 1 to get:

$$1 - \frac{V_{C,\,inst}}{\varepsilon} = e^{-t/RC}$$

Take natural log (ln) on both sides:

$$ln\left(1 - \frac{V_{C,\,inst}}{\varepsilon}\right) = -\frac{t}{RC}$$

Re-arranging:

$$ln\left(1 - \frac{V_{C,\,inst}}{\varepsilon}\right) = -\left(\frac{1}{RC}\right)(t) \qquad \text{..........(19)}$$

Eqn (19) is in the form of the equation of a straight line. Plotting the left hand side against time, we expect to get a straight line of negative slope. The magnitude of this slope is the reciprocal of the time constant. It should be clearly understood that it is the slope which is negative; not the time constant! Time constant is a physical quantity and cannot be negative.

Similar calculations for other three equations yield:

$$ln\left(\frac{I_{C,\,inst}}{I_{max}}\right) = -\left(\frac{1}{RC}\right)(t) \qquad \text{..........(20)}$$

and

$$ln\left(\frac{V_{C,\,inst}}{\varepsilon}\right) = -\left(\frac{1}{RC}\right)(t) \qquad \text{..........(21)}$$

and

$$ln\left(\frac{I_{C,\,inst}}{I_{max}}\right) = -\left(\frac{1}{RC}\right)(t) \qquad ....(22)$$

Please note that equations (10) and (16) are identical except for the negative sign. This is because the discharging current flows in a direction opposite to that of the charging current. The ammeter will show a negative sign. This may be recorded but should be left out in calculations.

Procedure

(a) Study of the Charging Mode

(1) Let us select a capacitor-resistor pair of rated values 440 μF and 230 $k\Omega$ The time constant of this pair will be about 100 seconds and a 17 volt supply will produce a maximum current of about 74 μA. Find the actual value of the resistor, using an ohmmeter, and the actual value of the capacitor, using an LCR meter; enter these values in the data sheet.

(2) Calculate the actual value of time constant and record in your data sheet.

(3) Set one multimeter to 20 V, DC and the other to 2.00 mA, DC.

(4) Measure the supply voltage ε and record its value. Calculate the actual value of I_{max} and enter in your data sheet.

(5) Set up the charging circuit, as shown in Fig (6), below.

(6) Set the stop watch to *zero*. Switch on the circuit and the stop watch ***simultaneously***. Record the voltmeter and ammeter readings every 20 seconds for the next 3 time constants. This amounts to 15 trials.

(7) Switch off the circuit and the stop watch. Reset the stop watch.

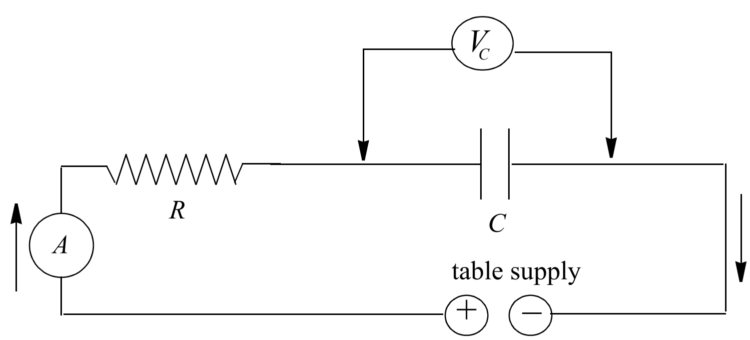

Fig (6) Circuit for the Charging of the Capacitor

(b) Study of the Discharging Mode

(8) We shall fully charge the capacitor and then start the discharge process. To fully charge the capacitor, set the resistance box to 1.00 $k\Omega$ and switch on the circuit. The capacitor will get charged immediately; the voltmeter should read the supply voltage ε and the ammeter should read just a few microamperes. *Leave the circuit on.*

(9) While the circuit is still on, set the resistance box to its original setting i.e. 230 $k\Omega$.

(10) Pull out the banana plug from the positive (red) terminal of the power supply and insert it in the negative (black) terminal of the power supply *and* start the timer simultaneously.

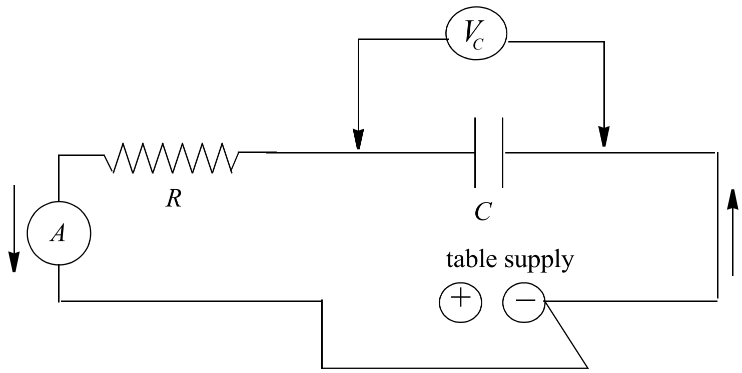

Fig (6) Circuit for the Discharging of the Capacitor

(11) Read and record the values of the ammeter and the voltmeter every 20 seconds for the next 3 time constants; (15 trials).

(12) The experiment ends. Switch off meters and disconnect all wires. Arrange apparatus neatly on the table.

Calculations & Graphs

Do all the calculations on the computer. For all four graphs to be plotted here, instruct the computer to fit second order polynomial curves to the data and print the equation to five decimal places.

(1) For all instantaneous charging voltages, calculate $ln\left(1 - \frac{V_{C, inst}}{\varepsilon}\right)$ and, ignoring the negative signs, plot on y-axis, against *time* (x-axis). This is Graph # 1.

(2) For all instantaneous charging currents, calculate $ln\left(\frac{I_{C, inst}}{I_{max}}\right)$ and, ignoring the negative signs, plot on y-axis against *time* (x-axis). This is Graph # 2

(3) For all instantaneous discharging voltages, calculate $ln\left(\frac{V_{C, inst}}{\varepsilon}\right)$ and, ignoring the negative signs, plot on y-axis against *time* (x-axis). This is graph # 3.

(4) For all instantaneous discharging currents, calculate $ln\left(\frac{I_{C, inst}}{I_{max}}\right)$ and, ignoring the negative signs, plot on y-axis against *time* (x-axis). This is graph # 4.

(5) The coefficient of x in these graphs is $1/(RC)$ or $1/\tau$. For each graph, find and fit the optimum order of polynomial to obtain the best value of $1/(RC)$. Note that $1/(RC)$ comes with a negative sign, which must be dropped. When the best value of $1/(RC)$ has been obtained, take its reciprocal. This is the time constant τ of our RC circuit. We shall get four of these. We would expect these four values to be equal to one another (more or

less), and also comparable to the value found in step (2) of Procedure. But you will find that this is not so. Time constants are larger than expected and are far from being equal.

(6) Find the average of the four time constants. This is τ'. Compare τ with τ' and find the percent difference (not percent error!, although the same formula is to be used). This will be part of *Results*.

(7) From the experimental value of τ' found in step (6), find the value of R', using Eqn (17). Next find the value of resistance r, the leakage resistance, using Eqn (18). Express r as percent of R and enter in *Results*.

(8) Compile *Results* using the information provided in steps (6) and (7) of this section.

Why Polynomials?

Eqns. (19) through (22) are all linear equations. Why did we plot polynomial graphs? It turns out that the leakage of electrical charge in capacitors induces nonlinearity. The effect is unpredictable. The nonlinearity is highly significant in some cases and moderately significant in others. One of the advantages of polynomial graphing is that it serves to remove impurities from the data and produces a *refined* value of whatever we are looking for. By fitting a second order polynomial, we get a distinctly improved value of the (reciprocal) of the time constant.

What if this improved value is still unacceptable? We have good news. We can ask the computer to fit a higher order of polynomial curve to our data! It is found that the level of refinement increases. We may try third, fourth, fifth and even sixth order polynomial curve fit. It is further found that the refinement does not continue to improve indefinitely. After a certain degree of refinement, the use of an increased order of polynomial, produces chaos. That's where we stop! Often best results are obtained with third or fourth order polynomials. But, in some cases, even a sixth order polynomial was found to produce genuine refinement.

Coefficients of higher powers of x (such as x^2, x^3, x^4, x^5, etc.) are all *impurities* and will be discarded. Luckily, these coefficients are found to be surprisingly small and insignificant. In many cases these remain zero up to fifth or even sixth place of decimal. This phenomenon further validates our methodology of extracting refined values of the (reciprocals) of time constants of our circuit. If, on the other hand, these coefficients emerged as massive entities, we would have had to abandon the technique.

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given.

Data & Data Tables

Name....................................   **Date**..............................
Instructor..............................   **Lab Section**....................
Partner..........................   **Table #**..........................

The Resistance R: (a) Nominal Value: 230 $k\Omega$
(b) Actual Value (using Ohmmeter) ($k\Omega$)

The Capacitor C (a) Nominal Value: 440 μF
(b) Actual Value (using LCR meter) (μF)

The Time Constant τ (a) Nominal Value: 230 x 10^3 x 440 x 10^{-6} = 101.20 sec

(b) Actual Value: sec

The Supply Voltage ε (as measured by a voltmeter) (volts)

The value of I_{max} (as found from ε/R) = (μA)

Study of Charging And Discharging Modes:

Table 3: Capacitor Voltages & Currents in Charging & Discharging Modes

	Charging Mode					Discharging Mode			
Trial # (1)	Time: (stop watch) (2)	Time (sec) (3)	$V_{C,inst}$ (volts) (4)	$I_{C,inst}$ (mA) (5)	Trial # (6)	Time (stop watch) (7)	Time (sec) (8)	$V_{C,inst}$ (volts) (9)	$I_{C,inst}$ (mA) (10)
1	00:00	0			1	00:00	0		
2	00:20	20			2	00:20	20		
3	00:40	40			3	00:40	40		
4	01:00	60			4	01:00	60		
5	01:20	80			5	01:20	80		
6	01:40	100			6	01:40	100		
7	02:00	120			7	02:00	120		
8	02:20	140			8	02:20	140		
9	02:40	160			9	02:40	160		
10	03:00	180			10	03:00	180		
11	03:20	200			11	03:20	200		
12	03:40	220			12	03:40	220		
13	04:00	240			13	04:00	240		
14	04:20	260			14	04:20	260		
15	04:40	280			15	04:40	280		
16	05:00	300			16	05:00	300		

Additional data or information (if any):

Calculation Tables **Name** **Date**

Recall: ε = (volts) I_{max} = $(mA\ )$

(a) Calculations For the Charging Mode

Table 4: Calculations For the Charging Mode

Serial #	$V_{C,\,inst}$ from Table 3 column 4	$\frac{V_{C,\,inst}}{\varepsilon}$	$1 - \frac{V_{C,\,inst}}{\varepsilon}$	$ln\left(1 - \frac{V_{C,\,inst}}{\varepsilon}\right)$	$I_{C,\,inst}$ from Table 3 column 5	$\frac{I_{C,\,inst}}{I_{max}}$	$ln\left(\frac{I_{C,\,inst}}{I_{max}}\right)$
1.							
2.							
3.							
4.							
5.							
6.							
7.							
8.							
9.							
10.							
11.							
12.							
13.							
14.							
15.							
16.							

(b) Calculations For the Discharging Mode;

Table 5: Calculations For the Discharging Mode

Serial #	$V_{C,\,inst}$ from Table 3 column 9	$\frac{V_{C,\,inst}}{\varepsilon}$	$ln\left(\frac{V_{C,\,inst}}{\varepsilon}\right)$	$I_{C,\,inst}$ from Table 3 column 10	$\frac{I_{C,\,inst}}{I_{max}}$	$ln\left(\frac{I_{C,\,inst}}{I_{max}}\right)$
1.						
2.						
3.						
4.						
5.						
6.						
7.						
8.						
9.						
10.						
11.						
12.						
13.						
14.						
15.						
16.						