

**Experiment # 10**

## Circuit Analysis - 2b

### Kirchhoff's Rules - Capacitors

PrinciplesKirchhoff's Rules

Kirchhoff's Rules for capacitors, do not differ from those for resistors. These were discussed adequately in a previous experiment, where the rules were applied to a circuit containing resistors and batteries only. As such, we will not repeat them. We shall, however, discuss the necessary changes in the circuit to cater for the special needs of capacitors.

Special Needs

Capacitors do not conduct electricity. However, when fed by an AC supply, pseudo current,  $I_C$ , flows which is capable of doing everything that a real current does. (A real current is one that flows through conductors.) This calls for the use of an AC power supply. Pseudo currents activate the property *reactance*  $\chi_C$ , of capacitors. We studied reactances extensively in two previous experiments. Reactances are frequency dependent and the placement of capacitors in an AC circuit produces a phase difference between the source voltage and its current. Again, multiple sources can never have identical frequencies and phases. Lack of coherence will most likely, ruin the experiment. All sources, therefore, must necessarily be of identical frequency and must be in phase with one another. These rather hard-to-comply-with conditions can (astonishingly) be met with, rather easily by using only one source, and splitting the output voltage into many *sources*.

Following are the loop and junction equations, suitable for circuits with capacitors and AC sources only (no resistors and no batteries). As you can see, there is only one change. We replaced  $R$  by  $\chi$ .

$$\sum_n V_{n, sources} = \sum_n \chi_n I_n \quad \dots\dots\dots(1)$$

$$\sum_n I_{in} = \sum_n I_{out} \quad \dots\dots\dots(2)$$

The reactance  $\chi_C$  of a capacitor is given by the equation:

$$\chi_C = \frac{1}{2\pi f C} = \frac{1}{\omega C} \quad \dots\dots\dots(3)$$

where, as previously stated:

$$\omega = 2\pi f \quad \dots\dots\dots(4)$$

The applicable Ohm's Law is:

$$V_C = \chi_C I_C = \frac{I_C}{\omega C} \quad \dots\dots\dots(5)$$

## Objectives of the Experiment

To study Kirchhoff's Rules for the circuit given below

- (i) *Direct Verification*: by verifying (a) all calculated values of currents and voltages, (b) all junction and loop equations
- (ii) *Indirect Verification*: by plotting a graph of total current against  $\epsilon_1$ .

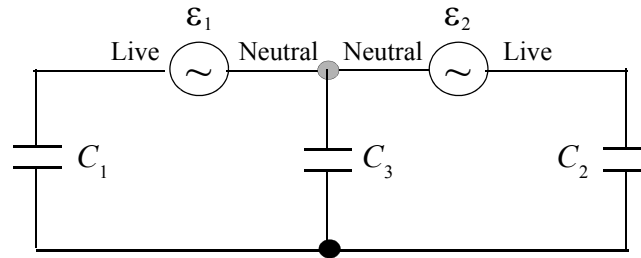


Fig (1) The Circuit

## Setting Up

### Applying Kirchhoff's Rules to the Assigned Circuit.

The circuit assigned for this experiment, is an exact parallel of the one we were assigned for resistors. This will allow us a direct comparison of the usefulness and applicability of Kirchhoff's rules for both types of components and both types of electricity. The circuit has three capacitors  $C_1$ ,  $C_2$  and  $C_3$  and two supply voltages  $\epsilon_1$  and  $\epsilon_2$ . Both voltages must be obtained from a single (stabilized) AC power supply (a function generator).

The circuit of Fig. (1) is reproduced in Fig. (2). We have replaced capacitors by their reactances and added some useful information.

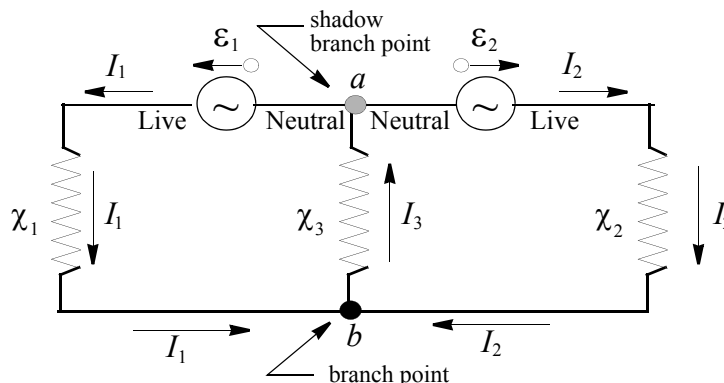


Fig (2) Circuit with Capacitors Replaced by Reactances

It should be pointed out that the *emf* directions and the directions of currents in Figs (2) show *instantaneous* voltages and currents, for some particular instant of time. This is because Kirchhoff's rules are valid only for instantaneous voltages and currents.

Analysis

The ensuing analysis is an exact parallel of that given in the experiment “Kirchhoff’s Rules - Resistors”, We shall, therefore, not repeat any of the arguments and only give the outlines of the analysis.

The junction (or the branch point) equation is:

$$I_1 + I_2 = I_3 \quad \text{.....(6)}$$

The two loop equations are:

$$\varepsilon_1 = \chi_1 I_1 + \chi_3 I_3 \quad \text{.....(7)}$$

$$\varepsilon_2 = \chi_2 I_2 + \chi_3 I_3 \quad \text{.....(8)}$$

Rearrange Eqns (7) and (8) to express  $I_1$  and  $I_2$  in terms of  $I_3$ . We get:

$$I_1 = \frac{\varepsilon_1}{\chi_1} - \frac{\chi_3 I_3}{\chi_1} \quad \text{.....(9)}$$

$$I_2 = \frac{\varepsilon_2}{\chi_2} - \frac{\chi_3 I_3}{\chi_2} \quad \text{.....(10)}$$

Plug in these values of  $I_1$  and  $I_2$  in Eqn (6) to get:

$$\left( \frac{\varepsilon_1}{\chi_1} - \frac{\chi_3 I_3}{\chi_1} \right) + \left( \frac{\varepsilon_2}{\chi_2} - \frac{\chi_3 I_3}{\chi_2} \right) = I_3$$

Solving for  $I_3$  we get:

$$I_3 = \frac{\left( \frac{\varepsilon_1}{\chi_1} + \frac{\varepsilon_2}{\chi_2} \right)}{\left( 1 + \frac{\chi_3}{\chi_1} + \frac{\chi_3}{\chi_2} \right)} \quad \text{.....(11)}$$

Unlike the experiment “Kirchhoff’s Rules - Resistors”, we cannot end our analysis here. We need to express the value of  $I_3$  in terms of capacitances and not reactances. Using Eqn (3) we find:

$$\frac{\varepsilon_1}{\chi_1} = (\omega C_1) \varepsilon_1 \quad \frac{\chi_3}{\chi_1} = \frac{\omega C_1}{\omega C_3} = \frac{C_1}{C_3} \quad \text{.....(12)}$$

$$\frac{\varepsilon_2}{\chi_2} = (\omega C_2) \varepsilon_2 \quad \frac{\chi_3}{\chi_2} = \frac{\omega C_2}{\omega C_3} = \frac{C_2}{C_3} \quad \text{.....(13)}$$

We insert these values in Eqn (11) to get:

$$I_3 = \frac{(\omega C_1) \varepsilon_1 + (\omega C_2) \varepsilon_2}{1 + \frac{C_1}{C_3} + \frac{C_2}{C_3}} = \frac{\omega(C_1 \varepsilon_1 + C_2 \varepsilon_2)}{\frac{C_1 + C_2 + C_3}{C_3}}$$

or

$$I_3 = \frac{\omega(C_1 \varepsilon_1 + C_2 \varepsilon_2)(C_3)}{C_1 + C_2 + C_3} \quad \text{.....(14)}$$

Eqn (14) gives us  $I_3$  in terms of the known parameters. Thus  $I_3$  stands determined. Next, we develop similar equations for  $I_1$  and  $I_2$ :

From Eqns (9) and (12), calculating for  $I_1$ , we get:

$$I_1 = \frac{\varepsilon_1}{\chi_1} - \frac{\chi_3 I_3}{\chi_1} = (\omega C_1) \varepsilon_1 - \left( \frac{C_1}{C_3} \right) I_3 \quad \text{.....(15)}$$

Similarly for  $I_2$ , using Eqn s (10) and (13), we get:

$$I_2 = \frac{\varepsilon_2}{\chi_2} - \frac{\chi_3 I_3}{\chi_2} = (\omega C_2) \varepsilon_2 - \left( \frac{C_2}{C_3} \right) I_3 \quad \text{.....(16)}$$

The voltages across the three capacitors may be found by using Ohm's law, (Eqn 5), Thus:

$$V_{C1} = \frac{I_1}{\omega C_1}, \quad V_{C2} = \frac{I_2}{\omega C_2}, \quad V_{C3} = \frac{I_3}{\omega C_3} \quad \text{.....(17)}$$

One can also write down the three loop equations in terms of voltages. Thus:

$$\varepsilon_1 = V_{C1} + V_{C3} \quad \text{.....(18)}$$

$$\varepsilon_2 = V_{C2} + V_{C3} \quad \text{.....(19)}$$

$$\varepsilon_1 - \varepsilon_2 = V_{C1} - V_{C2} \quad \text{.....(20)}$$

This completes our analysis in full. Equations (14) through (20) can all be verified by direct measurement of the circuit currents and voltages and percent errors can be calculated.

### (A) Direct Verification.

We shall set up the circuit of Fig (2) and

- (i) measure the 5 voltages:  $\varepsilon_1$ ,  $\varepsilon_2$ ,  $V_{C1}$ ,  $V_{C2}$ , and  $V_{C3}$ ; and
- (ii) measure the 3 currents:  $I_1$ ,  $I_2$ , and  $I_3$ .

### (B) Indirect Verification.

We shall now verify the use of Kirchoff's rules for our circuit, indirectly. As mandated, we shall plot a graph of total current against  $\varepsilon_1$ . Total current, in the circuit is  $I_3$  (see Eqn 6). So, we pick up Eqn (14) and convert it into a straight-line equation.

Rewrite Eqn (14) as:

$$I_3 = \frac{\omega(C_1 \varepsilon_1)(C_3)}{C_1 + C_2 + C_3} + \frac{\omega(C_2 \varepsilon_2)(C_3)}{C_1 + C_2 + C_3}$$

Rearranging,

$$I_3 = \left( \frac{\omega(C_2 C_3)(\varepsilon_2)}{C_1 + C_2 + C_3} \right) + \left( \frac{\omega(C_1 C_3)}{C_1 + C_2 + C_3} \right) (\varepsilon_1) \quad \text{.....(21)}$$

Equation (21) matches the equation of a straight line and is, therefore, the equation we were looking for. It has  $\varepsilon_1$  as independent variable and  $I_3$  as dependent variable (as was mandated)! The y-axis intercept:

$$\frac{\omega(C_2 C_3)(\varepsilon_2)}{(C_1 + C_2 + C_3)} \quad \text{.....(22)}$$

represents that fraction of  $I_3$  which is contributed by the voltage supply  $\varepsilon_2$ . This contribution stays constant for this part of the experiment. It can also be found by short-circuiting the voltage supply  $\varepsilon_1$ .

The slope:

$$\frac{\omega(C_1 C_3)}{(C_1 + C_2 + C_3)} \quad \text{.....(23)}$$

represents the equivalent conductance  $G_{eq}$ , of  $C_3$  (with respect to  $\epsilon_1$ ), when  $\epsilon_2$  is short-circuited. This is because with  $\epsilon_2$  shorted out, Eqn (21) will read:

$$I_3 = \left( \frac{\omega(C_1 C_3)}{C_1 + C_2 + C_3} \right) (\epsilon_1) = G_{eq} \epsilon_1 \quad \text{.....(24)}$$

A suitable circuit is shown in *Procedure*.

## Procedure

### (A) Measurement of Currents and Voltages.

- (1) We shall use capacitance substitution boxes for the three capacitors. Unless otherwise instructed, set  $C_1 = 0.08 \mu\text{F}$ ,  $C_2 = 0.03 \mu\text{F}$  and  $C_3 = 0.17 \mu\text{F}$ . Please note that the capacitor  $C_3$  is in the middle. Also make sure you did not, mistakenly, take  $C_3$  to be  $C_2$  or vice versa.
- (2) Check the values of the three capacitors using an LCR meter. If values are in error, consult the instructor. An LCR meter is on your table. Set it to 200 *nF* scale. When the value of the capacitor is being checked, the capacitor **must not** be in the circuit.
- (3) We shall use the function generator with built-in amplifier, for both sources in our circuit. Use one multimeter as voltmeter and set it to 20 *V*, *AC*.
- (4) Switch on the function generator. It should display a frequency of 1 *kHz*. Using the *amplitude control* knob of the generator, and the voltmeter, adjust the output voltage of the function generator to 5.00 *V* (as closely as possible). Disconnect the voltmeter.
- (5). Set up the circuit of Fig (3).
- (6) Adjust the positions of the sliding contacts in the two rheostats such that the output of the left hand side rheostat is 5.00 volts (as closely as possible) and that of the right hand side rheostat is 3.00 volts (also as closely as possible). Voltmeter in each case, should be connected between terminal 3 (live or positive) and terminal 2 (common or negative). Disconnect voltmeter after adjustment has been made. If the instructor suggests different voltages, follow the instructor.
- (7) Measure the five voltages:  $\epsilon_1$ ,  $\epsilon_2$ ,  $V_{C1}$ ,  $V_{C2}$ , and  $V_{C3}$ . Make sure that the leads of the voltmeter are connected properly. Fig (3) shows correct polarities of voltmeter lead wires. These polarities should be adhered to seriously.
- (8) Next step is to measure currents,  $I_1$ ,  $I_2$  and  $I_3$ , at the branch point (junction) *b*. Use the other multimeter as ammeter. Select *AC*, *A* and set to 2.00 *mA* range. (If the instructor advises differently, follow the instructor.)
- (9) Ammeter connections for the three measurements are shown in Fig (4). It should be recalled that an ammeter must be connected in series. This requires disrupting the circuit and inserting the ammeter in between. Fig (4) clearly illustrate the breaking up of the circuit and the insertion of the ammeter in between. Correct polarities have also been shown which must be followed.

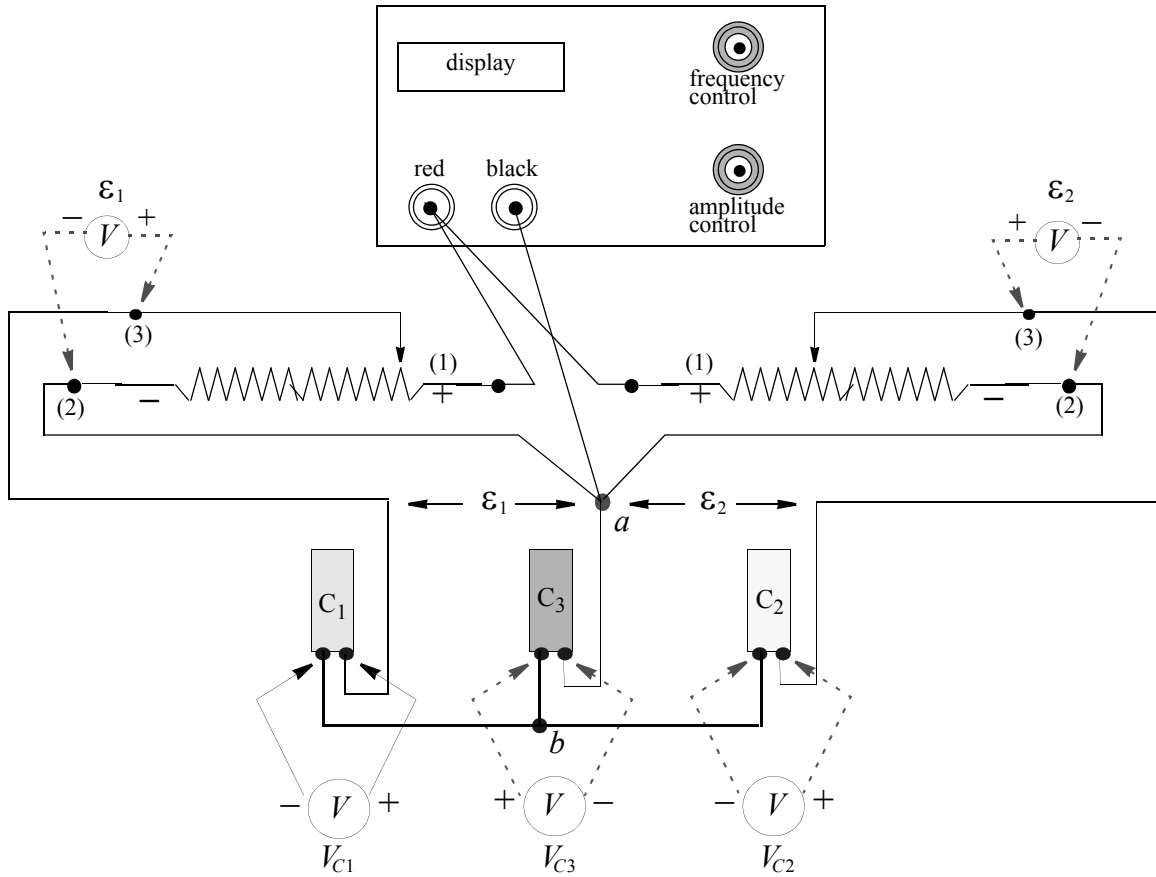


Fig (3) Assembling the Circuit and Measuring the Five Voltages

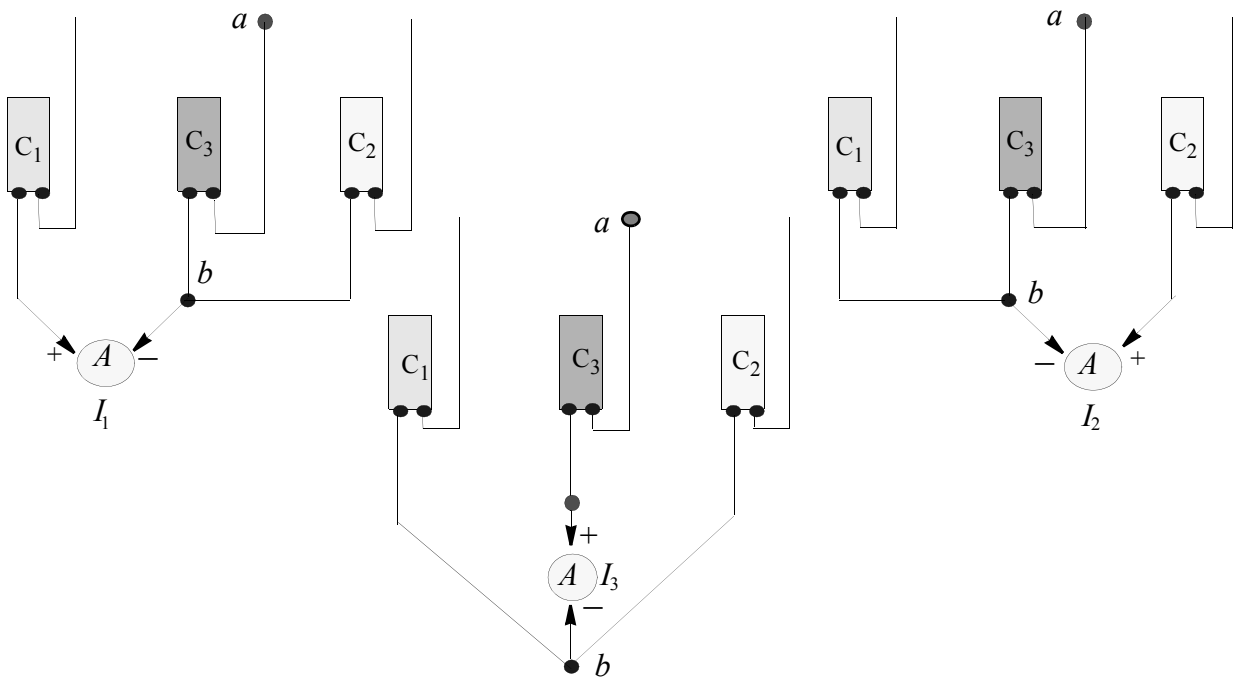


Fig (4) Measuring Currents  $I_1$ ,  $I_2$  and  $I_3$  at Junction  $b$

- (10) Next step is to measure loop voltages  $\epsilon_1$ ,  $\epsilon_2$ , and  $(\epsilon_1 - \epsilon_2)$ . The voltmeter placements for these measurements are shown in Fig (5). As always, follow the polarities very carefully.

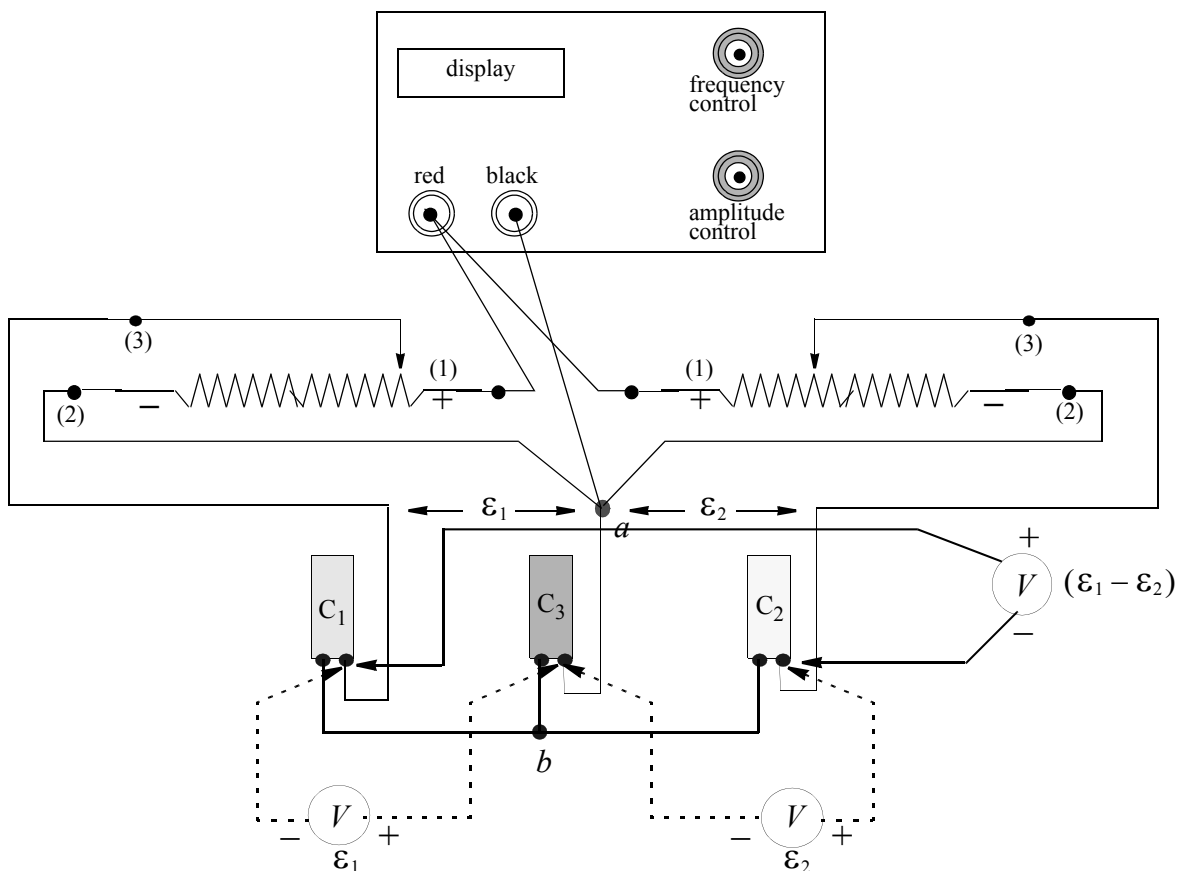


Fig (5) Measuring Loop Voltages

- (11) Next (and last) step is to measure the junction current  $I_1 + I_2$ . Follow Fig. (6) and enter the value in the data sheet. This completes the work for the first part of the experiment.

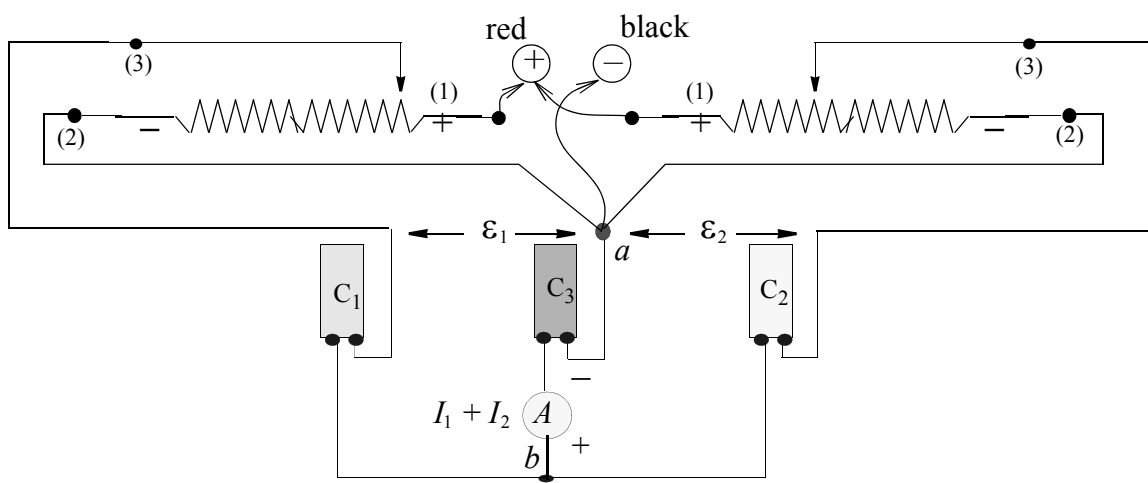


Fig (6) Measuring  $I_1 + I_2$

*(B) Plotting a graph of  $\epsilon_1$  against  $I_3$* 

- (1) No change in the circuit is required. We shall use the sliding contact of the left hand side rheostat to select values of  $\epsilon_1$ . As  $\epsilon_1$  is already at its maximum value, we shall only be reducing its value from approximately 5.00 volt to 0.00 volt, in 16 steps. This will require moving the sliding contact to the left of where ever it is now.
- (2) Use one multimeter as voltmeter and set it at 20 V, AC. Use it to measure  $\epsilon_1$  by connecting it to terminals (2) and (3) of the rheostat. Use the other multimeter as ammeter and set it to AC, A and select the 2.00 mA scale. Use it to measure the current  $I_3$ .
- (3) Switch on the function generator and for 16 different (arbitrary) positions of the sliding contact, read and record the values of  $\epsilon_1$  and  $I_3$ .
- (4) The experiment ends. Disconnect the circuit, switch off all meters and arrange every thing neatly on the table.

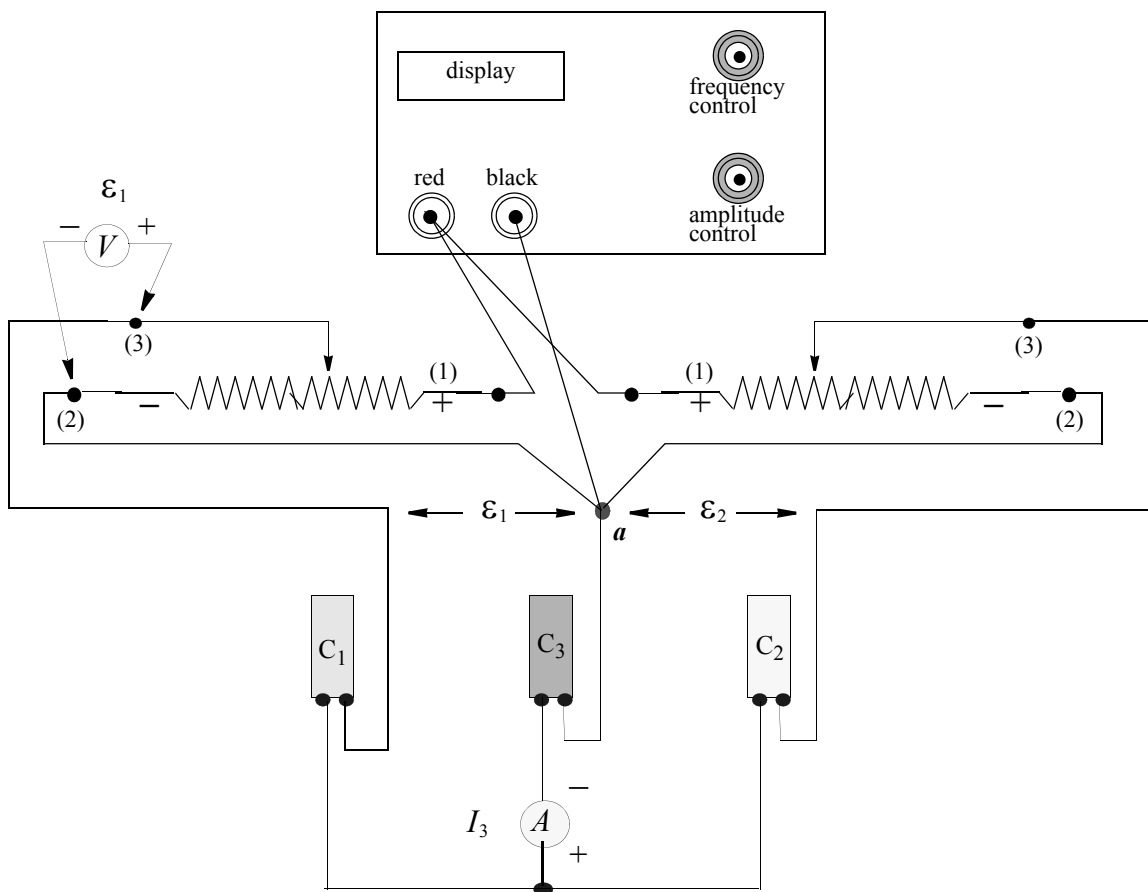


Fig (5) Circuit for Part (B) of the Experiment



## Calculations And Graphs

### (A) Verification of Eqns (14) through (20).

- (1) Plug in the measured values of  $\varepsilon_1$ ,  $\varepsilon_2$ ,  $C_1$ ,  $C_2$  and  $C_3$  in Eqn (14) and solve for  $I_3$ .
- (2) Use Eqns (15) and (16) to calculate the values of  $I_1$  and  $I_2$ .
- (3) Calculate  $V_{C1}$ ,  $V_{C2}$  and  $V_{C3}$  using Eqn (17).
- (4) Calculate  $\varepsilon_1$ ,  $\varepsilon_2$  and  $(\varepsilon_1 - \varepsilon_2)$  using Eqns (18), (19) and (20).
- (5) Enter expected values (from step (1) through step (4) above) in the proper column in the first *Results* table. Then enter the experimental values from Data Sheet in the next column. Find percent errors enter in the last column of this table.

### (B) The graph of $I_3$ against $\varepsilon_1$ .

- (1) Plot a graph of the 16 pairs of experimentally measured values of  $I_3$  (on y-axis) against the 16 selected values of  $\varepsilon_1$ , (on the x-axis), using a computer.
- (2) Calculate the intercept and slope using equations (22) and (23). These are *expected* values. Enter in proper columns in the second *Results* table.
- (3) Write the values of the slope and the y-axis intercept, from the computer print-out. These are *experimental* values. Enter in proper columns in the second *Results* table. Compare each with its expected value. Find percent errors and record in the table.

## Result Tables

A format for presenting the results of this experiment is given at the end.

## Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

## What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given



### Data & Data Tables

Name.....

Date.....

Instructor.....

Lab Section.....

Partner.....

Table #.....

#### (A) Measurement of Currents and Voltages.

AC source voltage  $\varepsilon_1$  :                      voltsAC source voltage  $\varepsilon_2$                       volts

**Table 1: Resistor Values**

Resistor	Nominal value (as set on the Capacitance Box) ( $\mu\text{F}$ )	Actual Value (as found using an Ohmmeter) ( $\mu\text{F}$ )
$C_1$		
$C_2$		
$C_3$		

**Table 2: Measured values of Parameters**

	Parameters	Measured Values
1	$V_{C1}$	
2	$V_{C2}$	
3	$V_{C3}$	
4	$I_1$	
5	$I_2$	
6	$I_3$	

**Table 2: Measured values of Parameters**

	Parameters	Measured Values	
7	$\epsilon_1$	a: step 7 b: step 9	
8	$\epsilon_2$	a: step 7 b: step 9	
9	$\epsilon_1 - \epsilon_2$	a: step 9	
10	$I_1 + I_2$	a: step 10	

(B) Plotting a graph of  $I_3$  against  $\epsilon_1$ .

**Table 3:**

Serial Number	$\epsilon_1$	$I_3$	Serial Number	$\epsilon_1$	$I_3$
1			9		
2			10		
3			11		
4			12		
5			13		
6			14		
7			15		
8			16		

Additional Data or Information (if any):

Table of Results

Name:

Date:

Table 4: (A) Direct Verification of Kirchhoff's Rules

#	Parameters	Calculated Values,		Experimental Values,		Error Percentage
		Eqn. #	magnitude (units)	magnitude (units)	Source	
1	$I_1$	(14)			data table line 4	
2	$I_2$	(15)			data table line 5	
3	$I_3$	(16)			data table line 6	
4	$V_{R1}$	(17)			data table line 1	
5	$V_{R2}$	(17)			data table line 2	
6	$V_{R3}$	(17)			data table line 3	
7	$\epsilon_1$	(18)			data table line 7	
8	$\epsilon_2$	(19)			data table line 8	
9	$\epsilon_1 - \epsilon_2$	(20)			data table line 9	
10	$I_1 + I_2$	(6)			data table line 10	

**Table 5: Indirect Verification of Kirchhoff's Rules**

Magnitude of $I_3$ when $\mathcal{E}_1$ is short- circuited (Expected)	Magnitude of $I_3$ when $\mathcal{E}_1$ is short- circuited (Experimental)	% error	Conductance $G_{eff}$ of $C_3$ when $\mathcal{E}_2$ is short-circuited (Expected)	Conductance $G_{eff}$ of $C_3$ when $\mathcal{E}_2$ is short-circuited (Experimental)	% error

Note: Conductance  $G_{eff}$  is, in fact,  $1/\chi_{eff}$ .

## Appendix

We would like to prove a statement we made while interpreting the slope of the graph for part B of the experiment. It was stated that the slope of the graph is effective conductance  $G_{eff}$  of capacitor  $C_3$  when  $\epsilon_2$  is shorted. We shall now prove this point by shorting  $\epsilon_2$  and calculating the current  $I_3$ . If we get the same value of for  $I_3$  as given by Eqn (24), the above point will be proved.

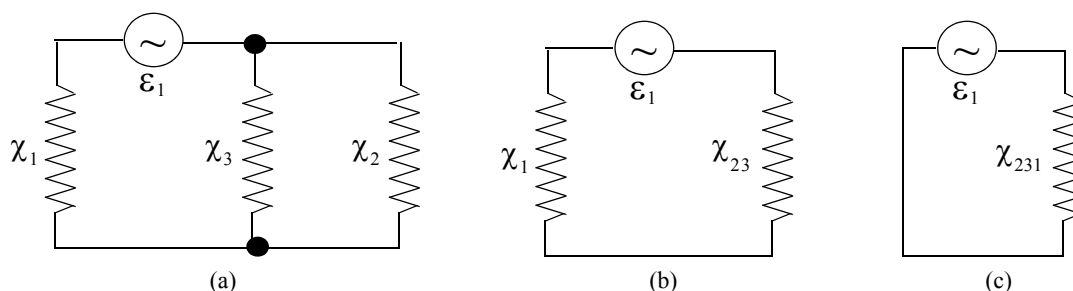


Fig (6) Crunching the Circuit of Fig (2)

(a) Combining  $\chi_2$  and  $\chi_3$  in parallel to get  $\chi_{23}$  :

$$\chi_{2,3} )_P = \frac{\chi_2 \chi_3}{\chi_2 + \chi_3} = \chi_{23}$$

This is shown in Fig (6-b). Substituting the values of  $\chi_2$  and  $\chi_3$  from Eqn (3), we get:

$$\chi_{23} = \frac{\left(\frac{1}{\omega C_2}\right)\left(\frac{1}{\omega C_3}\right)}{\left(\frac{1}{\omega C_2}\right) + \left(\frac{1}{\omega C_3}\right)} = \frac{\frac{1}{\omega^2 C_2 C_3}}{\frac{C_2 + C_3}{\omega(C_2 C_3)}} = \frac{1}{\omega^2 C_2 C_3} \times \frac{\omega(C_2 C_3)}{C_2 + C_3} = \frac{1}{\omega(C_2 + C_3)}$$

Next we combine  $\chi_{23}$  in series with  $\chi_1$  to get  $\chi_{231}$  :

$$\chi_{23,1} )_S = \chi_{23} + \chi_1 = \chi_{231}$$

This is shown in Fig (6-c). Substituting the values of  $\chi_{23}$  and  $\chi_1$  we get:

$$\chi_{231} = \frac{1}{\omega(C_2 + C_3)} + \frac{1}{\omega C_1} = \frac{C_1 + C_2 + C_3}{\omega(C_2 + C_3)(C_1)}$$

Now we shall make a table of voltages and currents through each reactance.

The table appears on the next page.

**Voltages and Currents for the Reactances of Fig (2)**

S or P	Reactances		$V_C = \chi_C I_C$	$I_C = \frac{V_C}{\chi_C}$
	$\chi_C$	values	voltages	currents
	$\chi_{231}$	$\frac{C_1 + C_2 + C_3}{\omega(C_2 + C_3)(C_1)}$	$\epsilon_1$	$\frac{\epsilon_1}{\frac{C_1 + C_2 + C_3}{\omega(C_2 + C_3)(C_1)}} =$ $\frac{\omega(C_2 + C_3)(C_1)}{C_1 + C_2 + C_3}(\epsilon_1)$
S	$\chi_1$	$\frac{1}{\omega C_1}$	$\frac{1}{\omega C_1} \times \frac{\omega(C_2 + C_3)(C_1)}{C_1 + C_2 + C_3}(\epsilon_1)$ $= V_{C1} = \left(\frac{C_2 + C_3}{C_1 + C_2 + C_3}\right)(\epsilon_1)$	$I_1 = \frac{\omega(C_2 + C_3)(C_1)}{C_1 + C_2 + C_3}(\epsilon_1)$
	$\chi_{23}$	$\frac{1}{\omega(C_2 + C_3)}$	$\frac{1}{\omega(C_2 + C_3)} \times \frac{\omega(C_2 + C_3)(C_1)}{C_1 + C_2 + C_3}(\epsilon_1)$ $= \left(\frac{C_1}{C_1 + C_2 + C_3}\right)(\epsilon_1)$	$\frac{\omega(C_2 + C_3)(C_1)}{C_1 + C_2 + C_3}(\epsilon_1)$
P	$\chi_2$	$\left(\frac{1}{\omega C_2}\right)$	$V_{C2} = \left(\frac{C_1}{C_1 + C_2 + C_3}\right)(\epsilon_1)$	$\frac{\left(\frac{C_1}{C_1 + C_2 + C_3}\right)(\epsilon_1)}{\left(\frac{1}{\omega C_2}\right)} =$ $I_2 = \left(\frac{\omega C_2 C_1}{C_1 + C_2 + C_3}\right)(\epsilon_1)$
	$\chi_3$	$\left(\frac{1}{\omega C_3}\right)$	$V_{C3} = \left(\frac{C_1}{C_1 + C_2 + C_3}\right)(\epsilon_1)$	$\frac{\left(\frac{C_1}{C_1 + C_2 + C_3}\right)(\epsilon_1)}{\left(\frac{1}{\omega C_3}\right)} =$ $I_3 = \left(\frac{\omega C_1 C_3}{C_1 + C_2 + C_3}\right)(\epsilon_1)$

The value of  $I_3$ , as found for  $\chi_3$  in the **last column of the last row** in the above table, is exactly the same as that found in Eqn (22). Hence the above mentioned point is proved. It is good to know that two different techniques (crunching & Kirchoff's rules) lead to identical results.