Experiment # 9

Circuit Analysis - 2a
Kirchhoff's Rules - Resistors

Principles

Defining Circuits & Circuit Analysis

Please refer to the discussion in either of the experiments "Circuit Analysis I"

The Need For Kirchhoff's Rules.

The crunching technique, as used in the preceding 2 experiments becomes ineffective if a circuit has multiple sources of electricity in multiple strings. This is because the equations used for crunching require that there should be a common current or a common voltage for the set of resistors or capacitors connected in series or parallel. With multiple sources in multiple strings, a set of resistors, apparently in series, may not share a common current! Similarly, components that are apparently in parallel, may not share a common voltage! In fact, failing to understand the effect of multiple sources in a circuit, is a guaranteed recipe for the "collapse" of the efforts to analyze the circuit.

Kirchhoff's Rules

Kirchhoff's rules are designed to cater for *all* types of circuits; from the very simple to the most complex ones. The rules enable us to analyze circuits with *any* number of components (passive, active or both) and *any* number of voltage sources (DC, AC or both), connected together in *any* mode. The said rules are based on (i) the law of conservation of energy and (ii) the principle of linear superposition.

According to Kirchhoff, every source in a multiple source circuit, supplies its own current and voltage to every component of that circuit. Thus if there are *n* sources in the circuit, there will be *n* different currents and *n* different voltages for *each* component! Kirchhoff discovered that the Principle of Superposition was applicable to these currents and voltages; i.e. the net current and voltage of a component could be found by superposing those *n* currents and *n* voltages. As an example of this principle, consider a circuit with six different components and three different sources. We ignore two of the three sources by short-circuiting them (replacing them by connecting wires) and solve for currents and voltages using the usual crunching rules. Next we ignore a different set of two sources and solve again. We shall have to do it three times because there are 3 sources. We shall end up with three different currents (positive or negative) and three different voltages (positive or negative) for each of the six components. *Linear superposition* then requires that we add these values scalerly to arrive at the net voltage across a component and the net current passing through it. This lengthy and laborious procedure is made simple and manageable by two rules formulated by G.R. Kirchhoff, some 150 years ago.

To apply Kirchhoff's rules to a circuit, we first identify (i) *branch points,* and (ii) *strings* in the circuit. Then we map out the *loops*. It is possible that some components or sources may simultaneously belong to two or more loops. Kirchhoff's Rules apply to loops and branch points:

(1) Kirchhoff's Loop Rule: *In a loop the sum of all voltages is zero.*

$$\sum_n V_n = 0 \qquad \text{..........(1)}$$

For the sum of all voltages to be zero, it is obvious that sum currents will be positive and some others will be negative. Kirchhoff defined source voltages as positive because they produce energy. Voltages across components were called negative because components consume (or absorb) energy. For this reason components are also known as *sinks*. Occasionally a source may be treated as a sink and a sink may become a source. This occurs due to conflicts in (i) the direction of current flow and (ii) the direction in which a loop is traversed.

Having divided voltages into *source-* and *sink-voltages,* we write:

$$\sum_n V_{n,\,sources} = \sum_n V_{n,\,\mathrm{sin}ks} \qquad \text{..........(2)}$$

DC sources of electricity are batteries. If a circuit consists of resistors and batteries only, we may apply Ohm's Law to the sink voltages and rewrite Eqn (2) as:

$$\sum_n V_{n,batteries} = \sum_n R_n\, I_n \qquad \text{..........(3)}$$

This is a *user-friendly* version of Kirchhoff's *Loop Rule*.

(2) Kirchhoff's Junction Rule: *At a junction (branch point), the sum of all currents is zero*.

$$\sum_n I_n = 0 \qquad \text{..........(4)}$$

Like voltages, we expect some currents to be positive and some others to be negative. Kirchhoff defined the currents directed towards a branch point as positive. Those that are directed outwards of the branch point are then deemed negative. Thus, at a junction, sum of incoming currents will be equal to the sum of the outgoing currents. We write:

$$\sum_n I_{in} = \sum_n I_{out} \qquad \text{..........(5)}$$

This is a *user-friendly* form of Kirchhoff's *Junction Rule.*

The purpose of analysis of a circuit as stated before, is to determine voltages across and currents passing through each component in the circuit. The use of Kirchhoff's rules, allows one to determine the currents. This requires simultaneously solving a set of junction and loop equations. Obviously, if there are *n* currents in the circuit then *n* equations will have to be solved simultaneously. Once currents are known, voltages may be calculated using Ohm's law.

Objectives of the Experiment

To study Kirchhoff's Rules for the circuit given below:

(i) Direct Verification: by verifying (a) all calculated values of currents and voltages, (b) all junction and loop equations, for the two modes in which the battery ε_2 *can be connected in the circuit.*

(ii) Indirect Verification: by plotting a graph of total current against ε_1.

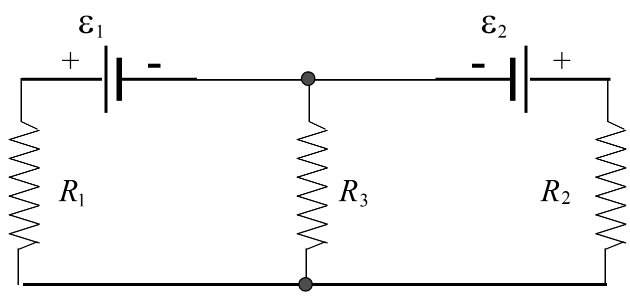

Fig (1) The Circuit

Setting Up

Applying Kirchhoff's Rules to the Assigned Circuit.

We shall apply Kirchhoff's rules to the assigned circuit. It is reproduced in Fig. (2) after adding some flesh to the skeleton: The circuit has two junctions: a and b. These two junctions (branch points) do not produce two independent junction equations (as can easily be seen). Hence, effectively, we have only one junction. The unused junction may be called *shadow* junction. A step-by-step procedure for the analysis of the circuit is given below

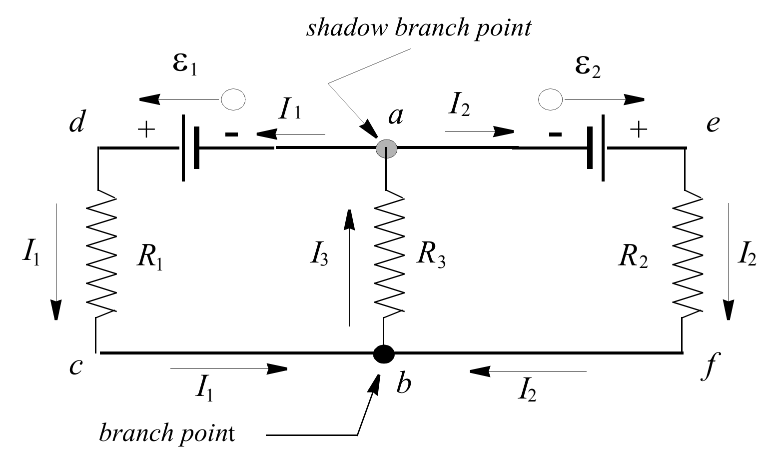

Fig (2) Circuit Ready for Analysis

(1) Select a junction. Let junction b be the *chosen* junction (branch point). Then junction a becomes the *shadow junction*.

(2) Draw *emf* arrows (directed from negative to the positive terminal) for all batteries. There are two batteries in this circuit. These are shown with their respective *emf* arrows and labeled ε_1 and ε_2.

(3) The current in a resistor must flow in the direction of the *emf* arrow. Thus *emf* arrows *dictate* the direction of current flow through resistors In our circuit, current I_1 from ε_1 flows through R_1 and current I_2 from ε_2 flows through R_2, obeying the directions of the respective *emf* arrows of the two batteries (as shown). Currents I_1 and I_2 meet at junction b, and combine to form I_3 which then flows through R_3. At the shadow branch point *a*, I_3 splits into I_1 and I_2 to return to their respective sources.

(4) Now that the directions of the three currents are all in place, write the junction equation for junction *b*. Looking at *b,* we find that currents I_1 and I_2 are travelling towards the junction. Hence these will be the *in* currents. Current I_3, on the other hand, is travelling away from the branch point. It will be the (only) *out* current. We write:

$$\sum_n I_n = I_1 + I_2 \qquad \sum_n I_{out} = I_3$$

The complete junction equation (vide Eqn 5) is:

$$I_1 + I_2 = I_3 \qquad \text{..........(6)}$$

(5) Identify the three loops. These are: *adcba*, *aefba* and *adcbfea*.

(6) To solve for the three currents, we need three equations. We have developed one, *two more are needed.* These will be loop equations. We have three loops and hence three equations are possible. We shall choose two. For a most economical solution, we select loops such that the left hand side currents I_1 and I_2 of Eqn (6) may be expressed in terms of the current I_3, of the right hand side. Thus we need a loop containing currents I_1 and I_3 and another containing currents I_2 and I_3. The respective loops are *adcba* and *aefba,* and the corresponding loop equations (vide Eqn 3) are:

$$\varepsilon_1 = I_1 R_1 + I_3 R_3 \qquad \text{..........(7)}$$

$$\varepsilon_2 = I_2 R_2 + I_3 R_3 \qquad \text{..........(8)}$$

(7) **Algebra**: Rearrange Eqns (7) and (8) to express I_1 and I_2 in terms of I_3. We get:

$$I_1 = \frac{\varepsilon_1}{R_1} - \frac{I_3 R_3}{R_1} \qquad \text{..........(9)}$$

$$I_2 = \frac{\varepsilon_2}{R_2} - \frac{I_3 R_3}{R_2} \qquad \text{..........(10)}$$

Plug in these values of I_1 and I_2 in Eqn (6) to get:

$$\left(\frac{\varepsilon_1}{R_1} - \frac{I_3 R_3}{R_1}\right) + \left(\frac{\varepsilon_2}{R_2} - \frac{I_3 R_3}{R_2}\right) = I_3$$

Collecting all the I_3 terms on the left hand side, we get:

$$\frac{I_3 R_3}{R_1} + \frac{I_3 R_3}{R_2} + I_3 = \frac{\varepsilon_1}{R_1} + \frac{\varepsilon_2}{R_2}$$

$$I_3\left(1 + \frac{R_3}{R_1} + \frac{R_3}{R_2}\right) = \frac{\varepsilon_1}{R_1} + \frac{\varepsilon_2}{R_2}$$

Solving for I_3, we get:

$$I_3 = \frac{\left(\frac{\varepsilon_1}{R_1} + \frac{\varepsilon_2}{R_2}\right)}{\left(1 + \frac{R_3}{R_1} + \frac{R_3}{R_2}\right)} = \frac{\varepsilon_1 R_2 + \varepsilon_2 R_1}{R_1 R_2 + R_2 R_3 + R_3 R_1} = \frac{\varepsilon_1 R_2 + \varepsilon_2 R_1}{P} \qquad \text{..........(11)}$$

where

$$P = R_1 R_2 + R_2 R_3 + R_3 R_1 \qquad \text{..........(12)}$$

Eqn (11) determines I_3 in terms of the known parameters. Thus I_3 stands determined. We can now go back to Eqns (9) and (10) and solve for the other two currents, I_1 and I_2.

(8) We may write down the third loop equation which was not used for the above analysis. It will, however, be needed for verification purposes. This is the (I_1 and I_2) loop:

$$\varepsilon_1 - \varepsilon_2 = I_1 R_1 - I_2 R_2 \qquad \text{..........(13)}$$

(9) To find the voltages across the three resistors, we now use Ohm's law:

$$V_{R1} = I_1 R_1 \qquad V_{R2} = I_2 R_2 \qquad \text{and} \qquad V_{R3} = I_3 R_3 \qquad \text{..........(14)}$$

(10) One can also write down the three loop equations in terms of voltages. Thus:

$$\varepsilon_1 = V_{R1} + V_{R3} \qquad \text{..........(15)}$$

$$\varepsilon_2 = V_{R2} + V_{R3} \qquad \text{..........(16)}$$

$$\varepsilon_1 - \varepsilon_2 = V_{R1} - V_{R2} \qquad \text{..........(17)}$$

Note (1): It should be noted that, in deriving Eqn (17), using loop *adcbfea*, we selected the counterclockwise direction for traversing the loop. The battery ε_2 (because of a clockwise *emf* direction) serves as an energy *user* (a sink) for this loop. What is actually happening here is that the battery ε_2 is getting *charged*; and as it is getting charged, it is using energy. It is for this reason that we call it a sink and assign a negative sign to it. Similarly, the resistor R_2 carries current in a clockwise direction. Hence instead of absorbing energy, it is *supplying* energy to this particular loop. A resistor, however, cannot *supply* energy. We, therefore, infer that less electrical energy is being used.

Note (2): It should be pointed out that the selection of the direction of traversal of a loop is entirely arbitrary. Had we selected a clockwise direction for traversing the said loop, the loop equation would have been:

$$\varepsilon_2 - \varepsilon_1 = V_{R2} - V_{R1} \qquad \text{..........(18)}$$

It is easily seen that Eqns.(17) and (18) are identical. Hence the choice of the direction for traversing a loop remains arbitrary.

Note (3): If, however, the battery ε_2 were to be connected with opposite polarity (negative terminal of ε_2 facing the positive terminal of ε_1, the loop equation would have been:

$$\varepsilon_1 + \varepsilon_2 = V_{R1} + V_{R2} \qquad \text{..........(19)}$$

Note (4): *Please note that for the situation of Note (3), Eqns (6), (7) and (8) will also have to be modified***.**

(11) Finally, a word about current directions. The directions of currents are chosen on the

hypothesis that current flows outwards from the positive terminals of a battery. But, in a circuit with more than one battery, a stronger current from one battery will force a current to flow in reverse direction in a weaker battery (and through associated resistors). Sometimes there is no way of knowing which current is stronger (or weaker) until such time that we have actually analyzed the circuit. The analysis has a built-in protection for such situations. It produces a negative sign for the current which flows in a direction opposite to the anticipated direction. No revisionary calculations are necessary We simply go back to our circuit diagram and reverse the arrows for such currents.

(A) Direct Verification.

For the assigned circuit, consisting of three resistors and two batteries, we shall calculate the currents passing through the three resistors, using Kirchhoff's rules (Eqns 11, 9 & 10). We shall then calculate the voltages across the resistors, using Ohm's law (Eqn 14), where we shall use the calculated values of currents. We shall also calculate voltages in each of the three loops using Kirchhoff's loop rule (Eqns 7, 8 &13), again using the calculated values of currents. Junction equation need not be calculated as that's where we started from. Thus there will be 9 calculated or *predicted values* for verification; or 10, if we do include the junction equation.

For greater satisfaction, we shall then redo the whole thing, *over again*, by reversing the polarity of battery ε_2 such that the two emf's now support each other. We shall obtain a second full set of *predicted* values. A complete *calculations guide* is provided in Appendix (i).

After the calculations are completed and a list of predicted values has been obtained, we shall set up the *actual* circuit of Fig (2) and measure the three currents and the three voltages for the three resistors, using an ammeter and a voltmeter. Loop voltages and the junction current will also be measured, using meters. We shall then reverse the polarity of ε_2 and obtain a second set of all 10 measured values.

A comparison of calculated and experimental values will establish the validity of the rules.

(B) Indirect Verification.

We shall use the circuit of Fig (2) with the polarities of the two batteries, as shown therein. Following are the necessary calculations for plotting a graph of total current I_3, against ε_1 (as required). From Eqn (11) we write:

$$I_3 = \frac{\varepsilon_1 R_2 + \varepsilon_2 R_1}{P} = \frac{\varepsilon_1 R_2}{P} + \frac{\varepsilon_2 R_1}{P}$$

As ε_1 is the designated *independent variable,* we separate it out, to get:

$$I_3 = \frac{\varepsilon_2 R_1}{P} + \left(\frac{R_2}{P}\right)\varepsilon_1 \qquad \text{..........(20)}$$

This equation is in the form of the equation of a straight line. If we plot a graph of I_3 against ε_1, we shall get a straight line. The y-axis intercept $(\varepsilon_2 R_1)/P$ will represent the fraction of current I_3 which is contributed by the battery ε_2. This contribution stays constant and can also be found by short-circuiting the battery ε_1. The slope R_2/P represents the effective conductance of R_3 when ε_2 is short-circuited. This is because I_3 flows through R_3, and if ε_2 were zero, Eqn (20) will read;

$$I_3 = \left(\frac{R_2}{P}\right)\varepsilon_1 = G_3\,\varepsilon_1 \qquad \text{..........(21)}$$

Procedure

(A) Measuring Currents and Voltages.

(1) We shall use resistance substitution boxes for the three resistances. Unless otherwise instructed, set $R_1 = 33\ k\Omega$, $R_2 = 56\ k\Omega$ and $R_3 = 82\ k\Omega$. Please note that R_3 is in the center. Also make sure you did not, mistakenly, take R_3 to be R_2 and vice versa.

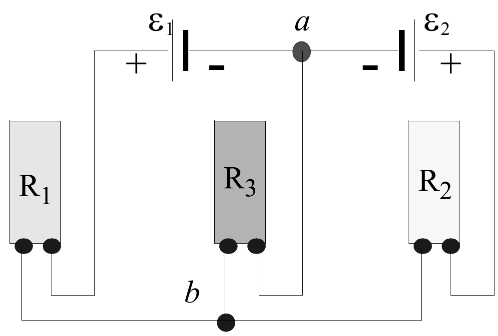

Fig (3) Assembling the Circuit and Measuring Voltages

(2) Check the values of the three resistances using a digital ohmmeter, that can read up to 2 decimal places in the 200 $k\Omega$ range. If values are in error, consult the instructor.

(3) We shall use the two outputs of the DC power supply, as ε_1 and ε_2, where ε_2 is a variable supply. Set ε_2 at 2.40 V (as closely as possible), using one of the multimeters as voltmeter, set to 20 V, DC range. The instructor may suggest a different voltage for ε_2.

(4) Set up the circuit as shown in Fig (3). Make sure that the terminals of the battery are connected as shown in the circuit diagram.

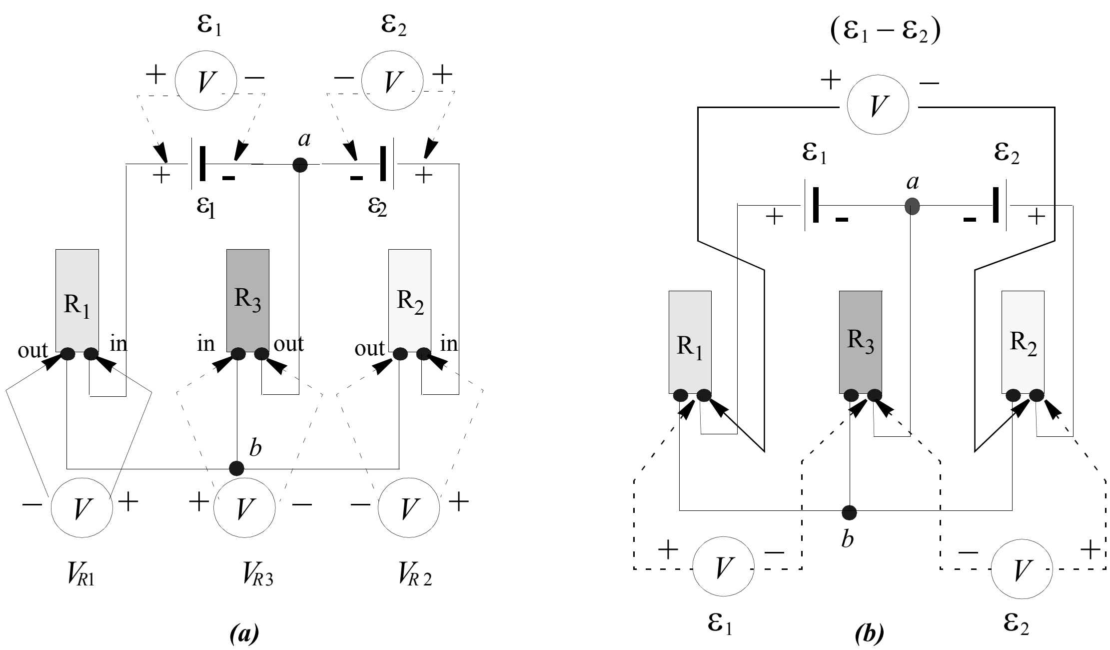

Fig (4) Measuring resistor voltages (a), and Loop voltages (b)

(5) When the instructor has checked the circuit, switch on the DC power supply and measure ε_1, ε_2, V_{R1}, V_{R2} and V_{R3}, (one at a time) using the voltmeter from Step (3). Fig (4a) shows correct polarities of the voltmeter for measuring these voltages. These polarities must be adhered to, seriously. The *plus* sign for voltmeter, signifies its *red* terminal.

(6) Now measure the loop voltages ε_1, ε_2, and $(\varepsilon_1 - \varepsilon_2)$. Voltmeter polarities are shown in Fig (4b).

(7) Next step is to measure currents, I_1, I_2 and I_3, at the branch point *b*. Use the multimeter that can read up to 3 decimal places of currents on the lowest *milliampere* range, such as the two-milliampere range. Even though all currents will be in microamperes and all multimeters offer one or more microampere ranges, we shall not use them! Such ranges produce unacceptable errors.

Ammeter connections for the three measurements are shown in Fig (5a, b, & c). It should be recalled that an ammeter must be connected in series. This requires disrupting the circuit and inserting the ammeter in between. Fig (5) illustrates the use of ammeter to measure these currents.

(8) Next step is to measure the junction current $I_1 + I_2$. Follow the diagram Fig. (5d). Enter values in data sheet, line 10.

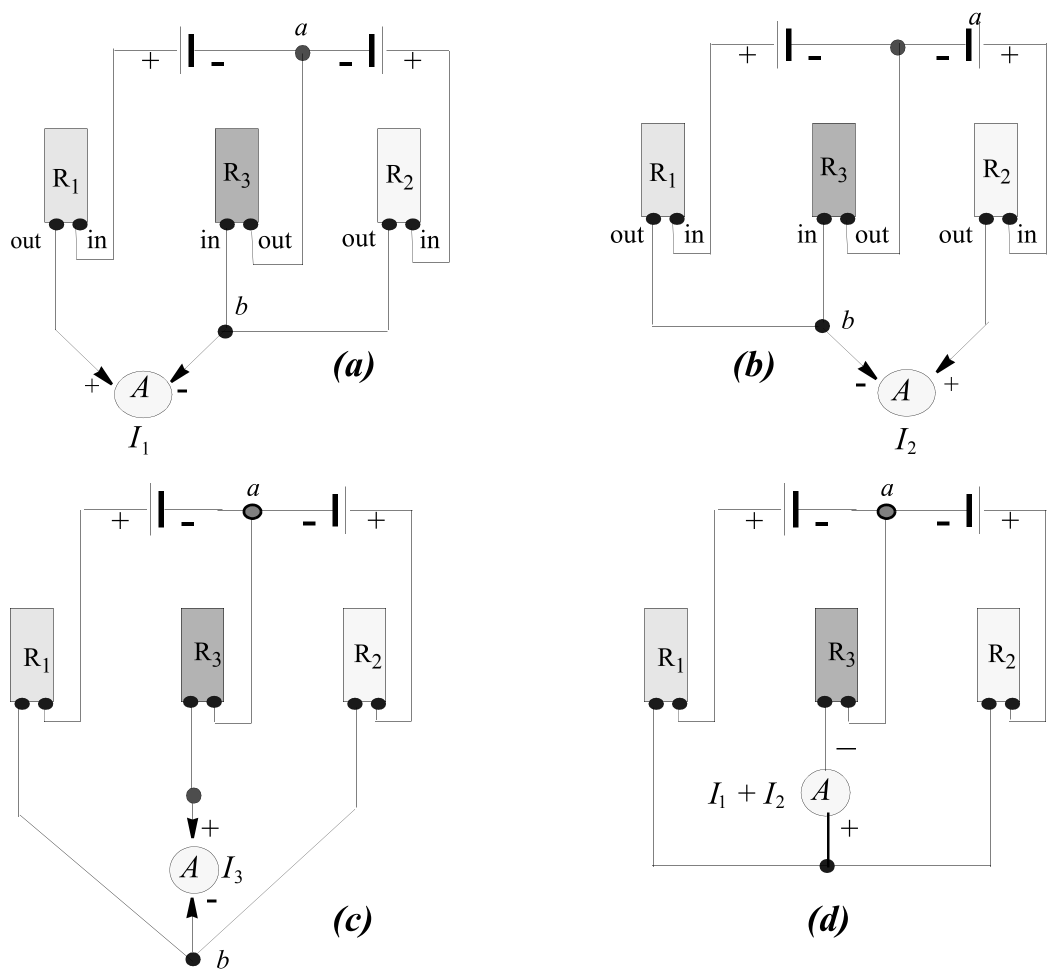

Fig (5) Measuring Currents

(9) Reverse the polarity of ε_2 by interchanging the wires in the positive and the negative terminals of ε_2. Repeat steps (5), (6), (7) and (8). In step (8) you will be measuring $I_1 + I_3$. Ammeter connection for this measurement are not shown. But you should be able to make necessary changes.

(10) This completes the first part of the experiment**.**

(11) Reset the polarity of ε_2 to its original setting.

(B) Plotting a graph of I_3 *against* ε_1

(12) Replace ε_1 by a voltage divider and set up the basic circuit of Fig (1), as shown in Fig (6). The sliding contact should be set for maximum resistance.

(13) Use one multimeter as voltmeter, to measure ε_1 by connecting it to terminals (2) and (3) of the rheostat. Use the other multimeter as ammeter and set it to a milliampere scale, as discussed in step (7) above. Use it to measure the current I_3

(14) Switch on the power supply and for 16 different (arbitrary) positions of the sliding contact, read and record the values of ε_1 and I_3**.** These positions should include both, the maximum resistance position and the minimum resistance position of the rheostat.

(15) The experiment ends. Disconnect the circuit, switch off all meters and arrange every thing neatly on the table.

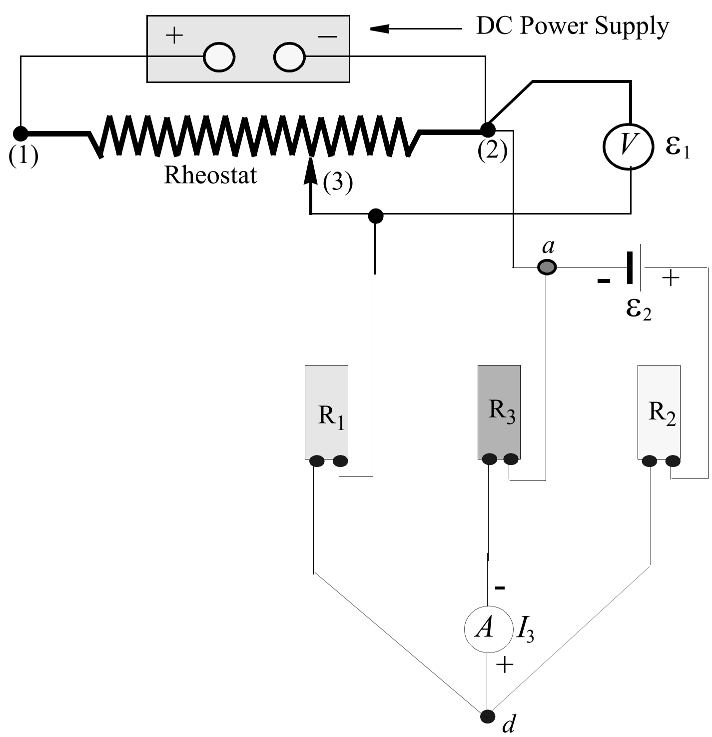

Fig (6) Circuit for ε_1*,* I_3 *Relationship*

Calculations And Graphs

(A) Verification of Currents & Voltages

(1) Plug in the measured values of ε_1, ε_2, R_1, R_2 and R_3 in Eqn (11) and solve for I_3.

(2) Go back to Eqns (9) and (10) and calculate the values of I_1 and I_2.

(3) Calculate V_{R1}, V_{R2} and V_{R3} using Eqn (14).

(4) Calculate ε_1, ε_2 and $(\varepsilon_1 - \varepsilon_2)$ using Eqns (15), (16) and (17).

(5) For the Reverse Polarity part of the experiment, one should really start from the beginning and carry out all the steps of the analysis. However, all major steps of the analysis have been worked out and given in Appendix (i), next page. You can plug-in the numbers and calculate all the expected values, for this part of the experiment.

(6) Having calculated the values of I_1, I_2 and I_3 and V_{R1}, V_{R2}, V_{R3}, ε_1, ε_2 and $(\varepsilon_1 + \varepsilon_2)$, compare them with those found experimentally and enter in the *Results Table*.

(B) The graph of I_3 against ε_1.

(7) Plot a graph of the 16 pairs of experimentally measured values of I_3 (on y-axis) against the 16 selected values of ε_1, (on the x-axis), using a computer.

(8) Calculate P, using Eqn (12) and use it to calculate R_1/P and R_2/P. Find $(R_1/P)\varepsilon_2$

(9) Record the values of the slope and the y-axis intercept, as printed out by the computer, and compare these with their expected values. The slope should match R_2/P while the intercept should match $(R_1/P)\varepsilon_2$, (as part of Eqn 20). Find percent errors. Enter in the *Results Table*. Please note that correct interpretations of R_2/P (slope) and $(R_1/P)\varepsilon_2$ (intercept) are given in the *Results Table*.

Conclusions and Discussions

Write your conclusions from the experiment and discuss them

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Result Tables

A format for presenting the results of this experiment is provided. Even though error percentages are not required in the first part, you may talk about them in *Conclusions* in a general manner.

Appendix (i)

The Circuit with emf's Supporting Each Other

The Circuit

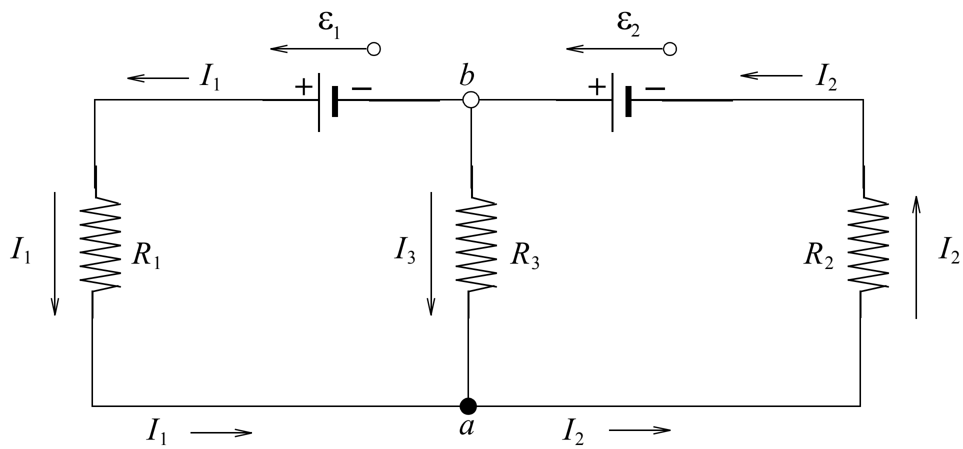

Fig (1) The Circuit with One Polarity Switched.

The Three Kirchhoff's Equations

junction equation: $I_1 + I_3 = I_2$ *(1)*

one loop equation: $\varepsilon_1 + \varepsilon_2 = R_1 I_1 + R_2 I_2$ *..........(2)*

solve for I_1: $I_1 = \dfrac{\varepsilon_1 + \varepsilon_2}{R_1} - \dfrac{R_2 I_2}{R_1}$

the other loop equation: $\varepsilon_2 = R_3 I_3 + R_2 I_2$ *..........(3)*

solve for I_3 : $I_3 = \dfrac{\varepsilon_2}{R_3} - \dfrac{R_2 I_2}{R_3}$

The Three Currents

$$I_2 = \frac{\dfrac{\varepsilon_1 + \varepsilon_2}{R_1} + \dfrac{\varepsilon_2}{R_3}}{\left(1 + \dfrac{R_2}{R_1} + \dfrac{R_2}{R_3}\right)} = \frac{(\varepsilon_1 + \varepsilon_2) R_3 + \varepsilon_2 R_1}{R_1 R_2 + R_2 R_3 + R_3 R_1} = \frac{(\varepsilon_1 + \varepsilon_2) R_3 + \varepsilon_2 R_1}{P} \quad \text{..........(4)}$$

where

$$P = R_1 R_2 + R_2 R_3 + R_3 R_1 \quad \text{..........(5)}$$

and

$$I_1 = \frac{\varepsilon_1 + \varepsilon_2}{R_1} - \frac{R_2 I_2}{R_1} \quad \text{..........(6)}$$

$$I_3 = \frac{\varepsilon_2}{R_3} - \frac{R_2 I_2}{R_3} \quad \text{..........(7)}$$

The Three Voltages

$$V_{R1} = V_1 = R_1 I_1 \qquad \text{..........(8)}$$

$$V_{R2} = V_2 = R_2 I_2 \qquad \text{..........(9)}$$

$$V_{R3} = V_3 = R_3 I_3 \qquad \text{..........(10)}$$

The Junction Equation

$$I_1 + I_3 = I_2 \qquad \text{..........(11)}$$

The Three Loop Equations

$$\varepsilon_1 + \varepsilon_2 = R_1 I_1 + R_2 I_2 = V_1 + V_2 \qquad \text{..........(12)}$$

$$\varepsilon_1 = R_1 I_1 - R_3 I_3 = V_1 - V_3 \qquad \text{..........(13)}$$

$$\varepsilon_2 = R_3 I_3 + R_2 I_2 = V_3 + V_2 \qquad \text{..........(14)}$$

Data & Data Tables

Name.................................... **Date**..............................

Instructor.............................. **Lab Section**....................

Partner........................... **Table #**..........................

(A) Measurement of Currents and Voltages.

First Supply Voltage: ε_1 : Volts (DC), as measured using a voltmeter

Second Supply Voltage: ε_2 Volts (DC), as measured using a voltmeter

Table 1: Resistor Values

Resistor	Nominal value (as set on the Resistance Box) ($k\Omega$)	Actual Value (as found using an Ohmmeter) ($k\Omega$)
R_1		
R_2		
R_3		

Table 2: Measured values of Parameters

	Parameters	Original Polarity of ε_2 Measured Values	Parameters	Reverse Polarity of ε_2 Measured Values
1	V_{R1}		V_{R1}	
2	V_{R2}		V_{R2}	
3	V_{R3}		V_{R3}	
4	I_1		I_1	
5	I_2		I_2	
6	I_3		I_3	

Table 2: Measured values of Parameters

	Parameters	Original Polarity of ε_2 Measured Values	Parameters	Reverse Polarity of ε_2 Measured Values
7	$I_1 + I_2$		$I_1 + I_3$	
8	ε_1		ε_1	
9	ε_2		ε_2	
10	$\varepsilon_1 - \varepsilon_2$		$\varepsilon_1 + \varepsilon_2$	

(B) Plotting a graph of I_3 against ε_1

Table 3: Data for Indirect Verification

Serial Number	ε_1	I_3	Serial Number	ε_1	I_3
1			9		
2			10		
3			11		
4			12		
5			13		
6			14		
7			15		
8			16		

Additional Data or Information (if any):

Pre-Graph Calculations Guide: (a) *The Case of Opposing emf's*

$R_1 = 32950\,\Omega$ $R_2 = 55790\,\Omega$ $R_3 = 81850\,\Omega$ $\varepsilon_1 = 5.03\ V$ $\varepsilon_2 = 2.39\ V$

	$R_1 =$ $R_2 =$ $R_3 =$ $\varepsilon_1 =$ $\varepsilon_2 =$		
	steps of calculation	example	your values
P	$R_1R_2 + R_2R_3 + R_3R_1 = P$	9.10165×10^{9}	
a	$\varepsilon_1 - \varepsilon_2 = a$	2.64 V	
b	$\varepsilon_1 \times R_2 = b$	280623.7	
c	$\varepsilon_2 \times R_1 = c$	78750.5	
d	$\boldsymbol{I_3 = (b + c)/P = d}$	39.4845×10^{-6} A	
e	$\varepsilon_1 / R_1 = e$	1.52655×10^{-4}	
f	$R_3/R_1 = f$	2.48407	
g	$\boldsymbol{I_1 = e - fd = g}$	54.5734×10^{-6} A	
h	$\varepsilon_2 / R_2 = h$	4.28392×10^{-5}	
i	$R_3/R_2 = i$	1.467109	
j	$\boldsymbol{I_2 = h - id = j}$	-15.08887×10^{-6} A	
k	$\boldsymbol{V_{R1} = V_1 = R_1 g = k}$	1.7982 V	
l	$\boldsymbol{V_{R2} = V_2 = R_2 j = l}$	– 0.84181 V	
m	$\boldsymbol{V_{R3} = V_3 = R_3 d = m}$	3.23181 V	
n	$\boldsymbol{g + j = d = n}$	39.4845×10^{-6} A	
o	$\boldsymbol{\varepsilon_1 = R_1 g + R_3 d = k + m = o}$	5.0300 V	
p	$\boldsymbol{\varepsilon_2 = R_2 j + R_3 d = m + l = p}$	2.5400 V	
q	$\boldsymbol{a = R_1 g - R_2 j = k - l = q}$	2.6400 V	

Pre-Graph Calculations Guide: (b) ***The Case of Supporting emf's***

$R_1 = 32950\,\Omega$ $R_2 = 55790\,\Omega$ $R_3 = 81850\,\Omega$ $\varepsilon_1 = 5.03\ V$ $\varepsilon_2 = 2.39\ V$

	$R_1 =$ $R_2 =$ $R_3 =$ $\varepsilon_1 =$ $\varepsilon_2 =$		
	Steps of Calculation	example	your values
P	$R_1R_2 + R_2R_3 + R_3R_1 = P$	9.10165×10^{9}	
a	$\varepsilon_1 + \varepsilon_2 = a$	7.42 V	
b	$a \times R_3 = b$	607327	
c	$\varepsilon_2 \times R_1 = c$	78750.5	
d	$\boldsymbol{I_2 = (b + c)/P = d}$	75.37946×10^{-6} A	
e	$a/R_1 = e$	2.251897×10^{-4}	
f	$R_2/R_1 = f$	1.6931714	
g	$\boldsymbol{I_1 = e - fd = g}$	97.5593×10^{-6} A	
h	$\varepsilon_2/R_3 = h$	2.919975×10^{-5}	
i	$R_2/R_3 = i$	0.6816127	
j	$\boldsymbol{I_3 = h - id = j}$	-22.17984×10^{-6} A	
k	$\boldsymbol{V_{R1} = V_1 = R_1 g = k}$	3.2145 V	
l	$\boldsymbol{V_{R2} = V_2 = R_2 d = l}$	4.2054 V	
m	$\boldsymbol{V_{R3} = V_3 = R_3 j = m}$	– 1.8154 V	
n	$\boldsymbol{g + j = d = n}$	75.3775×10^{-6} A	
o	$\boldsymbol{\varepsilon_1 = R_1 g - R_3 j = k - m = o}$	5.0299 V	
p	$\boldsymbol{\varepsilon_2 = R_3 j + R_2 d = m + l = p}$	2.390 V	
q	$\boldsymbol{a = R_1 g + R_2 d = k + l = q}$	7.4199 V	

Table of Results **Name:** **Date:**

Verifying Kirchhoff's Rules

(a) Direct Verification

Table 4: (A) Direct Verification of Kirchhoff's Rules

#	Para-meters	Calculated Values,		Experimental Values,		#	Para-meters	Calculated Values,		Experimental Values,	
		Eqn #	magnitude (units)	magnitude (units)	Source			Eqn #	magnitude (units)	magnitude (units)	Source
1	I_1	9			data table line 4	1	I_1	Ap 6			data table line 4
2	I_2	10			data table line 5	2	I_2	Ap 4			data table line 5
3	I_3	11			data table line 6	3	I_3	Ap 7			data table line 6
4	V_{R1}	14			data table line 1	4	V_{R1}	Ap 8			data table line 1
5	V_{R2}	14			data table line 2	5	V_{R2}	Ap 9			data table line 2
6	V_{R3}	14			data table line 3	6	V_{R3}	Ap 10			data table line 3
7	$I_1 + I_2$	6			data table line 7	7	$I_1 + I_3$	Ap 11			data table line 7
8	ε_1	15			data table line 8	8	ε_1	Ap 13			data table line 8
9	ε_2	16			data table line 9	9	ε_2	Ap 14			data table line 9
10	$\varepsilon_1 - \varepsilon_2$	17			data table line 10	10	$\varepsilon_1 + \varepsilon_2$	Ap 12			data table line 10

Note: Equation numbers appearing with 'Ap' in column 9, refer to the "Appendix (i)" equations, for the reverse polarity of the battery ε_2

(b) Indirect Verification

Table 5: Indirect Verification of Kirchhoff's Rules

Magnitude of I_3 when ε_1 is short-circuited (Calculated)	Magnitude of I_3 when ε_1 is short-circuited (Experimental)	% error	Conductance G_3 of R_3 when ε_2 is short-circuited (Calculated)	Conductance G_3 of R_3 when ε_2 is short-circuited (Experimental)	% error

Appendix (ii)

A remarkable confirmation of what has been said and done in this experiment is possible by applying the *crunching technique* to the circuit of Fig (1). We recall the following statement made earlier in the "Principles":

"Every source in a multiple source circuit, supplies its own current and voltage to every component of that circuit. Thus if there are "n" sources in the circuit, there will be "n" different currents and "n" different voltages for *each* component! Kirchhoff discovered that the Principle of Superposition was applicable to these currents and voltages; i.e. the net current and voltage of a component could be found by superposing those "n" currents and the "n" voltages."

In the circuit of Fig (1), we have two batteries and three resistors. With one battery shorted, we shall have only one battery in the network and three resistors in series-parallel combination. Crunching technique can now be applied and currents and voltages for all three resistors can be calculated. This can be repeated by short-circuiting the second battery, in place of the first, and recalculating the currents and voltages. The two sets of currents and voltages can be "superposed" (added scalerly) to get the values of currents and voltages when both batteries are present and active.

A characteristic difference between the two techniques is as follows. For Kirchhoff's rules treatment, we first calculate one current using values of given resistors and battery voltages (such as I_3, for the circuit of Fig 1, given by Eqn 11). The other two currents are then calculated using the value of this current. (See Eqns 9 & 10. They both use I_3.) As for the crunching technique, all three currents are found using the given values of the three resistors and the battery voltages. There is no dependence of any one current on other currents.

Determining Currents I_1, I_2 and I_3 With ε_2 Short-Circuited

We calculate the currents and voltages propagated in the circuit by the battery of emf ε_1.

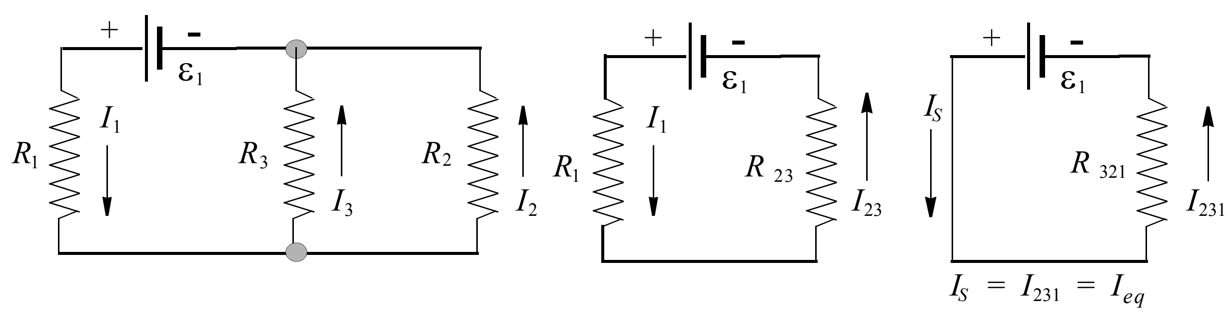

Fig (5) Circuit of Fig (1) with ε_2 Short Circuited

$$R_{2,3}]_P = \frac{R_2R_3}{R_2+R_3} = R_{23}$$

$$R_{23,1}]_S = \frac{R_2R_3}{R_2+R_3} + R_1 = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2+R_3} = \frac{P}{R_2+R_3} = R_{231} = R_{\text{eq}}$$

where $P = R_1R_2 + R_2R_3 + R_3R_1$

Table (1) Crunching The Circuit of Fig (5) Where ε_2 is Shorted

S or P		Resistors	$V_S = R\,I_S$	$I_S = \dfrac{V_S}{R}$
	R_{231}	$\dfrac{R}{R_2 + R_3}$	ε_1	$\dfrac{\varepsilon_1}{\dfrac{P}{R_2 + R_3}} = \left(\dfrac{R_2 + R_3}{P}\right)(\varepsilon_1)$ up
S	R_1	R_1	$(R_1)\left(\dfrac{R_2 + R_3}{P}\right)(\varepsilon_1) = \left(\dfrac{(R_1)(R_2 + R_3)}{P}\right)(\varepsilon_1)$	$\left(\dfrac{R_2 + R_3}{P}\right)(\varepsilon_1)$ down
	R_{23}	$\dfrac{R_2 R_3}{R_2 + R_3}$	$\dfrac{R_2 R_3}{R_2 + R_3} \times \dfrac{(R_2 + R_3)(\varepsilon_1)}{P} = \left(\dfrac{R_2 R_3}{P}\right)(\varepsilon_1)$	$\left(\dfrac{R_2 + R_3}{P}\right)(\varepsilon_1)$ up
P	R_3	R_3	$\left(\dfrac{R_2 R_3}{P}\right)(\varepsilon_1)$	$\dfrac{\left(\dfrac{R_2 R_3}{P}\right)(\varepsilon_1)}{R_3} = \left(\dfrac{R_2}{P}\right)(\varepsilon_1)$ up
	R_2	R_2	$\left(\dfrac{R_2 R_3}{P}\right)(\varepsilon_1)$	$\dfrac{\left(\dfrac{R_2 R_3}{P}\right)(\varepsilon_1)}{R_2} = \left(\dfrac{R_3}{P}\right)(\varepsilon_1)$ up

Determining Currents I_1, I_2 and I_3 With ε_1 Short-Circuited

We calculate the currents and voltages propagated in the circuit by the battery of emf ε_2.

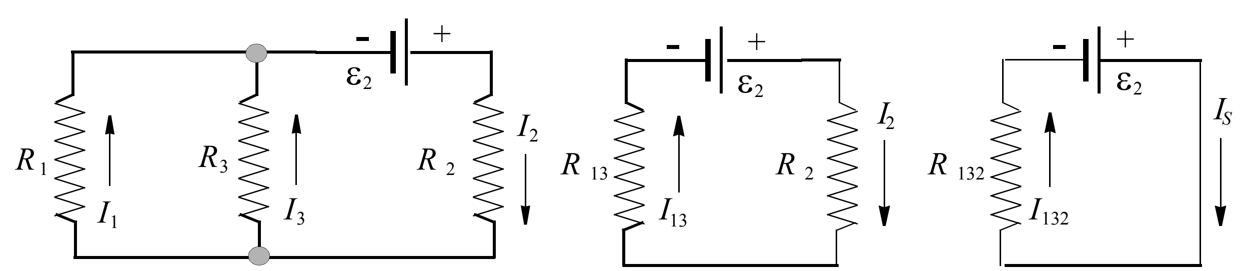

Fig (5) Circuit of Fig (1) with ε_1 Short Circuited

$$R_{1,3}]_P = \frac{R_1 R_3}{R_1 + R_3} = R_{13}$$

$$R_{13,2}]_S = \frac{R_1 R_3}{R_1 + R_3} + R_2 = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1 + R_3} = \frac{P}{R_1 + R_3} = R_{132} = R_{eq}$$

where $\quad P = R_1 R_2 + R_2 R_3 + R_3 R_1$

Table (2) Crunching The Circuit of Fig (6) Where ε_1 is Shorted

S or P		Resistors	$V_S = R I_S$	$I_S = \frac{V_S}{R}$
	R_{132}	$\frac{P}{R_1 + R_3}$	ε_2	$\frac{\varepsilon_2}{\frac{P}{R_1 + R_3}} = \left(\frac{R_1 + R_3}{P}\right)(\varepsilon_2)$ up
S	R_2	R_2	$(R_2)\left(\frac{R_1 + R_3}{P}\right)(\varepsilon_2) = \frac{(R_2)(R_1 + R_3)}{P}(\varepsilon_2)$	$\left(\frac{R_1 + R_3}{P}\right)(\varepsilon_2)$ down
	R_{13}	$\frac{R_1 R_3}{R_1 + R_3}$	$\left(\frac{R_1 R_3}{R_1 + R_3}\right)\left(\frac{R_1 + R_3}{P}\right)(\varepsilon_2) = \left(\frac{R_1 R_3}{P}\right)(\varepsilon_2)$	$\left(\frac{R_1 + R_3}{P}\right)(\varepsilon_2)$ up
P	R_3	R_3	$\left(\frac{R_1 R_3}{P}\right)(\varepsilon_2)$	$\frac{\left(\frac{R_1 R_3}{P}\right)(\varepsilon_2)}{R_3} = \left(\frac{R_1}{P}\right)(\varepsilon_2)$ up
	R_1	R_1	$\left(\frac{R_1 R_3}{P}\right)(\varepsilon_2)$	$\frac{\left(\frac{R_1 R_3}{R}\right)(\varepsilon_2)}{R_1} = \left(\frac{R_3}{P}\right)(\varepsilon_2)$ up

Superposing Currents for Each Resistor

To use the principle of superposition, we shall pick up the two contributions for each current, one from each table and add.

Table (3): The Currents Through the Three Resistors

	current	contribution from ε_1	contribution from ε_2	Net of the two contributions
R_1	I_1	$\left(\frac{R_2 + R_3}{P}\right)(\varepsilon_1)$ down	$\left(\frac{R_3}{P}\right)(\varepsilon_2)$ up	$\left(\frac{R_2 + R_3}{P}\right)(\varepsilon_1) - \left(\frac{R_3}{P}\right)(\varepsilon_2) = \frac{(R_2 + R_3)\,\varepsilon_1 - R_3\varepsilon_2}{P}$
R_2	I_2	$\left(\frac{R_3}{P}\right)(\varepsilon_1)$ up	$\left(\frac{(R_1 + R_3)}{P}\right)(\varepsilon_2)$ down	$\left(\frac{R_3}{P}\right)\varepsilon_1 - \left(\frac{R_1 + R_3}{P}\right)(\varepsilon_2) = \frac{R_3\,\varepsilon_1 - (R_1 + R_3)\varepsilon_2}{P}$
R_3	I_3	$\left(\frac{R_2}{P}\right)(\varepsilon_1)$ up	$\left(\frac{R_1}{P}\right)(\varepsilon_2)$ up	$\left(\frac{R_2}{P}\right)(\varepsilon_1) + \left(\frac{R_1}{P}\right)(\varepsilon_2) = \frac{R_2\,\varepsilon_1 + R_1\varepsilon_2}{P}$
	where $P = R_1R_2 + R_2R_3 + R_3R_1$			