Experiment # 5

Wheatstone's Bridge
Resistors

Principles

An Inherent Defect in Meters

Voltmeters and ammeters suffer from an inherent defect. They need energy to function and they draw this energy from the energy that they are given to measure. Please recall that voltage is energy per unit charge. The measured values of voltages and currents are, therefore, inaccurate. This inherent defect was a big problem in olden days when the technology wasn't developed enough to make reasonably accurate meters. The measurement errors were in excess of 10% which rendered precision work very difficult or impossible.

Consider a circuit for determining the resistance of a resistor, as shown in Fig (1). It uses a voltmeter and an ammeter and the value of the resistance is determined using Ohm's law.

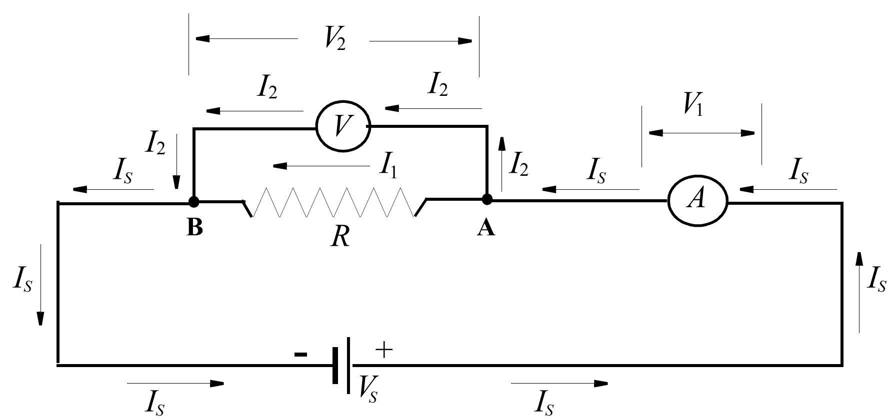


Fig (1) Resistance Measurement using Meters; Configuration #1

As we know, voltage divides in a series circuit. The series circuit, here, consists of the resistor and the ammeter. The source voltage, V_s divides into V_1 (across the ammeter) and V_2, (across both, the resistor and the voltmeter). Thus:

$$V_S = V_1 + V_2$$

Similarly, we know that current divides at a junction point, such as the point "A" in Fig (1). The source current I passes through the ammeter and then divides into I_1 and I_2 which respectively pass through the resistor and the voltmeter. The two currents combine at junction "B" to re-form the source current I:

$$I_S = I_1 + I_2$$

which then returns to the source.

It is easy to spot the error. The ammeter is monitoring the source current I and not the current I_1 that *actually* passes through the resistor! The voltmeter, however, is monitoring the correct voltage. When it comes to calculating the value of the resistance R, our instruments give us: V_2/I or $V_2/(I_1+I_2)$, instead of the correct value V_2/I_1.

An alternate circuit arrangement may be set up in the hope of correcting the error. This is shown in Fig (2).

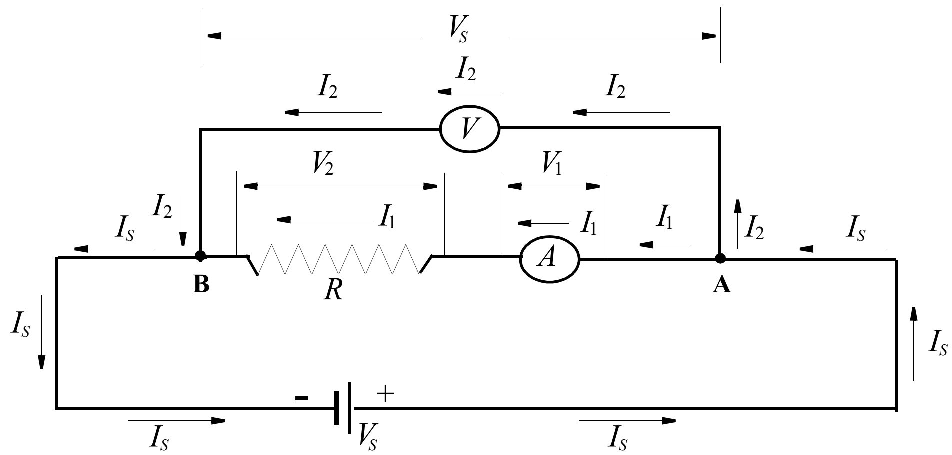


Fig (2) Resistance Measurement Using Meters; Configuration #2

The new configuration is not much help either. Here the ammeter is monitoring the actual current that passes through the resistor: I_1. The voltmeter, however, reads the source voltage V_S and not the voltage across the resistor V_2. When it comes to calculating the value of the resistance R, our instruments give us V_S/I_1 or $(V_1+V_2)/I_1$, instead of the correct value V_2/I_1.

This defect is inherent and cannot be corrected. Even the best of the present day meters will exhibit some error even though it may be infinitesimally small. Sometime such an effect does show up when, in the middle of a series of measurements, one changes the range of the meter.

The "Meter-less" Measurement: Wheatstone's Bridge

When direct methods fail, people tend to use indirect methods. One such method is that of *ratio proportions*. Mathematically one writes; $A : B = C : D$ or $A/B = C/D$. One *compares* the unknown entity with one whose value is accurately known (a *standard)*. Such a technique was developed by Mr. Wheatstone for finding values of resistors without using meters. The circuit designed by him is called a *Wheatstone's Bridge*. It is shown in Fig (3). The circuit uses four resistances, one each for A, B, C,and D: the four components of the ratio-proportion equation. One of these is a standard resistor: R_S and one of course, is the *unknown* resistor: R_X. The other two resistors are called R_1 and R_2. A galvanometer is also used to detect the *null* point during measurements and represents the sign of equality in the above equation. Wheatstone thus converted a mathematical equation into an electrical circuit, with sign of equality being the *null* point!

Rigorous analysis of the circuit requires the use of Kirchhoff's Rules. Since these rules have not been introduced yet, such an analysis is postponed to a later experiment. A simpler analysis will be given here. It is based on logical reasoning which enables us to bypass the algebra that the use of Kirchhoff's rules requires.

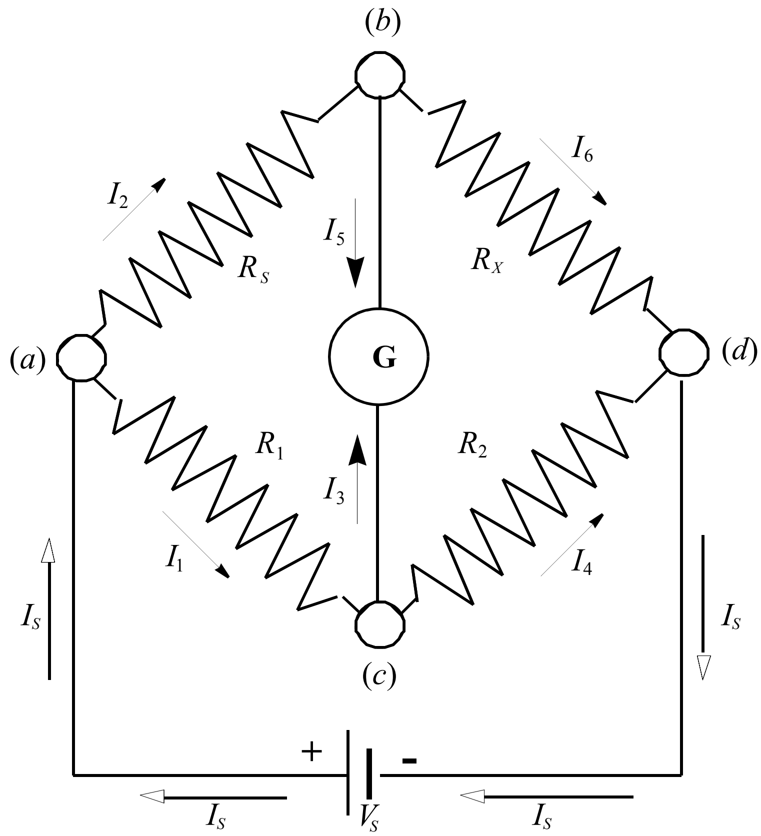


Fig (3) The Wheatstone's Bridge

Analysis

The circuit has four branch points or junctions; marked: *a*, *b*, *c*, and *d*. The source current I_S breaks up into two parts: I_1 and I_2 at the first branch point *a*. These currents respectively pass through resistors R_1 and R_S. Current I_1 subdivides into currents I_3 and I_4 at the branch point marked *c*. Of these, current I_4 passes through R_2 while the current I_3 travels toward the galvanometer. Likewise, current I_2 also subdivides into currents I_5 and I_6 at the branch point *b*. Of these, current I_6 passes through R_X while the current I_5 travels towards the galvanometer. The respective directions of flow of these currents are shown in the circuit diagram. Finally, currents I_4 and I_6 travelling towards the branch point *d*, combine at *d* (instead of subdividing) and re-form the source current I_S; which then flows back to the source.

Please note that the currents I_3 and I_5 flow into the galvanometer from opposite directions. The net current flowing into the galvanometer is, therefore, the difference of the two. The galvanometer is usually a center-zero meter. Let us suppose that $I_3 > I_5$ and that the net current $(I_3 - I_5)$ produces a deflection in the galvanometer, towards the right hand side of the center-zero. Then, if I_5 were to be greater than I_3, then the net current $(I_5 - I_3)$ will produce a deflection in the galvanometer towards the left hand side of the center-zero. And if the two currents happened to be equal, net current would be zero and there will be no deflection in the galvanometer at all. The galvanometer needle will stay at the position of rest, i.e. *zero*. This state is called *galvanometer null*; and the bridge, in this situation, is said to be *balanced*. In the *galvanometer null* position, no current flows between branch points *b* and *d* with the result that *b* and *d* cease to function as branch points. In order to obtain the galvanometer null condition, the values of currents I_3 and I_5

are varied till they become equal in magnitude. This is done by varying one or more of R_1, R_2 and R_S. For a given R_S, one may vary R_1 and R_2 to obtain a galvanometer null.

It will now be shown that for a particular set of resistance values that cause the bridge to be balanced, the ratio R_S/R_X equals the ratio R_1/R_2. This equality can be solved for the unknown resistor R_X. This is the meter-less method for determining the values of unknown resistances.

Since for a current to flow there must be a potential difference, a galvanometer null necessarily implies that there is no potential difference between the junction points *b* and *c*; or that $V_b = V_c$. This does not mean that V_b and V_c are zero. It is the *difference* that is zero! Thus if V_b happens to be + 3.24 volts then V_c must also be + 3.24 volts. Or if V_b is - 4.77 volts then V_c is also - 4.77 volts.

When V_b is equal to V_c , it implies that junctions *b* and *c* are behaving as a single junction. This causes R_S to be in parallel with R_1: their left hand side terminals meet at junction *a* and their right hand terminals meet at junction *bc*. Similarly R_X will be found to be in parallel with R_2. When two resistors are in parallel, they share a common voltage. Thus

$$V_{RS} = V_{R1}$$

and

$$V_{RX} = V_{R2}$$

Applying Ohm's law to the four voltages, we get:

$$R_S I_2 = R_1 I_1 \qquad \text{..........(1)}$$

$$R_X I_6 = R_2 I_4 \qquad \text{..........(2)}$$

Now, since *b* and *c* are not branch points any more, currents I_1 and I_2 do not get subdivided. Therefore I_1 continues to flow beyond junction point *c* as I_1 and passes through R_2 . Similarly, I_2 continues to flow beyond the junction point *b* as I_2 and passes through R_X. Thus:

$$I_6 = I_2 \text{ and } I_4 = I_1$$

We rewrite Eqn (2) as:

$$R_X I_2 = R_2 I_1 \qquad \text{..........(3)}$$

Divide Eqn (1) by Eqn (3); the currents cancel out and we are left with:

$$\frac{R_S}{R_X} = \frac{R_1}{R_2} \qquad \text{..........(4)}$$

Eqn (4) may now be solved for R_X.

One may doubt the meter-less-ness of the Bridge because of the presence of the galvanometer. It should be noted, however, that no measurements are made unless the bridge is balanced. In this state the galvanometer reads zero. No current flows between the points *b* and *c*. In this state, therefore, the galvanometer is really not being used. The method remains a meter-less method.

Objectives of the Experiment

(a) To determine the value of an unknown resistor, using Wheatstone's Bridge.

(b) To find the equivalent resistance of two resistors when connected (i) in series (ii) in parallel.

Setting Up

Of the three resistances R_S, R_1 and R_2 whose values are varied in order to balance the bridge, one may leave one of them fixed and vary only the remaining two. It is customary to select a fixed value for R_S and use variable resistors for R_1 and R_2. With two variable resistors, one would need two controls to vary their resistances. One clever way of varying two resistances with one control knob is to have a resistance with a movable tapping. The two parts of the resistance, on the two sides of the tapping, will serve as two distinct resistors. As the position of tapping is altered, one resistance will increase in value with simultaneous decrease in the value of the other resistance. Such a resistance is usually in the form of a long wire and the tapping is in the form of a *sliding contact*. Fig (4) shows such an arrangement.

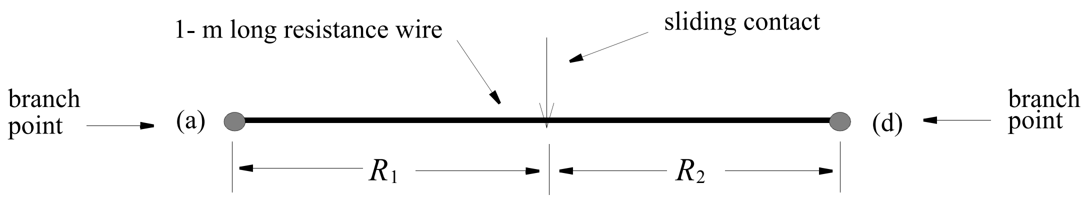


Fig (4) Two Resistances Varied Simultaneously by One Control Knob

For a given value of the standard resistor R_S, one would adjust the position of the sliding contact until the bridge is balanced and the galvanometer shows zero deflection. An apparatus that uses this technique is called a Meter Bridge. It consists of R_1 and R_2, as shown in Fig (4) and a long conductor, in the form of a metallic strip (with *zero* resistance), to serve as branch point *b*. It has three terminals attached to it, so that the remaining components of the circuit may be connected easily. Please note that the entire strip is just one contact point: the branch point *b.* The complete meter bridge is shown in Fig (5).

As stated above, the resistances R_1 and R_2 are in the form of one long wire. The wire used in the meter bridge apparatus shown above, is one meter long and is placed over an actual meter stick in such a way that its end-to-end length is exactly one meter. If this wire be of uniform dimensions, then its resistance over its entire length will be uniform. The resistance of a segment of the wire will be proportional to its length.Thus:

$$R_1 \propto l_1 \text{ and } R_2 \propto l_2$$

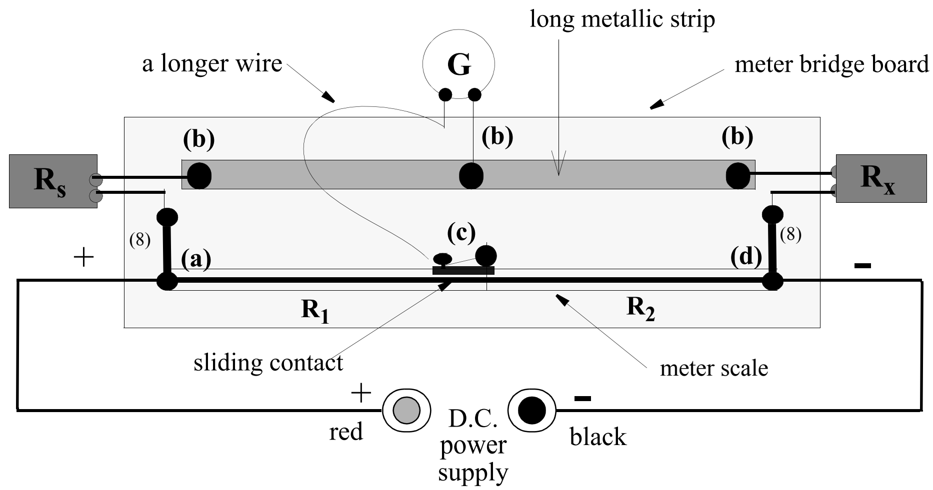
Dividing:

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} \qquad \text{..........(5)}$$

Plug-in this value of R_1/R_2 in Eqn (4) to get:

$$\frac{R_S}{R_X} = \frac{l_1}{l_2}$$

This yields a rather simple method of finding R_1 and R_2! All we need do is to measure the respective lengths of the wire for the two resistors, on the meter stick, placed under the wire. In fact one needs read only the left hand side length (the zero of the meter stick is on the left side); the other is found by subtracting the first from "100"



(*) Some models do not have this extended terminal, on the two sides of the meter stick.

Fig (5) A Complete Wheatstone's Bridge Circuit

Solving for R_S, we get:

$$R_S = R_X\left(\frac{l_1}{l_2}\right) \qquad \text{..........(6)}$$

We find that Eqn (6) is in the form of the equation of a straight line and hence it can be used to determine the value of an unknown resistance by plotting a graph.

Series and Parallel Combination of Resistors

Following are the formulae for the equivalent resistance R_{eq}, of a set of resistances connected together in series and in parallel.

Series combination of two resistors

$$R_{eq} = R_1 + R_2 \qquad \text{..........(7)}$$

Parallel combination of two resistors:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{..........(8)}$$

Adding fractions, we get;

$$R_{eq} = \frac{1}{(1/R_1 + 1/R_2)} = \frac{R_1 R_2}{R_1 + R_2} \qquad \text{..........(9)}$$

Eqns (7) and (8) can be extended to calculate the equivalent resistance of more than two resistors. Eqn (9), however, is used only when two resistors are connected together in parallel

Procedure

(A) Determining of the Value of the Given (Unknown) Resistor R_X

(1) Set a value of 1200 Ω (or the value assigned by the Instructor) on the R_X resistance box and check its exact value using the small multimeter as ohmmeter. Enter this value (with correct units) in the Data sheet. Before checking the resistance, make sure that the resistance box is not connected to the circuit.

(2) Set up the circuit given in Fig (5), using the given DC Power Supply. The supply comes with an off/on switch. This switch will be kept in the off position so that no current flows in the circuit. Use a second resistance box for R_S.

(3) Select following values of R_S, one at a time: 700 Ω, 800 Ω, 900 Ω..........1800 Ω. These will be twelve trials for this part of the experiment.

(4) For each R_S, switch on the circuit and move the sliding contact back and forth till the bridge is balanced. Switch off the circuit. Depress the sliding contact again and carefully read and record the length l_1.

Note (1): While balancing the bridge, *do not drag* the sliding contact over the meter bridge wire. This will eventually deform the wire and its resistance per unit length will become erratic, and so will your results. The best way of moving the sliding contact is to release it before moving.

Note (2): Keep the circuit switched on for the minimum time. The meter bridge wire may become hot; its resistance will then change. This will also affect your result unfavorably.

(5) When all 12 trials have been completed, switch off the circuit.

(B) Determining the Equivalent Resistance of Two Resistors, Connected in Series

(1) Use the third resistance box on your table together with the one used as R_X, as the two given resistances and connect them in series. Set the two at 1000 Ω and 800 Ω respectively (or as instructed by the Instructor). Find the actual value of this combination using the ohmmeter. While using the ohmmeter, make sure that the two resistance boxes have been disconnected from the circuit.

(2) Select the following values of R_S: 1300 Ω, 1400 Ω........2400 Ω for 12 trials.

(3) Repeat steps (4) and (5) of part (A) of this experiment.

(C) Determining the Equivalent Resistance of Two Resistors, Connected in Parallel.

(1) Connect the two resistances in parallel; without changing their resistance values. Find the actual value of this combination using the ohmmeter. While using the ohmmeter, make sure that the two resistance boxes have been disconnected from the circuit.

(2) Select the following values for R_S: 200 Ω, 250 Ω.........750Ω for 12 trials.

(3) Repeat steps (4) and (5) of part (A) of this experiment

Calculations and Graphs.

(1) For each part and for each trial find (i) l_2 as (100 - l_1), and (ii) l_1 / l_2

(2) For each part, plot R_S on y-axis and l_1 / l_2 on the x-axis; using the computer. There will be three graphs altogether. Record values of slope in each case and compare with their expected (those found using ohmmeter) values. Find percent errors.

(3) Complete the report with "Results",

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Additional (Extra Credit) Assignment

(if approved by the instructor)

(a) Show that the balance points (i.e. the galvanometer-null points), l_1, are given by:

$$l_1 = \frac{(100)(R_S)}{(R_S + R_X)}$$

(b) Other than cheating, how can the calculations be used gainfully?

Data & Data Tables

Name....................................   **Date**..............................
Instructor..............................   **Lab Section**....................
Partner..........................   **Table #**..........................

(A) Determination of the value of the given (un-known) resistance.

(a) Value of R set on the Resistance Box; Nominal value: (Ω)
(b) Actual value as found by using the ohmmeter (Ω)

Table 1: Determining the Value of an Unknown Resistance

Serial #	R_S (Ω)	l_1 (cm)	$l_2 = 100 - l_1$ (cm)	Serial #	R_S (Ω)	l_1 (cm)	$l_2 = 100 - l_1$ (cm)
1				7			
2				8			
3				9			
4				10			
5				11			
6				12			

(B) Equivalent resistance of two resistances, connected in Series.

(a) Values of R set on the two resistance boxes:
Nominal values(i) Ω (ii) Ω
(b) Actual value of the combination, as found by using an ohmmeter Ω

Table 2: Determining the Effective Resistance of Two resistors Combined in Series

Serial #	R_S (Ω)	l_1 (cm)	$l_2 = 100 - l_1$ (cm)	Serial #	R_S (Ω)	l_1 (cm)	$l_2 = 100 - l_1$ (cm)
1				7			
2				8			
3				9			
4				10			
5				11			
6				12			

(C) Equivalent resistance of two resistances, connected in Parallel.

(a) Values of R set on the two resistance boxes:
Nominal values (i) Ω (ii) Ω

(b) Actual value of the combination, as found by using an ohmmeter Ω

Table 3: Determining the Effective Resistance of Two Resistors Combined in Parallel

Serial #	R_S (Ω)	l_1 (cm)	$l_2 = 100 - l_1$ (cm)	Serial #	R_S (Ω)	l_1 (cm)	$l_2 = 100 - l_1$ (cm)
1				7			
2				8			
3				9			
4				10			
5				11			
6				12			