Experiment # 4

Batteries: Electro-motive Forces and Internal Resistances

Principles

A Battery

A battery is a source of DC electricity. Valence electrons, having been pulled out of suitable atoms, are stored here with maximum allowable electrical potential energy. These electrons are in a "combat ready" state and rush outwards in a circuit when "switched on". Having performed work in an electrical device, such as a lamp, these electrons return to the battery completely exhausted; their potential energies being exactly zero. Every single electron (millions of them, to be modest) returns. None is lost or mis-placed in the circuit. The battery re-energizes these electrons to their full energy and sends them back in the circuit. The process continues as long as the circuit remains switched on.

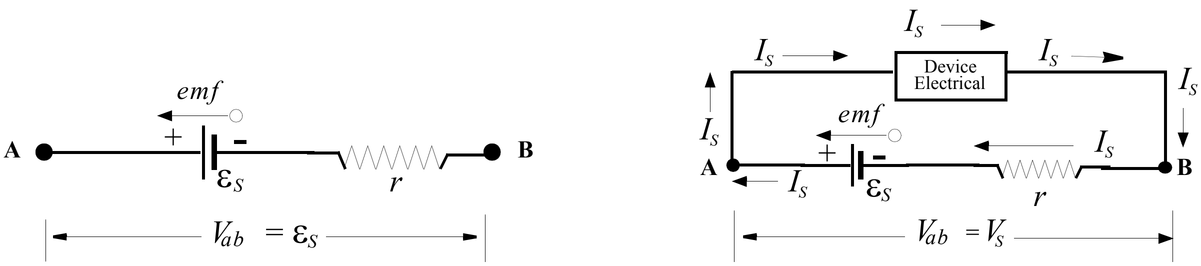
A battery converts chemical energy into electrical energy. It is based on the principle of dissolution of metals in a solvent. When a metal (such as zinc) is dipped in a solvent (such as sulfuric acid), it begins to dissolve at a rate specific of the metal and the solvent. In the process of dissolution the metal is ionized by the solvent, atom by atom; positive ions are drawn in the solvent and the valence electrons are left behind in the body of the metal. The number of these ions cannot increase indefinitely because the electrostatic repulsive force of these ions prevents further ionization. The solvent, at the same time, becomes positively charged. If we now introduce a second metal in the solvent (such as carbon) the positively charged solvent will pull out electrons from it and the metal itself will be left with positive ions. A battery has, thus, been created. This is of course, a simplified description of the principle; actual batteries are far more involved.

*The **emf** of a Battery*

The act of keeping negative electrons and positive ions apart against their natural tendencies, imparts them energy. This energy is potential energy because it depends on the positions of the charges and is called electrical potential energy. The terminal with excess electron is said to be at lower potential and that with excess of ions is deemed to be at a higher potential. A "potential difference" is created (and maintained) by the battery by converting chemical energy into electrical potential energy. Electrons are said to travel from lower potential to higher potential. This is unlike an object in gravitation which travels from higher potential to the lower potential.

A pair of metals always produces a battery of a specific potential difference, *irrespective* of their geometrical shape and size *and* the amount and strength of the chemicals used. This is primarily because of the specific rate of dissolution of a metal in a solvent and the electrostatic equilibrium condition. Batteries made of carbon and zinc will always produce a potential difference of 1.5 volt. A mercury, zinc battery produces a potential difference of 1.3 volt.

The electrical potential energy of electrons, per unit charge, inside a battery is defined as the "electro-motive force" or "*emf*" of the battery. The symbol is ε_S and the unit is *volt*: V . As we know now, electro-motive force (*emf*) is not a force. It is the potential difference of the terminals of a battery when no current is being drawn from it; i.e. when the battery is not in use. An arrow with a hollow circle at the tail end represents the direction of flow of electricity from positive terminal, outward.



(a) An open-ended battery (battery not in use)
Terminal potential difference, $V_{ab} = \varepsilon_S$

(b) Battery in use. Current flows in the circuit
Terminal potential difference, $V_{ab} = V_S$

Fig (1) A battery: (a) not in use (b) in use

Internal Resistance of a Battery

As stated above, the battery re-energizes the exhausted electrons and tries to maintain the "rated" level of electrical potential energy by converting more chemical energy into electrical energy. In the process ions travel through the solvent (electrolyte). The motion is not entirely friction free and the movement is, to some extent, sluggish. The sluggishness may be viewed as friction or resistance. Since this resistance is internal to the battery and is built into its structure, it is called the "internal resistance" of the battery. We write "r " for it and measure it in ohms. We cannot see this resistance nor can we measure it directly. The schematic diagram of the battery including the internal resistance is given in Fig (1).

The Battery Equation

When the battery is not in use, no current flows and the internal resistance remains inactive. The potential difference across its terminals V_{ab} , is its electro-motive force: ε_S ; as shown in Fig (1a). But when the battery is in use, it supplies some current I_S in the external circuit to a device. The internal resistance now becomes active. Some electrical energy is converted into thermal energy. The battery fails to maintain the rated potential difference ε_S . The loss of potential (voltage) equals in magnitude to $I_S r$ (by Ohm's Law). The voltage across the terminals of the battery is now less than ε_S ! The reduced voltage across the battery terminals is called "the terminal voltage of the battery" or simply V_{ab} and is given by the formula:

$$V_{ab} = \varepsilon_S - Ir \qquad \text{..........(1)}$$

This is the battery equation. Here I_S is given by:

$$I_S = \frac{\varepsilon_S}{R_{eq}} \qquad \text{..........(2)}$$

where R_{eq} is the total (or net or equivalent) resistance in the circuit, including the internal resistance of the battery.

It should be noted that the loss of energy inside a battery is directly related to the current I_S. The greater the current, the greater will be the term $I_S r$ and the smaller will be the value of V_{ab}. In the limiting case, when the numerical value of $I_S r$ equals ε, then V_{ab} will become zero. The value of the battery current I_S for which V_{ab} is rendered zero, is called I_{max}. Such a current will be drawn only if there is no resistance in the external circuit. This occurs when the battery is shorted i.e. short-circuited by connecting the two terminals with a connecting wire. A battery cannot supply such a current for any length of time and gets *discharged* in few short moments. If we do this to a flashlight battery, the battery will get discharged in few seconds and will become very hot indeed. If we do this to a fully charged car battery, the battery may explode.

The Dilemma

The dilemma is how to determine the *emf* of a battery. One may suggest the use of a voltmeter. But, as pointed out in an earlier experiment, voltmeters require energy to function and they take this energy from the energy they are set out to measure. When connected across a battery to measure its *emf*, a voltmeter will draw some current from the battery in order to function. As soon as a current is drawn, the potential across the terminals of the battery switches from its *emf* ε_S to V_{ab} ! The effort defeats the purpose and we conclude that a direct measurement is inherently impossible.

We cannot measure r either, as it is part of the structure of the battery and hence unreachable. Similarly, we cannot determine I_{max} because it will burn out both: the battery and the meter. The only parameters that can possibly be measured in Eqn (1) is the terminal voltage V_{ab} for different circuit currents I_S. Can this measurement lead us to a knowledge of ε_S, r and I_{max}, is a very good question.

Objectives of the Experiment

To determine the emf: ε_S, the internal resistance r and the maximum current, I_{max} for the given (i) D type, (ii) C type and (iii) AA type dry batteries.

Setting Up

When direct methods fail, one looks for indirect ones; the more popular of these being the *ratio proportion* method. Like the Wheatstone's Bridge philosophy, we may attempt comparing an unknown *emf* with a known or standard *emf*. For reasons of establishing standards, a standard cell has been developed whose *emf* remains constant (if adequately used) and serve as a "standard" for voltage measurements. The *emf* of such a cell lies in the range: 1.0182 V to 1.0187 V. Each cell comes with its own value of the *emf*. Not only that the *emf* is known accurately to several places of decimal, the variation from one cell to another occurs in the fourth place of decimal.

A Circuit

Knowing that a *standard* of *emf* is available, one can proceed and design a circuit for comparison. One rather ingenious circuit is shown in Fig (2). It uses a variable resistor with a sliding contact and a center-zero galvanometer, quite like the ones used for the Wheatstone's Bridge. The arrangement is called a *potentiometer*.

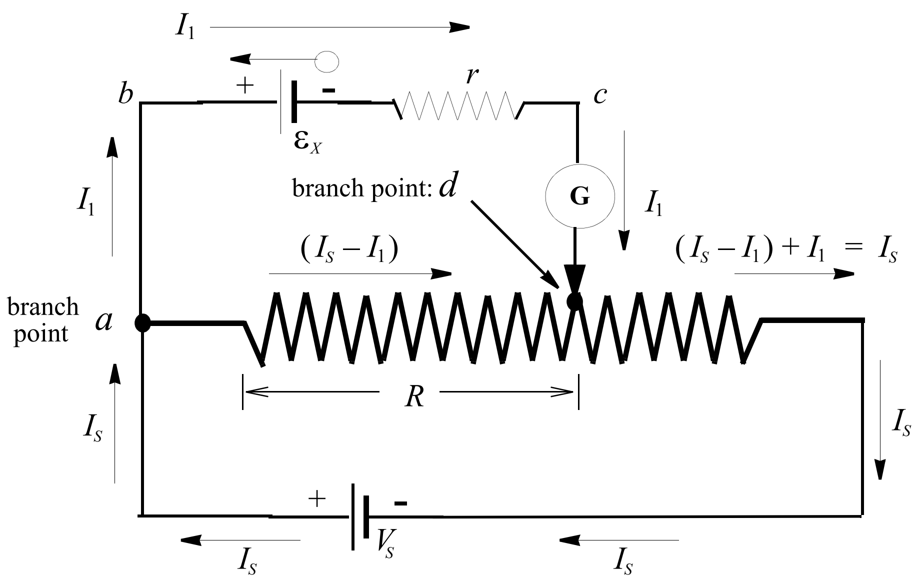


Fig (2) Determination of "emf" of a Battery Using a Potentiometer

The Analysis

The circuit shows the break up of the source current I_S into I_1 and $(I_S - I_1)$ at the branch point *a*. These currents respectively pass through the battery of unknown *emf* and the variable resistor R . The two meet again at branch point *d* to re-form the source current I which returns to the source. Consider the loop *abcda* and apply the Kirchhoff's loop rule.

$$-\varepsilon_X - I_1 r + (I_S - I_1)R = 0 \qquad \text{..........(3)}$$

Removing the parentheses we get:

$$-\varepsilon_X - I_1 r + I_S R - I_1 R = 0$$

Separating out I_1 we get:

$$I_1 (r + R) = I_S R - \varepsilon_X \qquad \text{..........(4)}$$

For the potential difference across the given battery to be its emf, I_1 must be zero. Eqn. (4) suggests that if ε_X were to be equal to $I_S r$, the right hand side of this equation will vanish, thereby forcing I_1 to become zero. Now R is a variable resistance. By varying its magnitude we should be able to change $I_S r$ to the extent that it equals ε_X. Let R_X be the adjusted value of R such that:

$$\varepsilon_X = I_S R_X \qquad \text{..........(5)}$$

and $I_1 = 0$. Such a situation will be very easy to recognize because the center-zero galvanometer will read zero. In other words, a galvanometer null will be indicative of the fact that $I_1 = 0$.

We have expressed the unknown *emf*: ε_X in terms of the resistance R_X. We may do so with the *emf* of the standard cell (also called ε_S) as well by replacing the given battery by the standard cell and readjusting the variable resistance. Let us suppose that for a value R_S of R, the galvanometer null has once again been reached. Then:

$$\varepsilon_S = I_S R_S \qquad \text{..........(6)}$$

From Eqns. (5) and (6), by dividing, we get rid of the source current I_S. One finds:

$$\varepsilon_X / \varepsilon_S = R_X / R_S \qquad \text{..........(7)}$$

This result permits us to find the emf of the given battery.

If we use a long resistance wire for R, whose resistance per unit length is constant, then the resistance of a length of this wire will be proportional to the length. Let $R_X \propto l_X$ and $R_S \propto l_S$, then we may rewrite Eqn. (7) as:

$$\varepsilon_X / \varepsilon_S = l_X / l_S \qquad \text{..........(8)}$$

"Internal Resistance" and "Maximum Current" of a Battery

The above described circuit does not cater for the other two parameters of the battery, *viz*. the internal resistance r of the battery and the maximum current I_{max} that may be drawn from it. One probably never uses a battery to the limit, because it cannot sustain such currents for any reasonable period of time, but a knowledge of I_{max} would help one to compare it with similar other batteries. For example one may be interested in finding the difference between a type *D*, a type *C* and a type *AA* battery; all of which have the same rated *emf* of 1.5 volt. Obviously because of large differences in their respective sizes one would expect them to have different current capacities in terms of I_{max}.

One way of finding all three characteristics of a battery is to plot its voltage-current relationship in a moderate range of current that will not overload the battery. If very large currents are drawn for a substantial duration of the experiment, the battery will be permanently damaged. Consider the battery equation, Eqn. (1), and re-write it in a form more easily recognizable as the equation of a straight line:

$$\begin{aligned} V_{ab} &= \varepsilon_S - r I_S \\ y &= b + mx \end{aligned} \qquad \text{..........(9)}$$

A comparison suggests that a plot of V_{ab} against I_S should yield a straight line of negative slope. The magnitude of the slope will be the internal resistance r of the battery. A y-axis intercept (obtained by extrapolation) will equal the emf ε_S of the given battery. The equation also tells us that as current drawn from the battery increases, the potential difference across its terminals, V_{ab} decreases. In the limit when maximum current I_{max}, is drawn from the battery, $V_{ab} = 0$. Setting V_{ab} equal to zero in the graph, we shall be on the x-axis. The extrapolated current reading on the x-axis will be our I_{max}. Alternatively one could set V_{ab} equal to zero in Eqn. (9), and calculate I_{max}:

$$I_{max} = \varepsilon_S / r \qquad \text{..........(10)}$$

Or may be one should do both. A suitable circuit is shown in Fig (4) which uses a current control arrangement for varying the current drawn from the given battery. Corresponding voltages are recorded and one plots voltages against currents for the above mentioned study.

Procedure

We shall use a meter bridge of the Wheatstone's Bridge experiment but call it a potentiometer. This will permit the determination of the emfs of the three given batteries using the comparison method. All the details of the bridge have been retained, except that the arm for housing the unknown resistance is not used any more. This is the first part of the experiment.

(A) Determining *emf's* of batteries using a potentiometer

(1) Set up the circuit of Fig (3) with one of the given batteries connected between terminals (a) and (b). To keep the battery voltage off, keep the crocodile clip on the plastic part of the red wire from the battery holder. To switch on the battery voltage, move the clip to the bare wire at the end of the red wire. Similarly, to keep the (table) supply voltage off, do not connect the wire from terminal (a) of the potentiometer to the red terminal of the supply voltage, at the side of the table.

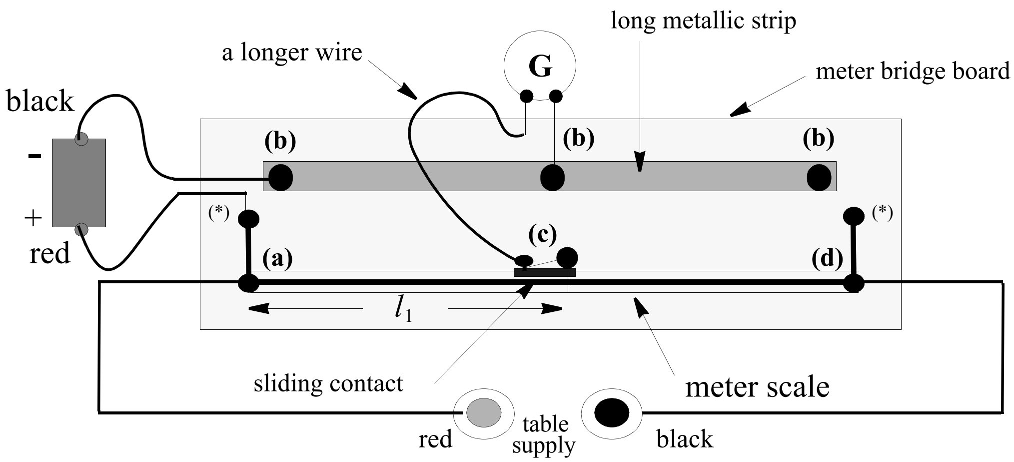


Fig (3) The Potentiometer Circuit

(2) Switch on both: the table supply and the battery voltages. Adjust the position of the sliding contact to get a galvanometer null. While scanning the length of the potentiometer wire for the null point, it is a good idea to move the sliding contact in one direction only (say from right to left) and stop when the null point is reached. As soon as the null point is reached, switch off both: the battery voltage and the (table) supply voltage. Read and record the length l_{X1} of the wire. To adjust the position of the sliding contact, do not drag it over the wire.

(3) Replace the battery by the standard cell and repeat step (2). Let the length be called l_{S1}.

(4) Repeat steps (2) and (3) for the second and the third battery. The respective lengths will be l_{X2}, l_{S2} and l_{X3}, l_{S3}

(5) Disconnect the circuit.

NOTE: The *standard cell* must be handled very carefully. For one thing, it *must be kept upright*. It should not be allowed to fall on its side. Secondly, one *must never try to measure its voltage* using a voltmeter. Such an attempt will permanently damage the cell. Finally, the positive and the negative *terminals must be connected as per the diagram* Wrong polarity will spoil the cell. *You have been warned*.

(B) Determining ε_S, r and I_{max} of batteries, treating ε_S of batteries as ε_X

(1) Set up the circuit given in Fig (4); using a rheostat, a voltmeter and an ammeter. Keep the circuit switched off by placing the crocodile clip on the plastic part of the red wire connected to the positive terminal of the battery. Please note that table supply is not used in this part of the experiment.

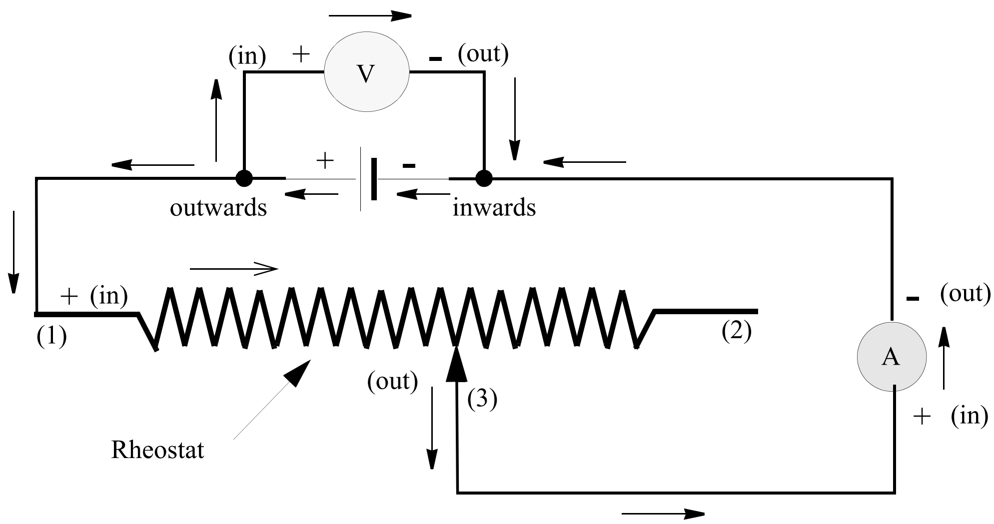


Fig (4) A circuit for Determining the Three Parameters of a Battery

(2) Use one multimeter as ammeter and set it to 200 mA DC. Make sure it reads up to 3 decimal places on this scale. If possible, use the 20 mA scale but remember you cannot change scale half way through in an experiment. The entire data should be collected on one scale only. Use the other multimeter as voltmeter and set to 2 V DC range.

Set the rheostat for maximum resistance.

Following steps should be completed rather quickly to avoid undesirable consequences of drawing large current from the battery. Proceed carefully with your trials. Remember you cannot go back and repeat a trial!

(3) Switch on the circuit by shifting the crocodile clip to the bare metal part of the red wire. Record the values of voltage V_{ab} (in volts, with three decimal places) and current I_S (in milliamperes, with three decimal places). This is your first trial.

(4) Slide the sliding contact of the rheostat by about a cm and record the fresh values of voltage and current as indicated in the respective meters. This is your second trial.

(5) Repeat step (4) for 14 more trials for a total of 16 trials.

(6) Repeat steps (3) to (5) for the other two batteries.

(7) The experiment ends. Disconnect the circuit and place everything in an orderly manner on the table. The wires should be put neatly in the box.

Calculations & Graphs

(A) Determining *emf's* of batteries using a potentiometer

(1) Find the average of l_{S1}, l_{S2} and l_{S3}. Call it l_S.

(2) Use $\varepsilon_S = 1.0184$ volt, or as instructed by the instructed.

(2) Calculate the emfs of the three batteries using Eqn (8). The lengths l_{X1}, l_{X2} and l_{X3} are each to be treated as l_X for the purposes of Eqn (8).

(B) Determining ε_S, r and I_{max} of batteries, treating ε_S of batteries as ε_X

(1) For each of the three batteries plot a graph with voltages on the y-axis and currents on the x-axis; with the help of a computer. Print out the graph with the equation of the straight line with at least 4 decimal places.

(2) Read and record the slopes and intercepts from the three computer print outs. You will notice that the slopes have negative signs. This was expected; but remember that it is the slope that is negative. The internal resistance r is a physical quantity and cannot be negative. The y-axis intercepts are the *emf's* of the batteries.

(3) To get the value of I_{max}, set $y = 0$ in the equation, printed out by the computer and solve it for x. This will be your I_{max}.

(4) Calculate I_{max} using Eqn. (10).

(5) Compare values of *emf*s as found in two different ways. Compare the values of I_{max}, as found in two different ways. Do not find errors or deviations.

(6) Complete the report with "Results",

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Data & Data Tables

Name.................................... **Date**..............................

Instructor.............................. **Lab Section**....................

Partner.......................... **Table #**..........................

(A) Determination of emfs of batteries using a potentiometer

Battery Type D	l_{X1}	(cm)	l_{S1}	(cm)
Battery Type C	l_{X2}	(cm)	l_{S2}	(cm)
Battery Type AA	l_{X3}	(cm)	l_{S3}	(cm)

(B) Determining ε_S, r *and* I_{max} *of batteries, treating* ε_S *of batteries as* ε_X

Table 1: $V-I$ Characteristics of Batteries

Serial Number	V_{ab} (V)	I_S (A)	Serial Number	V_{ab} (V)	I_S (A)
D-1			C-1		
D-2			C-2		
D-3			C-3		
D-4			C-4		
D-5			C-5		
D-6			C-6		
D-7			C-7		
D-8			C-8		
D-9			C-9		
D-10			C-10		
D-11			C-11		
D-12			C-12		
D-13			C-13		
D-14			C-14		
D-15			C-15		
D-16			C-16		

Table 1: $V - I$ Characteristics of Batteries

Serial Number	V_{ab} (V)	I_S (A)	Serial Number	V_{ab} (V)	I_S (A)
AA-1			-1		
AA-2			-2		
AA-3			-3		
AA-4			-4		
AA-5			-5		
AA-6			-6		
AA-7			-7		
AA-8			-8		
AA-9			-9		
AA-10			-10		
AA-11			-11		
AA-12			-12		
AA-13			-13		
AA_14			-14		
AA-15			-15		
AA-16			-16		

“Results”:

Table 2: Values of ε_S, r and I_{max} of batteries

Battery (type)	*emf* ε_S using the Potentio-meter method (V)	Experimental Values using the V-I Characteristics method			Maximum Current I_{max} (expected) (A)
		emf ε_S (V)	Internal Resistances r (Ω)	Maximum Current I_{max} (A)	
D					
C					
AA					