Experiment # 20

Density of Solids
Archimedes' Principle

Principles

Introducing "Density"

All objects, especially rigid ones, made from materials and substances, have specific and well-defined geometrical shapes and sizes. They also carry specific and well-defined quantities of matter or mass. It is common knowledge that objects (made from the same material or substance) that are larger in shapes and sizes have more mass, and vice versa. It is also found that objects of identical shapes and sizes but made using different materials or substance, have different masses. We use the words *heavier* and *lighter* for them. For example if we take two identical bricks (rectangular blocks), one made of lead and the other of gold, the one made of gold will be found to be far heavier than that made of lead. Let us analyze the situation scientifically using some material, such as oakwood.

(i) If we take two blocks of oakwood that are both identical to one another, except that one is longer than the other, we shall find that the one with longer length l, will have larger mass m. We conclude:

$$m \propto l \qquad ....(1)$$

(ii) If we take two blocks of the same oakwood material, that are both identical to one another, except that one is wider than the other, we shall find that the one with larger width w, will have larger mass m. We conclude:

$$m \propto w \qquad ....(2)$$

(iii) If we take two blocks of our oakwood that are both identical to one another, except that one is thicker than the other, we shall find that the one with greater thickness t, will have larger mass m. We conclude:

$$m \propto t \qquad ....(3)$$

Combining these observations we find that:

$$m \propto (l)(w)(t) \qquad ....(4)$$

Now, the product of length, width and thickness altogether, is *volume*. We write V for it.

$$(l)(w)(t) = V \qquad ....(5)$$

From (4) and (5)

$$m \propto V \qquad ....(6)$$

Introducing a constant of proportionality ρ, we get:

$$m = \rho V \qquad ....(7)$$

The constant of proportionality ρ is called *density,* density of wood, in this case. Its units are kg/m^3. If we repeat the experiment with (say) iron, we shall get a very different value of ρ.

Role of Density

(a) To Predict Weight

Basically *density* tells us how *densely* matter is packed in an object. One type of material will pack matter more densely in a given volume than the other. If we consider an empty box that is 1 m long, 1 m wide, and 1 m thick (or high) then the material lead will pack 11,300 kg of lead in this box while the material gold will pack 19,300 kg! Thus gold has packed matter more densely than lead and hence the mass of gold per unit volume is more than that of lead. Or *the density of gold is greater than the density of lead*. Thus, a knowledge of density allows us to predict the comparative masses (and hence weights) of objects that are identical in shapes and sizes. Again, given the shape and size of an object, we can predict its mass (and hence its weight) from a knowledge of its density.

(b) To Be Independent of Shapes and Sizes

A knowledge of density makes us independent of the shapes and sizes of objects. Knowing densities, we can always tell (for example) that objects made of aluminium will be lighter than those made of iron. As you can see, no specific shape or size has been mentioned. This aspect of density is particularly useful for fluids (liquids and gases) that do not have fixed shapes and sizes. They must adapt to the shape of the container. In case of liquids, the volume doesn't change from container to container but the shape does. As the volume doesn't change, we know from Eqn. (7) that the mass (and hence weight) of a liquid will not change from container to container, either.

Specific Gravity

It is found that the density of pure water at 20°C is exactly 1000 kg/m^3. Some one came up with the neat idea of expressing all densities in units of density of water (a very convenient round number). Such a ratio is called *Specific Gravity* and is written as *SG*. It has (obviously) no units. Thus specific gravity of gold will be 19.30, that of aluminum 2.60, that of copper 8.90, and so on. In order to convert density into specific gravity, we need divide by 1000, and to convert specific gravity into density, we multiply by 1000. We write:

$$SG = \frac{\rho_{object}}{\rho_{water}} = \frac{\rho_{object}}{1000} \quad \text{..........(8)}$$

Density of Mixtures

When two or more materials (or substances) of different densities are mixed together, the density of the resulting material (or substance) is given by the equation:

$$\rho_{mixture} = \frac{\text{total mass}}{\text{total volume}} = \frac{m_1 + m_2 + ...}{V_1 + V_2 + ...} \quad \text{..........(9)}$$

Archimedes Principles

Of considerable interest is the situation in which a solid is placed in a liquid, such as water. We commonly observe three possibilities. (i) the solid sinks in the fluid, such as a piece of rock in water, (ii) the solid is partially dipped in the fluid, such as ships and boats in water, and (iii) the solid floats on the surface of the fluid, such as a leaf on the surface of water. We shall call these as (i) fully submerged, (ii) partially submerged, and (iii) unsubmerged situations. The basic formula and the principle of flotation was developed by the Greek Scientist *Archimedes* and is known as *Archimedes's Principle*. Following are the necessary formulae for each situation:

(i) Fully Submerged solid (Sinking)

In this case the density of the solid is definitely greater than the density of the fluid. the solid sinks to the bottom of the container. The fluid, however, exerts a force, called the *force of buoyancy* or the *buoyant force* F_B on the solid. This force is directed opposite the weight force F_g of the object. The net or the resultant of these two forces is called the *apparent weight* of the solid in that fluid. If we write F_g' for it, we get the following equation:

$$F_{net} = F_g' = F_g - F_B \qquad \text{.........(10)}$$

where

$$F_B = \rho_{fl}\, g\, V_{fl} = \rho_{fl}\, g\, V_{solid} = \rho_{fl}\, g\, V \qquad \text{.........(11)}$$

The volume V used above is essentially the volume of the solid. But since the solid displaces an equal volume of fluid when placed in fluid, we may say that it is also the volume of the *displaced* fluid. In general, therefore, we do not need a subscript for V.

(ii) Partially submerged Solid

When a solid is only partially submerged in a fluid, the force of buoyancy F_B exerted by the fluid on the submerged part of the solid, equals the entire weight force F_g of the solid. If the height h' of the solid (where h' is smaller than the full height h of the solid) is submerged, the buoyant force on it will be:

$$F_B = \rho_{fl}\, g A h'$$

where we have replaced V by Ah.

As F_B balances F_g, we write:

$$\rho_{fl}\, g A h' = \rho_{solid}\, gAh$$

Simplifying, we get:

$$\rho_{fl}\, h' = \rho_{solid}\, h \qquad \text{.........(12)}$$

(iii) Unsubmerged Solid (Flotation), Critical flotation

If the density of the solid is less than that of the fluid, the solid will float on the surface of the fluid and no part of it will be submerged. Of interest is the case when the two densities are equal. The solid will be in equilibrium with the fluid, as if it were a part of the fluid. It will stay at rest, in any part of the fluid, at any depth. The necessary equation is:

$$\rho_{solid} = \rho_{fl} \qquad \text{.........(13)}$$

Often one deals with a mixture of solids that will be in equilibrium with a given fluid (or mixture of fluids). In such cases, we first determine the densities of mixtures (of solids or of liquids or of both) using Eqn. (9) and then apply Eqn. (13).

Objectives of the Experiment

To determine the specific gravity of the given metal.

Setting Up

In order to determine the *specific gravity* directly, we shall use water as the fluid. As the densities of metals are greater than that of water, we expect the metals to be fully submerged in water. We, therefore, recall Eqn. (10):

$$F_g' = F_g - F_B$$

Rearranging:

$$F_g - F_g' = F_B \qquad \text{..........(14)}$$

Now, $F_g = mg$; so let $F_g' = m'g$, where m' is to be interpreted as *apparent mass* of the solid in the fluid. Remember that the *apparent mass* of a solid is always less than its actual mass. Inserting the values we get:

$$mg - m'g = \rho_{fl}\, g\, V_{solid}$$

where we have taken the value of F_B from Eqn. (11). Cancelling out g from the two sides we get

$$m - m' = \rho_{fl}\, V_{solid}$$

Taking reciprocal on both sides, we get:

$$\frac{1}{m - m'} = \frac{1}{\rho_{fl}\, V_{solid}}$$

Multiply both sides by m, to get:

$$\frac{m}{m - m'} = \frac{m}{\rho_{fl}\, V_{solid}}$$

We shall replace m on the right hand side of the equation, by $\rho_{solid}\, V_{solid}$, as per Eqn. (7):

$$\frac{m}{m - m'} = \frac{\rho_{solid}\, V_{solid}}{\rho_{fl}\, V_{solid}}$$

Cancelling out V_{solid} from the top and bottom, on the right hand side of the equation:

$$\frac{m}{m - m'} = \frac{\rho_{solid}}{\rho_{fl}}$$

If the fluid used is water, then ρ_{solid} / ρ_{fl} will be SG_{solid}. We get:

$$\frac{m}{m - m'} = SG_{solid} \qquad \text{..........(15)}$$

Eqn. (15) is suitable for solving problems. For the purposes of the experiment, we need to reduce it to the form of the equation of a straight line:

$$y = b + mx$$

To this effect, we first we cross multiply the two sides in Eqn. (15):

$$m\, SG_{solid} - m' SG_{solid} = m$$

Rearranging and taking m outside the parentheses as a common term, we get

$$m' SG_{solid} = m(SG_{solid} - 1)$$

$$m' = \left(\frac{SG_{solid} - 1}{SG_{solid}}\right) m = \left(1 - \frac{1}{SG_{solid}}\right) m \qquad \text{..........(16)}$$

Eqn. (16) tells us that we should choose different values of mass m of the given metal, and for each, find its apparent mass m' in water. A graph of m' on y-axis against m (on x-axis) will yield a straight line of slope:

$$slope = 1 - \frac{1}{SG_{solid}} \qquad \text{..........(17)}$$

Eqn (17) can easily be solved for the specific gravity of the given metal.

Apparatus Required

1. A set of 4 masses such as $5g$, $10g$, $20g$ and $40g$.
2. A mass hanger of the same metal as above, or one of very small mass, (one made of plastic may be).
3. A digital balance capable of reading up to two decimal places of 1 gram,
4. A 1000 ml beaker, to be filled nearly to the top, with (preferably) distilled water,
5. A frictionless double pulley system,
6. A set of standard masses including a *one* gram mass, a *two* gram mass, and some small paper clips, some staples from a stapler, and a mass hanger,
7. Accessories such as stand, clamp, cord, etc.

Procedure

1. Set-up the apparatus as shown.

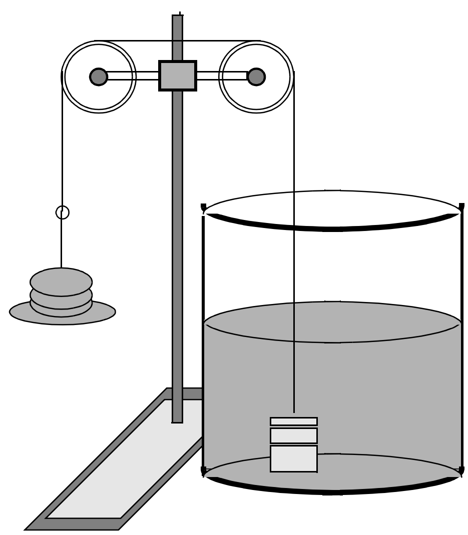

Fig (1) Determining the Specific Gravity of the Given Solid

2. Find the exact mass of each of the four given masses (up to 2 decimal places) and also an exact mass of the mass holder. Record these in Table (1) of Data Sheet. We shall be using various combinations of these masses for our experiment to build up larger masses. But once they have been in water, we cannot weigh them again to find their total masses.

3. Spread the *given* masses on a paper towel and write their determined masses underneath each. We shall pick them up for use, then put them back in the same place, for subsequent use.
4. Place the first (smallest) given mass on the mass hanger and find its total mass using the digital balance. and record it in your data sheet in the column for m. The mass will be in grams.
5. Attach the above mass to one side of the cord and let it sink in water in the beaker. It should sit on the bottom of the beaker. The cord itself will pass over the two pulleys (as shown).
6. Attach the standard mass holder to the other end of the cord and place enough standard masses upon it such that the mass in the beaker is *just about* lifted from the bottom of the beaker. It should not fall back and it should not rise upward either.
7. Disconnect the *standard mass and mass holder* assembly from the cord and find its mass using the digital balance. This is m'. Enter in the proper column in the data sheet.
8. The smallest available standard mass is a *one gram* mass. If you find that adding the *one gram* mass to the standard mass causes the given mass to go up, while *not* adding it, causes the given mass to go down, then it means that you need fractional standard masses. In the absence of such masses, we shall use lightweight paper clips and even staples from a stapler. Exact mass (found by using the digital balance) that balances the sunken mass, should be recorded in the data sheet.
9. Place the second *given mass* on the mass hanger. Repeat steps 5, 6, 7 & 8.
10. Continue with rest of the masses and their combination, as described in the data table.
11. When all trials have been completed, take out all four *given* masses from water and wipe them with paper towel. Place them, and all other apparatus neatly on the table. Take care that the distilled water (if used) in the beaker is not thrown out. The next lab group may use it.

Calculations & Graph

1. From your data sheet, plot m' values on y-axis and m values on x-axis. Draw a best fit straight line using a computer. Have the computer print the equation of the straight line to 5 decimal digits. The computer should also print the r^2 value.
2. Plug in the value of slope in Eqn. (17) and solve for SG_{solid}.
3. Compare with the actual value of SG_{solid} (if given) and find percent error. If *actual value* is not provided by the instructor, just record your findings. You may, in the *Discussion*, give your reasons for your findings to be trustworthy.
4. Complete the report with ***Results***.

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Data & Data Tables

Name................................... **Date**.............................

Instructor............................. **Lab Section**...................

Partner.......................... **Table #**..........................

Density of the Given Solid.................................kg/m³ (if given)

Table 1. Actual Values of Masses in the Set of *Given* Masses

given mass	**stated mass**	**actual mass**	given mass	**stated mass**	**actual mass**
smallest #1			and the next larger: #3		
next larger #2			largest #4		

Table 2. Values of m and m' from Experiment

mass selection	m	m'	mass selection	m	m'
mass # 1			mass # 2 + 4		
mass # 2			mass # 3 + 4		
mass # 3			mass # 1 + 2 + 3		
mass # 4			mass # 1 + 2 + 4		
mass # 1 + 2			mass # 1 + 3 + 4		
mass # 1 + 3			mass # 2 + 3 + 4		
mass # 1 + 4			mass # 1 + 2 + 3 + 4		
mass # 2 + 3					

Another set of data tables, just in case.............

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mass # 3			mass # 1 + 2 + 3		
mass # 4			mass # 1 + 2 + 4		
mass # 1 + 2			mass # 1 + 3 + 4		
mass # 1 + 3			mass # 2 + 3 + 4		
mass # 1 + 4			mass # 1 + 2 + 3 + 4		
mass # 2 + 3					