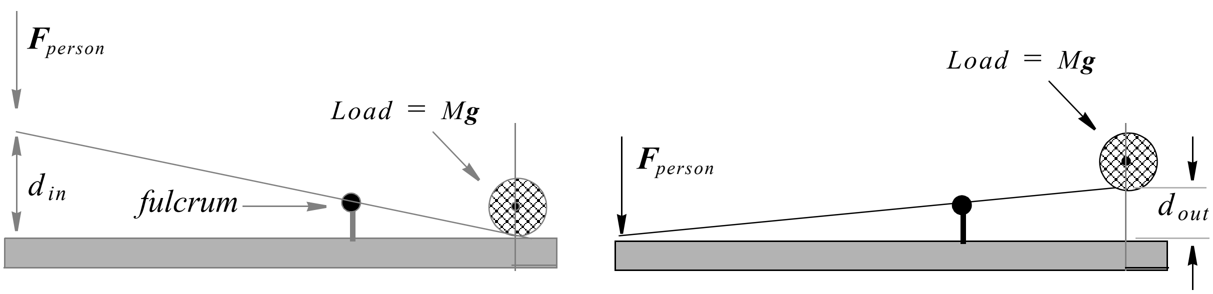
Experiment # 19

Efficiency of Simple Machines

Principles

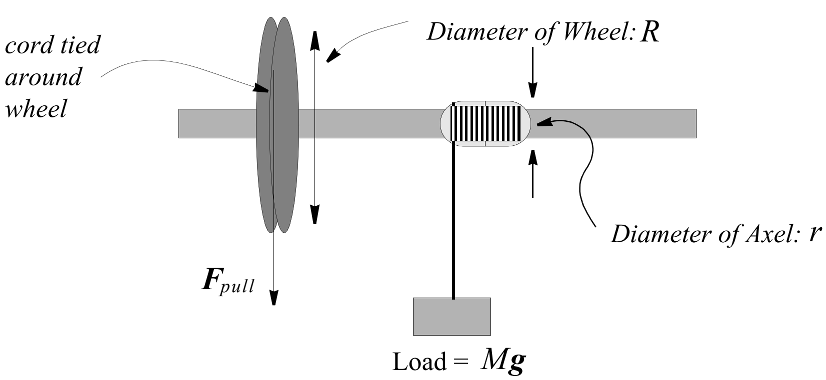
Simple Machines

Machines are devices to lift loads to elevated levels. One uses a machine to reduce one's effort, or to lift heavier loads that wouldn't otherwise be lifted using man-power alone. Examples of simple machines are levers, wheel-and-axle, pulleys, inclined planes, and the like. Following are illustrations of four simple machines.



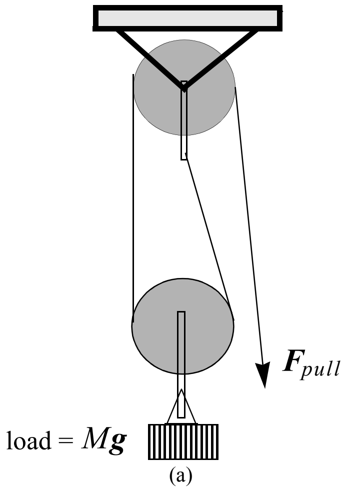
A **small** force acting through a **large** distance lifts a **large** load through a **small** distance

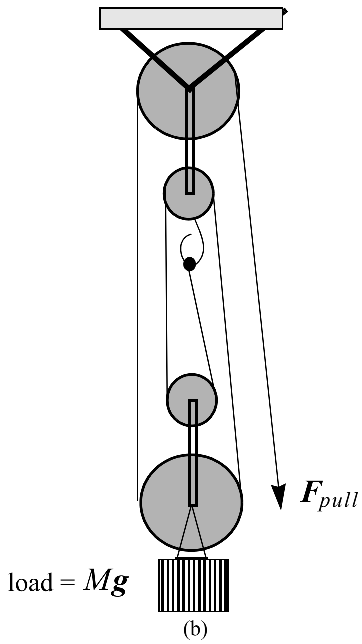
Fig (1) The Lever System

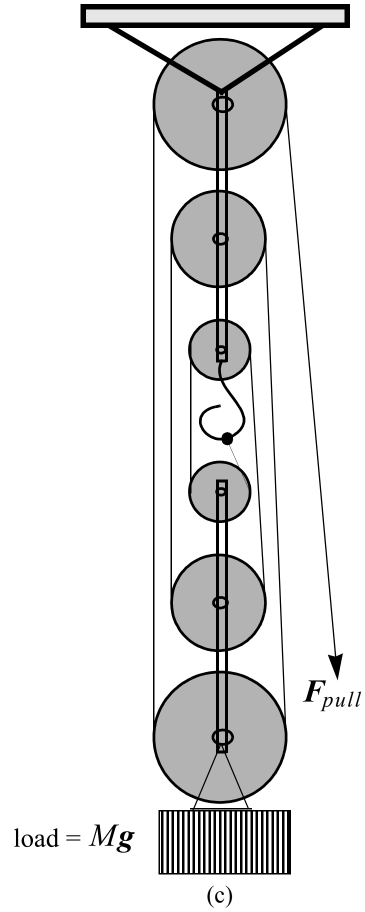


When the cord is pulled through one circumference (=$2\pi R$*)*
the load moves up through one circumference (= $2\pi r$*)*

Fig (2) The Wheel & Axle System





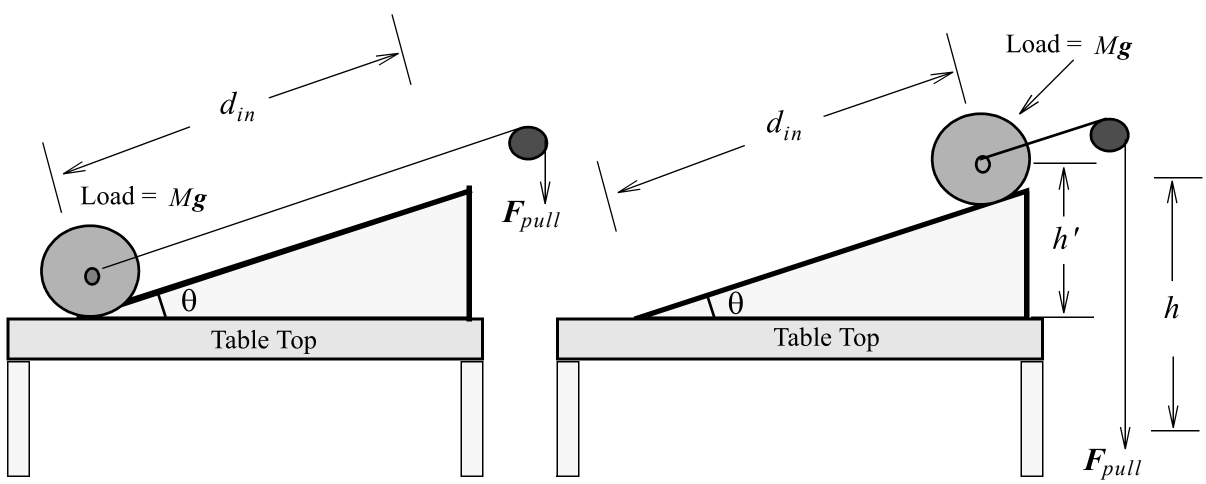


(a) When one pulley *rolls* upward then the load moves up by $d_{out} = (1/2)\, d_{in}$

(b) When two pulleys *roll* upward then the load moves up by $d_{out} = (1/4)\, d_{in}$

(c) When three pulleys *roll* upward then the load moves up by $d_{out} = (1/6)\, d_{in}$

Fig (3) Systems of Pulleys



As $\boldsymbol{F}_{pull}$ pulls the cord through h the load $M\boldsymbol{g}$ moves up through h'

Fig (4) Inclined Plane

The Role of Machines.

A machine is an intermediate agency between the *effort of a person* and the *work done.* We may say that *work done by a person directly on a load* is replaced by the *work done by a person on the machine* and *the work done by the machine on the load.* As has been shown in the foregoing four examples, the person applies a smaller force through a larger distance and the machine applies a larger force through a smaller distance. The force applied by the machine is always the weight force of the load.

Efficiency of Simple Machines

In real life, no system is an ideal system. A part of our effort is always lost due to the presence of dissipative factors. We, therefore, end up doing more work than what was actually needed to be done. If we call our effort as W_{in} and the work that was actually needed to be done, as W_{out}, then it will be found that $W_{out} < W_{in}$. The difference will be large in some cases and small in others but it will never be zero. Best way to estimate the loss is to calculate the ratio of *work-that-was-accomplished* to *effort-that-was-applied*. This is ***Efficiency***! It is defined as: W_{out} / W_{in} .

In case the work is done with the help of a machine, the concept of efficiency will not change. We shall redefine W_{in} as the work done by the person on the machine, and W_{out} as the work done by the machine on the load.

$$E = W_{out} / W_{in}$$

By definition
$$W = (F)(d)(\cos\phi)$$

If force and displacement were to be in the same direction then $\cos\phi = 1$, so we write:

$$W = Fd$$

Then:

$$W_{in} = F_{in} d_{in} \quad \text{and} \quad W_{out} = F_{out} d_{out}$$

We get:

$$E = F_{out} d_{out} / F_{in} d_{in} \qquad \text{.........(1)}$$

Mathematical Analysis

Rewrite Equation (1) as:

$$E = \frac{F_{out}}{F_{in}} \frac{d_{out}}{d_{in}} = \frac{F_{out} / F_{in}}{d_{in} / d_{out}} = \frac{AMA}{IMA} \qquad \text{..........(2)}$$

Define:

$F_{out} / F_{in} = AMA =$ Actual Mechanical Advantage

$d_{in} / d_{out} = IMA =$ Ideal Mechanical Advantage.

AMA is the factor by which the machine *actually* multiplies (or amplifies) the effort. IMA, on the other hand, is the factor by which the machine *would have* multiplied (or amplified) our effort had there been no dissipative forces! Please note that IMA is always greater than one. AMA is usually greater than one but sometime it can be less than one also. In any case, however, AMA is *always* less than IMA and hence efficiency is *always* less than one or less than 100%.

The quantity F_{out} is interpreted as the load which the machine lifts upward with uniform speed. If M be the mass of the load, then:

$$F_{out} = Mg$$

We interpret d_{out} as the height through which the machine lifts the load. Similarly F_{in} is to be interpreted as the effort of the person which may be in the form of $\boldsymbol{F_{pull}}$. Finally, we interpret d_{in} as the distance (height) through which F_{in} does work.

It is always possible to calculate IMA for a simple machine. As for AMA, one needs to know the effect of dissipative forces on a given system. One can determine the effort F_{in} for a given load either experimentally or calculate it from efficiency, provided that the efficiency is specified.

Finding IMA

We shall calculate IMA for each of the four simple machines shown above.

(a) Lever:

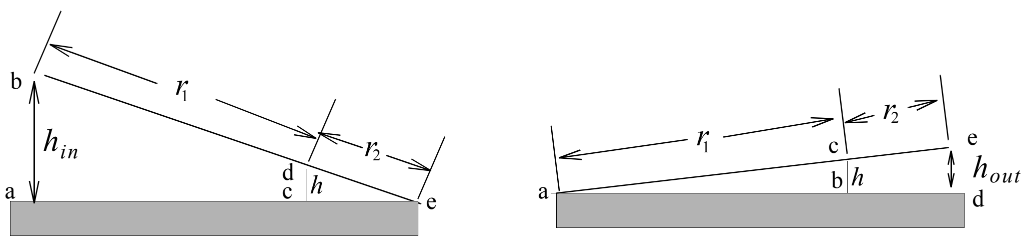


Fig (5a) Initial State of lever

Fig (5b) Final State of lever

Fig (5) The Lever: IMA

From the similarity of triangles *abe* and *cde* (Fig 5a), we find

$$\frac{h_{in}}{r_1 + r_2} = \frac{h}{r_2} \qquad \text{or} \qquad h_{in} = \frac{h(r_1 + r_2)}{r_2} \qquad \text{..........(3)}$$

Similarly from the similarity of triangles *abc* and *ade* Fig (5b):

$$\frac{h_{out}}{r_1 + r_2} = \frac{h}{r_1} \qquad \text{or} \qquad h_{out} = \frac{h(r_1 + r_2)}{r_1} \qquad \text{..........(4)}$$

where we have written:

$$h_{in} \text{ for } d_{in} \qquad \text{and} \qquad h_{out} \text{ for } d_{out}.$$

From (3) and (4), by dividing:

$$\frac{d_{in}}{d_{out}} = \frac{h_{in}}{h_{out}} = \frac{h(r_1 + r_2)}{r_2}\frac{r_1}{h(r_1 + r_2)} = \frac{r_1}{r_2} \qquad \text{..........(5)}$$

Hence

$$IMA \text{ (lever system)} = r_1 / r_2 \qquad \text{..........(6)}$$

(b) Wheel & Axle:

As shown in Fig (2), F_{pull} pulls the cord by one circumference of the wheel and the load moves up by one circumference of the axle. Thus:

$$d_{in} = h_{in} = 2\pi R; \quad \text{where} \quad R = \text{radius of the wheel}$$

And

$$d_{out} = h_{out} = 2\pi r; \quad \text{where} \quad r = \text{radius of the axle}$$

Dividing:

$$\frac{d_{in}}{d_{out}} = \frac{h_{in}}{h_{out}} = \frac{2\pi R}{2\pi r} = \frac{R}{r} \qquad \text{..........(7)}$$

Hence:

$$IMA \text{ (wheel \& axle)} = R/r \qquad \text{..........(8)}$$

(c) Pulleys:

From Fig (3a), we find that the ratios of d_{in} to d_{out} is 2. This is because the pulley "rolls". For a rolling object, the velocity of the center of mass of the pulley is one half of the linear velocity of the cord that unwinds from it. Hence

$$\frac{d_{in}}{d_{out}} = 2 \qquad \text{..........(9)}$$

Hence

$$IMA \text{ (one pulley rolling up)} = 2 \qquad \text{..........(10)}$$

Similarly, for pulley systems in Fig (3b) and (3c), we have:

$$IMA \text{ (two pulleys rolling up)} = 4 \qquad \text{..........(11)}$$

$$IMA \text{ (three pulleys rolling up)} = 6 \qquad \text{..........(12)}$$

(d) Inclined Plane:

From Fig (4) it is seen that as F_{pull} pulls the cord through a vertical height h, the load moves up through a vertical height h'. Thus:

$$d_{in} = h \quad \text{and} \quad d_{out} = h'$$

Hence

$$\frac{d_{in}}{d_{out}} = \frac{h}{h'},$$

Now

$$h' = d\sin\theta \quad \text{and} \quad h = d$$

Therefore:

$$\frac{d_{in}}{d_{out}} = \frac{h}{h'} = \frac{d}{d\sin\theta} = \frac{1}{\sin\theta} \qquad \text{..........(13)}$$

Hence0

$$IMA \text{ (inclined plane)} = \frac{1}{\sin\theta} \qquad \text{..........(14)}$$

Finding AMA

To find the actual mechanical advantage, one would set up the load (a given weight) and find experimentally, the minimum (suspended) weight that will pull the load up. Such a weight will result in a uniform (un-accelerated) motion of the load upward. The ratio of load to the suspended weight will be the AMA of the given system. The load is always the weight $M\mathbf{g}$, and the suspended weight (found experimentally) may be expressed as $m\mathbf{g}$,. Then AMA is given as:

$$AMA = M\mathbf{g}/m\mathbf{g} = M/m \qquad \text{..........(15)}$$

Determining Efficiency

Efficiency is given by Eqn (3):

$$E = (AMA)/(IMA) \qquad \text{..........(16)}$$

and can be found from a knowledge of AMA and IMA. It may be expressed either as a fraction or in percent form.

Objectives of the Experiment

To determine the efficiency of the given (a) inclined plane system (b) system of pulleys; the efficiency to be expressed in percent form.

Setting up

We shall select a roller of mass M as the *load* to be lifted up using the given inclined plane. Using Equation (16) for AMA and Eqn (15) for the IMA of an inclined plane, we can set up the equation for the efficiency as:

$$E = \frac{AMA}{IMA} = \frac{M/m}{1/\sin\theta}$$

Rearranging, we get:

$$\left(\frac{M}{m}\right) = E\left(\frac{1}{\sin\theta}\right) \qquad \text{..........(17)}$$

Or

$$M\sin\theta = E\,m \qquad \text{..........(18)}$$

Or

$$m = (M/E)\sin\theta \qquad \text{..........(19)}$$

Eqns (18) and (19) both corresponds to the equation of a straight line: $y = mx + b$ with $b = 0$. We find that θ is the independent variable while m is the dependent variable. We may select several values of θ and for each, determine experimentally a value of the suspended mass m which will pull the load *upwards* with **uniform** speed. If Eqn (18) is used, we shall plot $M\sin\theta$ against m and get E directly as the slope of the best fit straight line. If Eqn (19) is used, we shall plot m against $\sin\theta$ and extract E from the slope. Use of Eqn (18) is, to some extent, unscientific and its use signifies *laziness* of the workers which may be tolerable. The instructor should tell you, in advance, as to the equation to be used for the experiment.

As for the given system of pulleys, the value of IMA is fixed and need not be calculated. One would select several values of the load M,and determine experimentally the corresponding values of the suspended masses m that will lift the load upwards with uniform speed. Such a mass will be a *minimum* mass. Plotting M against m, one would get AMA as the slope of the best fit straight line. Knowing the number of pulleys that *roll* upward as the load is lifted, one can write down the IMA and calculate the efficiency. Please recall that *rolling upward* means *rotating and climbing up.*

Apparatus Required

(1) Inclined plane apparatus with magnetically supporting *wall*.

(2) An inclined plane with attached protractor and plumb line, to be supported magnetically on the *wall*.

(3) Massless and frictionless pulley, to be magnetically supported on the *wall*.

(4) Set of masses (with hooks), to be used as *load.*

(5) Two sets of masses

(6) Cord, mass holder and other accessories

(7) Two blocks of pulleys; with three pulleys in each block,

(8) Table clamp and rod to make a vertical stand for the pulleys

(9) Clamp for clamping the pulleys to the rod.

(10) Cord, meter stick and other accessories, as required.

Procedure

(A) The Inclined Plane.

(1) Find the mass of the roller using the digital balance and call it M.

(2) Select angles θ as 10^o, 15^o, 20^o, 24^o, 28^o, 32^o, 36^o, 40^o, 45^o, 50^o, 55^o, & 60^o.

(3) Set the inclined plane at each of these angles (one at a time) with the help of the plumb line and set the pulley in such a way that the cord is parallel to the inclined plane.

(4) For each θ determine the magnitude of the total suspended mass m (using the digital balance) such that the roller moves up with uniform speed.

(5) This is the end of this part of the experiment. Set the inclined plane at 0^o and place the pulley in a reasonable position.

(B) The System of Pulleys

(1) Record the number of pulleys in your system of pulleys that roll up as the load is lifted. Call it N.

(2) Select values of M as 60 g, 100 g, 140 g,...........500 g (12 trials).

(3) For each load determine experimentally the magnitude of the total suspended mass m that will lift the load up with uniform speed.

(4) The experiment ends. Arrange all apparatus neatly on the table. Leave the system of pulleys with some weights so as to keep it in good shape.

Calculations & Graphs

Inclined Plane

(1) If using Eqn (18), calculate $M \sin\theta$ for each trial. If using Eqn (19), do not calculate $M \sin\theta$.

(2) If using Eqn (18), Plot $M \sin\theta$ on y-axis and m on the x-axis. If using Eqn (19), plot $\sin\theta$ on x-axis and m on the y-axis. In either case, draw a best fit straight line, using a computer, and find the efficiency E of the given inclined plane from its slope. Express it in percent form.

System of Pulleys

(1) Multiply N by 2. This is the IMA of the given system of pulleys.

(2) Plot M on y-axis and m on the x-axis. Draw the best fit straight line using a computer, and find its slope. This is AMA of the given system of pulleys.

(3) Calculate the efficiency and express in percent form.

(4) Complete the report with ***Results.***

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Data & Data Tables

Name.................................... **Date**..............................

Instructor.............................. **Lab Section**....................

Partner.......................... **Table #**..........................

Efficiency of the Inclined Plane

(a) Mass of the Roller: M (g)

Total suspended mass (mass placed on the mass holder *plus* the mass of the mass holder) should be determined using the digital balance.

Table 1: Data for Determining the *Efficiency* of an Inclined Plane

Trial #	Angles: θ (degrees)	Total Suspended Mass: m (g)
1	10	
2	15	
3	20	
4	24	
5	28	
6	32	
7	36	
8	40	
9	45	
10	50	
11	55	
12	60	

Additional Information or Data (if any)

Efficiency of the System of Pulleys

Total suspended mass (mass placed on the mass holder *plus* the mass of the mass holder) should be determined using the digital balance.

Number of *rolling* pulleys:

Table 2: Data For Determining the *AMA* of the Given System of Pulleys

Trial #	Mass to be placed on holder (g)	Mass of Load with holder: M (g)	Total Suspended Mass: m (g)
1	60		
2	100		
3	140		
4	180		
5	220		
6	260		
7	300		
8	340		
9	380		
10	420		
11	460		
12	500		

Additional Information or Data (if any)