Experiment # 16

Rotational Dynamics (2)
Rotational Equilibrium

Principles

Much of the underlying principles has been discussed in the experiment on Rotational Inertia. The material may be briefly reviewed before doing this experiment. For the specifics of this experiment, we need talk about rotational equilibrium only.

Rotational equilibrium is a parallel of linear equilibrium. It is indicative of either *state of rest* or of s*tate of uniform rotation* i.e. rotation with constant (or uniform) angular velocity.

If an object is in a state of rotational equilibrium, then either (a) it is experiencing no torque at all, or (b) it is under the influence of at least two torques that mutually neutralize each other, thereby causing the net torque to be zero. It is very educative to compare the situation with linear equilibrium where we only have one possibility! (which one and why only one?)

The situation where no torques are acting on the object is of little interest; (think of a book placed on a table). In the case where two or more torques are acting on the object and are causing the object to be in equilibrium, we may write:

$$\Sigma\tau = 0 \qquad \text{..........(1)}$$

As the torques rotate an object (or tend to rotate it) either clockwise (cw) or counterclockwise (ccw), Eqn. (1) can be written as:

$$\Sigma\tau_{cw} = \Sigma\tau_{ccw} \qquad \text{..........(2)}$$

Thus for an object to be purely in a state of rotational equilibrium, the algebraic sum of clockwise torques acting upon the object should match the algebraic sum of counterclockwise torques acting upon it.

It will be appropriate, at this time, to recall the (structural) definition of a torque:

$$\tau = Fr\sin\theta \qquad \text{..........(3)}$$

As a useful corollary of this definition, we notice that if for a given force, the moment arm $r\sin\theta$ were to be zero, then the torque will also be zero. Another way of saying the same thing is that if a force acts at the axis of rotation *itself*, then it exerts no torque on that object.

Objective of the Experiment

To Study the condition for rotational equilibrium for a **meter stick** *as the object, and hence find its (a) mass (b) center of mass (position).*

Setting up

To study the rotational equilibrium of a meter stick, such as the one shown in Fig (1), we need to establish that it is indeed *capable* of rotating about some axis.

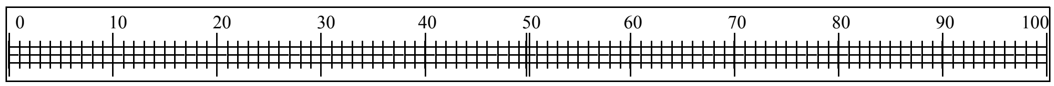

Fig (1) A Meter Stick

The meter stick does not have to go round and round. It doesn't even have to oscillate. It will be quite enough for it to be *able* to rotate about an axis on a fulcrum. Fig (2) demonstrates the rotatibility of the meter stick when it is placed on a traditional fulcrum. The weight force of the meter stick exerts a torque and the meter stick rotates until it hits the table.

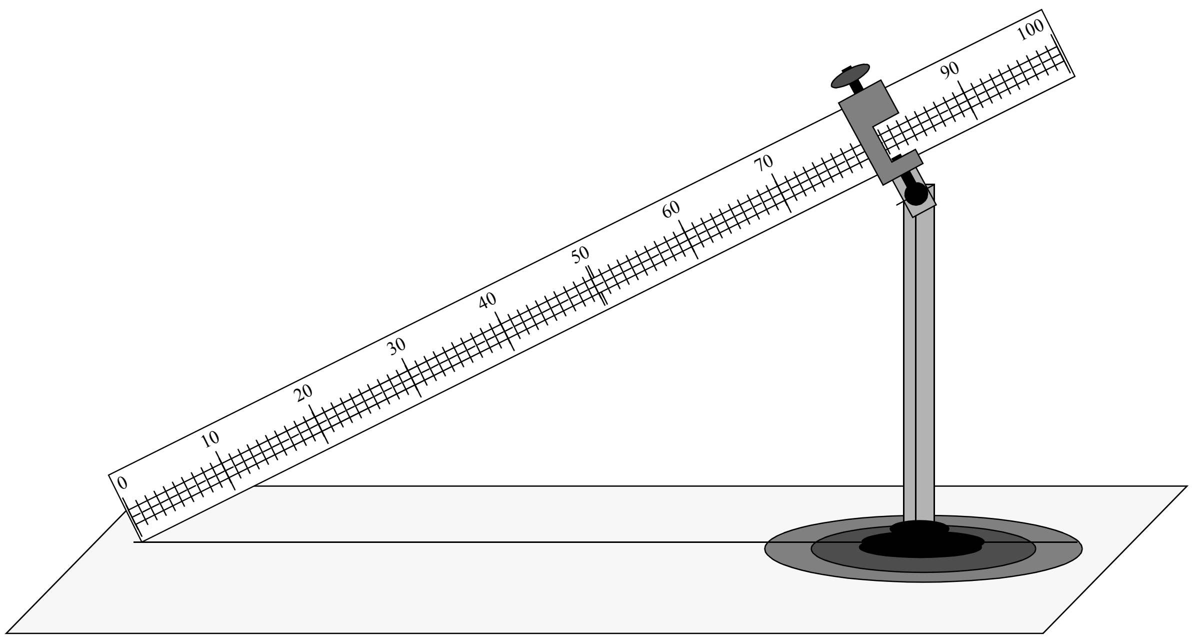

Fig (2) Meter stick is being stopped from rotating by the table.

Having shown that the meter stick is rotatable, we shall study its rotational equilibrium. Currently only the weight force of the meter stick is exerting a torque on it. The meter stick will be in rotational equilibrium if this torque could be turned zero. This, of course, can be accomplished knowing that (i) the earth exerts a force of pull *only* on the center-of-mass of the object, and that (ii) a torque is zero if its moment arm is zero. So, if we place the fulcrum at the center-of-mass of the meter stick, the moment arm will become zero, the torque will become zero, and the meter stick will be in rotational equilibrium. This is shown in Fig (3).

Fig (3), in fact, is an elegant proof that the earth exerts a force of pull *only* on the center-of-mass of objects, whatever shape and size they may have.

It will also, quite easily, be possible to apply any number of additional torques on the meter stick by suspending any number of weights on either side of the fulcrum. It will be an interesting proposition to play around with them, adjusting their magnitudes and positions, in search of an overall equilibrium of the meter stick.

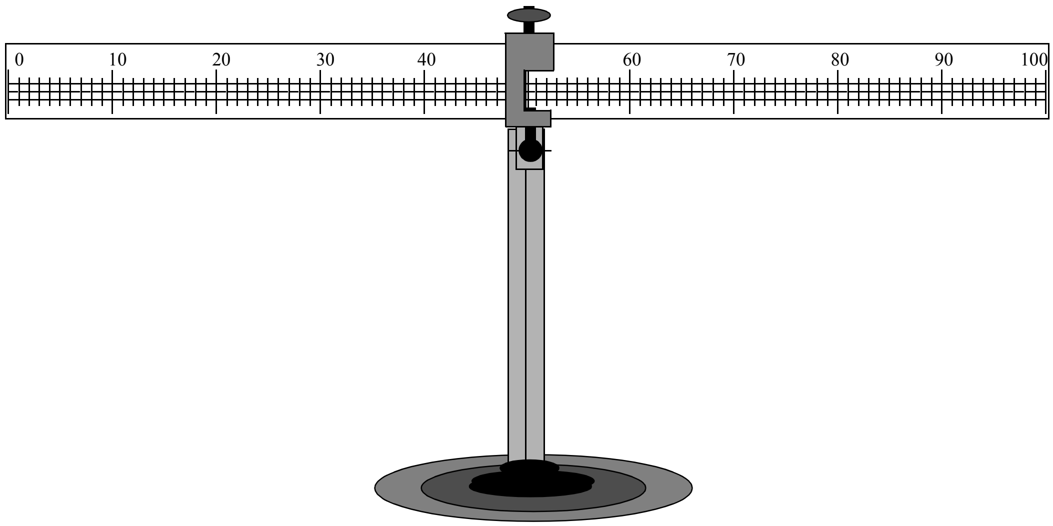

Fig (3) A Meter Stick Balanced on a Fulcrum (Equilibrium)

It will be more interesting if we activate the torque of the weight force of the meter stick also and let it exert an additional torque. We would do so simply by repositioning the fulcrum, away from the center-of-mass of the meter stick.

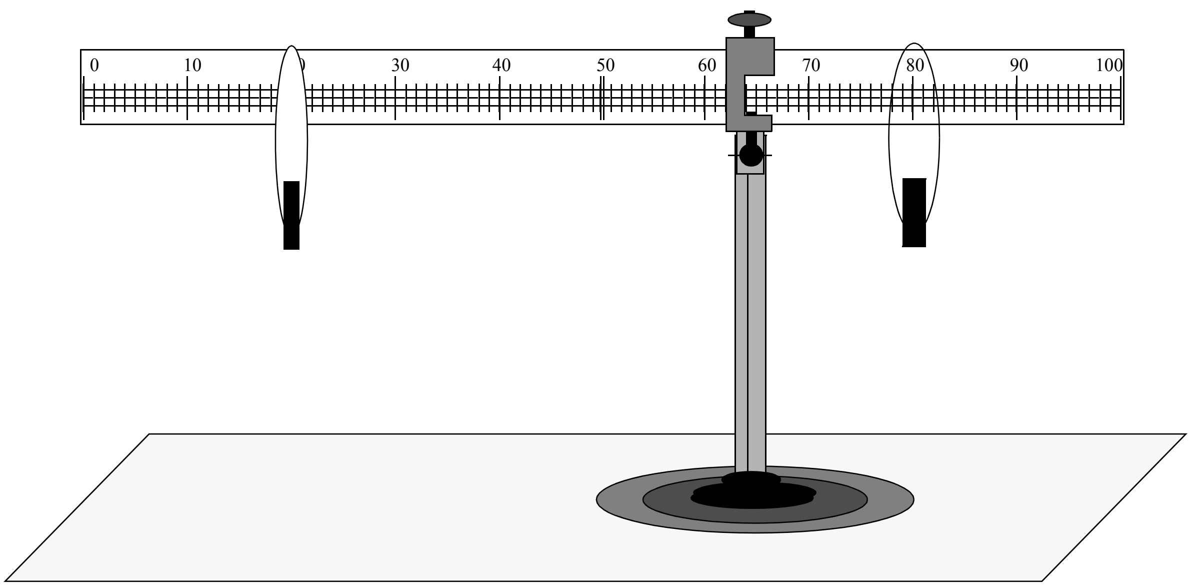

Fig 4) The Meter Stick in Rotational Equilibrium

For a simple but still effective study in the laboratory, we shall narrow down the number of externally applied torques (suspended weights) to just two: one on each side of the fulcrum. This constraint will not limit the scope of work because even with two suspended weights around a given fulcrum, a multitude of combinations of *magnitudes of weights* and their *moment arms* will produce a multitude of distinct rotational equilibria. Please be advised that the torque of the weight force will be an additional (third) torque. Even though the magnitude of its weight force will stay constant, we shall have, at our disposal, a wide range of selectable moment arms for its torque.

The Plan

We have two unknowns for the *objectives* of the experiment: (i) the mass, and (ii) the center-of-mass. We need two equations that should be solved simultaneously to achieve the objectives. But we have only one equation at our disposal (Eqn 2). We, therefore, have no choice but to use this equation twice, by performing the experiment twice (in two parts).

The First Part

A meter stick, in general, is of uniform dimensions. We expect its center-of-mass to be around the 50 cm mark. Let us choose the position of the fulcrum some distance d away from the 50 cm mark, say on its right hand side. This will cause the weight force of the meter stick to exert a torque τ_w on the meter stick, *on the left hand side of the fulcrum*. We then apply the two torques τ_1 and τ_2 (that were proposed earlier) on the meter stick by using weight forces $m_1\boldsymbol{g}$ and $m_2\boldsymbol{g}$ by choosing the masses m_1 and m_2. Let τ_1 be applied on the right hand side of the fulcrum and τ_2 on its left hand side. We shall have two torques on the left side of the fulcrum and one on its right side. If τ_1 and τ_2 be so chosen that the three torques τ_1, τ_2 and τ_w together freeze the meter stick (establish rotational equilibrium), then we may write down the equilibrium equation as follows: Note that the right hand side of the meter stick appears on the left hand side of this equation.

$$(m_1\boldsymbol{g})(l_1)(\sin 90) = (M\boldsymbol{g})(L)(\sin 90) + (m_2\boldsymbol{g})(l_2)(\sin 90)$$

Dividing throughout by $\boldsymbol{g}$ and knowing that $\sin 90 = 1$, we get

$$m_1 l_1 = ML + m_2 l_2 \qquad \text{..........(4)}$$

Here the left hand side of the equation represents the torque on the right hand side of the fulcrum and the right hand side of the equation represents the torques on the left hand side of the fulcrum. This equation matches the equation of a straight line

$$y = b + mx \qquad \text{..........(5)}$$

and hence adequately qualifies to be used for the first part of the experiment. By plotting $m_1 l_1$ against l_2, we shall get a value of m_2 from the slope of the best fit straight line. The y-axis intercept b will represent the torque ML, exerted by the weight force of the meter stick.

Following is a complete description of all terms appearing in the above equations:

m_1 = mass for the right-hand-side torque

m_2 = mass for the left-hand-side torque

M = mass of the meter stick

d = distance of the fulcrum from the 50 cm mark on the meter stick.

L = arm length of the weight force Mg. It is the distance of the position of the center-of-mass of the meter stick from the fulcrum. (We, however, do not know this position.) This distance will be on the left side of fulcrum and will be invisible to us.

Please note that L and d are two different lengths!

l_1 = arm length of the weight force $m_1\boldsymbol{g}$. It is the distance of m_1 from the fulcrum. This distance will be on the right side of fulcrum

l_2 = arm length of the weight force $m_2\boldsymbol{g}$. It is the distance of m_2 from the fulcrum. This distance will be on the left side of fulcrum

A term by term comparison of Eqns (4) and (5) dictates that if Eqn (4) is to produce a straight line, then ML and m_2 must be treated as the two constants of the equation of the straight line. Thus for the entire duration of the first part of the experiment, we are *obligated* to keep M, L and m_2 constant. We shall, therefore, select one value each for L and m_2. These values will not be disturbed during the course of the experiment. Please note that M is already a constant quantity.

The Second Part

For the second part of the experiment, we choose fulcrum to be some distance d' away from the 50 cm mark on its *left* hand side. This will cause the weight force of the meter stick to exert the torque τ_w on the meter stick, *on the right hand side of the fulcrum*. We shall now have two torques on the right hand side of the fulcrum and one on its left hand side. Without loss of generality, it may be advisable (though not necessary) to take d' to be equal to d. We obtain an equation similar to Eqn (4), for the equilibrium of the meter stick. Note that the right hand side of the meter stick now appears on the right hand side of the equation.

$$m_1 l_1' = ML' + m_2 l_2' \qquad \text{.........(6)}$$

where l_1', l_2' and L' are the corresponding arm lengths for the new position of the fulcrum. Please note that m_2 is now placed on the right side of the fulcrum and m_1 is placed on the left side. Please also note that the left hand side of the equation *now* represents the torque on the left hand side of the fulcrum and the right hand side of the equation represents the torques on the right hand side of the fulcrum.

If we plot Eqn (6) directly, we shall end up getting a second value of m_2. This will be of little interest to us. We, therefore, rearrange Eqn (6) as:

$$m_2 l_2' = -ML' + m_1 l_1' \qquad \text{.........(7)}$$

By plotting $m_2 l_2'$ on y-axis against l_1' we shall *now* get a value of m_1 from the slope of the best fit straight line! The y-axis intercept b' will represent the torque ML', exerted by the weight force of the meter stick. We find that b' has a negative sign. This is because we plotted Eqn (7) instead of Eqn (6). Rest assured b' is really a positive torque and, while calculating the center-of-mass of the meter stick, we shall ignore the negative sign.

A term by term comparison of Eqns (5) and (7) now directs us to keep m_1 constant for the entire duration of the experiment. The net outcome of the comparison of Eqns (4) and (7) with Eqn (5) is that, in the interest of getting straight lines, the values of m_1 and m_2 have gotten frozen! Our freedom of being able to choose *a multitude of magnitudes of weights* has evaporated. This, however, is a blessing in disguise because we are now left with only two free or unconstrained parameters: l_1 and l_2. These will now be used as the *independent* and the *dependent* parameters.

Technically, values of m_1 and m_2 are frozen within the individual parts of the experiment only. One need not use their values from the first part of the experiment, into the second part. For the sake of convenience and simplicity, however, we may just *import* their magnitudes from the first part into the second part. Thus going from the first part of the experiment to the second, as we switch the masses m_1 and m_2 around, from right side to the left side and from left side to the right side, we shall not alter their magnitudes.

It should further be noted that m_1 will have to be larger than m_2 because it has to balance two torques on the other side of the fulcrum. The values of l_1 will then be smaller than those of

l_2. For plotting Eqns (4) and (7), we should really select lengths of l_2 and experimentally determine the corresponding lengths of l_1. But in the interest of greater accuracy, we shall set the smaller lengths (i.e. l_1) on the meter stick and experimentally determine the larger ones.

Mathematical Analysis

The slopes of the two graphs will just be the values of m_2 and m_1 respectively. These can, now be compared directly with their actual (preselected) values. A reasonable agreement will establish our faith in the design of the experiment, and serve as *authenticity* tests.

The intercepts of the two graphs are the magnitudes of the torques of the weight force of the meter stick, for the two fulcrum positions. We shall use the two intercepts to determine the two objectives of the experiment.

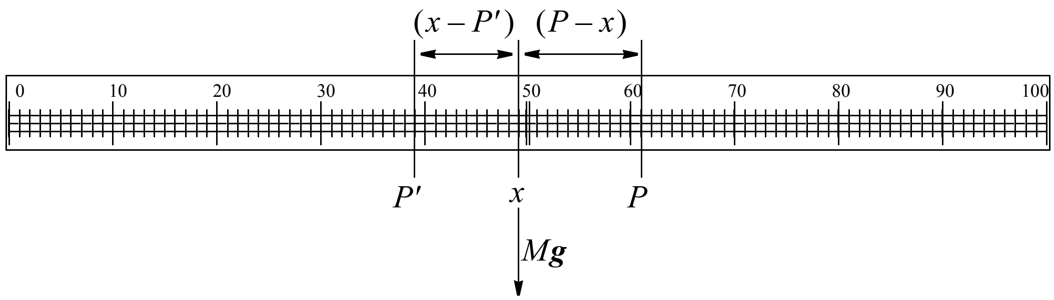

Fig (5) The Torque of the Weight Force of the Meter Stick with respect to its Center of Mass

Let x be the unknown position of the center-of-mass of the meter stick (as read directly on the meter stick) and P be the position of the right hand side fulcrum (as read directly on the meter stick), then

$$P = (50 + d) \qquad \text{..........(8)}$$

and

$$L = (P - x)$$

The intercept $b = ML$ can now be written as:

$$b = M(P - x) \qquad \text{..........(9)}$$

Likewise, for P' being the position of the left hand side fulcrum (as read directly on the meter stick), we find from Fig (5):

$$P' = (50 - d') \qquad \text{..........(10)}$$

and

$$L' = (x - P')$$

The intercept $b' = ML'$ can now be written as:

$$b' = M(x - P') \qquad \text{..........(11)}$$

From Eqns (9) & (11), we get by dividing:

$$\frac{b}{b'} = \frac{M(P - x)}{M(x - P')}$$

or

$$\frac{b}{b'} = \frac{(P - x)}{(x - P')}$$

Cross multiplication yields:

$$bx - bP' = b'P - b'x$$

Solving for x, the position of the center-of-mass, we get:

$$x = \frac{(bP' + b'P)}{(b + b')} \qquad \text{..........(12)}$$

As for the mass of the meter stick, it can now be determined either from Eqn (9) or (11) or both. It is recommended that both equations be used to yield two values of M. The average of the two values should then be regarded as the *experimental* value of the mass of the meter stick.

We have thus accomplished the two objectives of the experiment using only one mathematical equation! This is a good example of scientific ingenuity.

Apparatus Required

(1) A meter stick

(2) A meter stick clamp

(3) A traditional fulcrum

(4) Masses to make up m_1 and m_2. The masses should have holes at their centers. Suggested values of m_1 and m_2 are: $120g$ and $50g$.

(5) Loops of cord, that pass through the holes at the centers of the masses.

(6) Digital balance

Procedure

Please note that the values of parameters used in this Procedure are not binding. The instructor may change some or all values at his/her discretion.

(1) We shall set up the apparatus in the manner shown in Fig (4).

(2) Balance the meter stick carefully on the fulcrum as shown in Fig (3). When in the horizontal position, record the position of the fulcrum, as read directly on the meter stick, in centimeters, millimeters and fractions thereof. This is the position of the center-of-mass of the meter stick.

(3) Find the mass M of the meter stick using the digital balance.

Be sure to use the correct data sheet. If you are using procedure ***B*** ($d \neq d'$), then you should choose the data sheet according to the mass M of your meter stick. For $M \leq 100$ g**,** use data sheet on page 189. If, on the other hand, $M > 100$ g**,** use the data sheet on page 190. In any case, only one data sheet will be used.

(4) Select 120 g for the right hand side torque and 50 g for the left hand side torque. Thus we are setting $m_1 = 120$ g and $m_2 = 50$ g. Please note that no mass holders are being used. The masses are suspended with the help of (massless) cords. Find the *actual* value of these two masses using the digital balance. These are their *expected* values.

(A) Performing the Experiment with $d = d'$

(5) Select $d = 10$ cm. This means that we shall set the right hand side fulcrum at the 60 cm mark on the meter stick. Use data sheet on page 187 or page 188.

(6) Select the following values of l_1 (in cm): 15, 16.5, 18, 19.5,......36 cm. These are 15 trials. The actual positions of m_1 will be 75 cm, 76.5, 78,......96 cm.

(7) For each l_1, adjust the position of m_2 such that the meter stick is in rotational equilibrium (will be in the horizontal position). Patience, precision and concentration are very important here. Record the equilibrium positions of m_2, as read directly on the meter stick. We shall use these positions to calculate l_2.

(8) This completes the first part of the experiment.

(9) Select $d' = 10cm$. This means that we shall set the left hand side fulcrum at 40 cm mark on the meter stick. Switch m_1 and m_2. Place m_1 on the left hand side and m_2 on the right hand side of the fulcrum.

(10) Select the same 15 values for l_1' as were selected for l_1 in step (5). The actual positions of m_1 will be 25 cm, 23.5, 22, 20.5,.......4 cm.

(11) Repeat step (7) and record the equilibrium positions of m_2 as read directly on the meter stick. We shall use these positions to calculate l_2' .

(B) Performing the Experiment with $d \neq d'$

(5a) Select $d = 8.50$ cm. This means that we shall set the right hand side fulcrum at the 58.5 cm mark on the meter stick.

(6a) (i) For $M \leq 100$ g, use data sheet on page 189. Select the following values of l_1: 7.5 cm, 9, 10.5, 12,......28.5 cm. These are 15 trials. The actual positions of m_1 will be 66 cm, 67.5, 69,......87 cm.

(ii) For $M > 100$ g, use data sheet on page 190. Select the following values of l_1: 13 cm, 14.5, 16, 17.5,......34 cm. These are 15 trials. The actual positions of m_1 will be 71.5 cm, 73, 74.5,......92.5 cm.

(7a) For each l_1, adjust the position of m_2 such that the meter stick is in rotational equilibrium (will be in the horizontal position). Patience, precision and concentration are very important here. Record the equilibrium positions of m_2, as read directly on the meter stick. We shall use these positions to calculate l_2.

(8a) This completes the first part of the experiment.

(9a) Select $d' = 12.50$ cm. This means that we shall set the left hand side fulcrum at the 37.5 cm mark on the meter stick. Switch m_1 and m_2. Place m_1 on the left hand side and m_2 on the right hand side of the fulcrum.

(10a) For both: $M \leq 100$ g and $M > 100$ g, select the following values of l_1' : 17.5 cm, 18.5, 19.5, 20.5,......31.5 cm. The actual positions of m_1 will be 20 cm, 19, 18, 17,.......6 cm.

(11a) Repeat step (7a) and record the equilibrium positions of m_2 as read directly on the meter stick. We shall use these positions to calculate l_2' .

(12) The experiment ends. Remove the masses from the meter stick and arrange everything neatly on the table.

Calculations & Graphs

Following instructions apply to both, (A) and (B), data:

Fulcrum on the Right Side of the 50 cm Mark

(1) From your data table, calculate $m_1 l_1$ for all 15 trials. Please note that the values of l_1 are to be picked up from the *magnitude* column of the Data Table and not from the *position* column.

(2) Plot $m_1 l_1$ on the y-axis against the corresponding l_2 values (on the x-axis). The values of l_2 are, likewise, should be taken from the *magnitude* column of the Data Table.

(3) Draw the best fit straight line using the computer and write down the values of the slope and the y-axis intercept. These must contain at least 3 decimal digits.

(4) The slope here, represents the mass m_2. Compare with its *actual* value from step (4). Calculate percent error. Enter in the ***Results*** table.

(5) Record the value of the y-axis intercept as b.

Fulcrum on the Left Side of the 50 cm Mark

(6) From your data tables, calculate $m_2 l_2'$ for all 15 trials, using values of l_2' from the relevant *magnitude* column of the Data Table.

(7) Plot $m_2 l_2'$ on the y-axis against the corresponding l_1' values (on the x-axis). Again l_1' values are to be taken from the relevant *magnitude* column of the Data Table.

(8) Draw the best fit straight line using the computer and write down the values of the slope and the y-axis intercept. These must contain at least 3 decimal digits.

(9) The slope here, represents the mass m_1. Compare with its value from step (4). Calculate percent error. Enter in the ***Results*** table.

(10) Record the absolute value of the y-axis intercept as b'. As stated before, we shall ignore the negative sign of the intercept.

The Mass of the Meter Stick & the Position of its Center-of-Mass

(11) Calculate the value of P using Eqn. (8) and that of P' using Eqn (10). Please note that d and d' may or may not be equal. It depends on the option being used. Refer to step (5) or (5a) of "Procedure".

(12) Calculate x, the position of the center-of-mass of the meter stick using Eqn (12). Do not forget that both b and b' are positive intercepts. The answer will directly be the number in centimeters (and fractions) as read on the meter stick and will be indicative of

the location of its center-of-mass. It is expected to be in the vicinity of the 50 cm mark. Compare with the position of the center-of-mass as found in step (2) of *Procedure.* Calculate percent error and enter in the ***Results*** table.

(13) Now that the value of x is known, go back to Eqn (9) and Eqn (11) and calculate the mass of the meter stick M. Find the average value of M. Compare this value with the one found in step (2) of *Procedure* using digital balance. Calculate percent error and enter in the ***Results*** table.

(14) Complete the report with ***Results.***

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Data & Data Tables

Name.................................. **Date**.............................

Instructor............................. **Lab Section**...................

Partner.......................... **Table #**..........................

Position of the center of mass of the meter stick: *cm*

Mass of the meter stick *g*

Mass for the R.H.S. torque m_1 *g* Mass for the L.H.S. torque m_2 *g*

Position of the R.H.S. fulcrum 60 *cm* Position of the L.H.S. fulcrum 40 *cm*

Magnitude of d and d' as set for the experiment: 10.00 *cm*

Table 1. Investigating Rotational Equilibrium of a Meter Stick

#	Fulcrum to the Right of the 50 cm Mark				#	Fulcrum to the Left of the 50 cm Mark			
	l_1 in (cm)		l_2 in (cm)			l'_1 in (cm)		l'_2 in (cm)	
	Magnitude	Position	Position	Magnitude(*)		Magnitude	Position	Position	Magnitude(**)
1	15	75			1	15	25		
2	16.5	76.5			2	16.5	23.5		
3	18	78			3	18	22		
4	19.5	79.5			4	19.5	20.5		
5	21	81			5	21	19		
6	22.5	82.5			6	22.5	17.5		
7	24	84			7	24	16		
8	25.5	85.5			8	25.5	14.5		
9	27	87			9	27	13		
10	28.5	88.5			10	28.5	11.5		
11	30	90			11	30	10		
12	31.5	91.5			12	31.5	8.5		
13	33	93			13	33	7		
14	34.5	94.5			14	34.5	5.5		
15	36	96			15	36	4		
(*) To find the "Magnitudes" for this column, subtract "Position" from 60 cm					(**) To find the "Magnitudes" for this column, subtract 40 cm from "Position"				

Following is an extra Data Table, in case the instructor chooses to change the parameters of the experiment

Position of the center of mass of the meter stick: *cm*

Mass of the meter stick *g*

Mass for the R.H.S. torque m_1 *g* Mass for the L.H.S. torque m_2 *g*

Position of the R.H.S. fulcrum: *cm* Position of the L.H.S. fulcrum: *cm*

Magnitude of $d =$ *cm* Magnitude of $d' =$ *cm*

Value of P in Fig (5) = $(50 + d)$ *cm* Value of P' in Fig (5) = $50 - d'$ *cm*

Table 1. Investigating Rotational Equilibrium of a Meter Stick

#	Fulcrum to the Right of the 50 cm Mark				#	Fulcrum to the Left of the 50 cm Mark			
	l_1 in (cm)		l_2 in (cm)			l'_1 in (cm)		l'_2 in (cm)	
	Magni-tude	Posi-tion	Posi-tion	Magni-tude(*)		Magni-tude	Posi-tion	Posi-tion	Magni-tude(**)
1					1				
2					2				
3					3				
4					4				
5					5				
6					6				
7					7				
8					8				
9					9				
10					10				
11					11				
12					12				
13					13				
14					14				
15					15				
(*) To find the "Magnitudes" for this column, subtract "Position" from $(50 + d)$ cm					(**) To find the "Magnitudes" for this column, subtract $(50 - d')$ cm from "Position"				

Data Table for option (B) of the experiment $d \neq d'$ **Mass of meterstick:** $M \leq 100$ g

Position of the center of mass of the meter stick: cm

Mass of the meter stick g

Mass for the R.H.S. torque m_1 g Mass for the L.H.S. torque m_2 g

Position of the R.H.S. fulcrum: cm Position of the L.H.S. fulcrum: cm

Magnitude of d = 8.50 cm Magnitude of d' = 12.50 cm

Value of P in Fig (5) = 58.50 cm Value of P' in Fig (5) = 37.50 cm

Table 1. Investigating Rotational Equilibrium of a Meter Stick

#	Fulcrum to the Right of the 50 cm Mark				#	Fulcrum to the Left of the 50 cm Mark			
	l_1 in (cm)		l_2 in (cm)			l_1' in (cm)		l_2' in (cm)	
	Magni-tude	Posi-tion	Posi-tion	Magni-tude(*)		Magni-tude	Posi-tion	Posi-tion	Magni-tude(**)
1	7.5	66			1	17.5	20		
2	9	67.5			2	18.5	19		
3	10.5	69			3	19.5	18		
4	12	70.5			4	20.5	17		
5	13.5	72			5	21.5	16		
6	15	73.5			6	22.5	15		
7	16.5	75			7	23.5	14		
8	18	76.5			8	24.5	13		
9	19.5	78			9	25.5	12		
10	21	79.5			10	26.5	11		
11	22.5	81			11	27.5	10		
12	24	82.5			12	28.5	9		
13	25.5	84			13	29.5	8		
14	27	85.5			14	305	7		
15	28.5	87			15	31.5	6		
(*) To find the "Magnitudes" for this column, subtract "Position" from (58.50) cm					(**) To find the "Magnitudes" for this column, subtract (37.50) cm from "Position"				

Data Table for option (B) of the experiment $d \neq d'$ **Mass of meterstick:** $M > 100$ g

Position of the center of mass of the meter stick: cm

Mass of the meter stick g

Mass for the R.H.S. torque m_1 g Mass for the L.H.S. torque m_2 g

Position of the R.H.S. fulcrum: 58.5 cm Position of the L.H.S. fulcrum: 37.5 cm

Magnitude of d = 8.50 cm Magnitude of d' = 12.5 cm

Value of P in Fig (5) = 58.50 cm Value of P' in Fig (5) = 37.50 cm

Table 1. Investigating Rotational Equilibrium of a Meter Stick

#	Fulcrum to the Right of the 50 cm Mark				#	Fulcrum to the Left of the 50 cm Mark			
	l_1 in (cm)		l_2 in (cm)			l'_1 in (cm)		l'_2 in (cm)	
	Magnitude	Position	Position	Magnitude(*)		Magnitude	Position	Position	Magnitude(**)
1	13	71.5			1	17.5	20		
2	14.5	73			2	18.5	19		
3	16	74.5			3	19.5	18		
4	17.5	76			4	20.5	17		
5	19	77.5			5	21.5	16		
6	20.5	79			6	22.5	15		
7	22	80.5			7	23.5	14		
8	23.5	82			8	24.5	13		
9	25	83.5			9	25.5	12		
10	26.5	85			10	26.5	11		
11	28	86.5			11	27.5	10		
12	29.5	88			12	28.5	9		
13	31	89.5			13	29.5	8		
14	32.5	91			14	305	7		
15	34	92.5			15	31.5	6		

(*) To find the "Magnitudes" for this column, subtract "Position" from 58.5 cm

(**) To find the "Magnitudes" for this column, subtract 37.5 cm from "Position"