Experiment # 14

Momentum Conservation (1)
Elastic Collisions

Principles

Definition

Momentum is a property of objects that are *in motion*. It is defined by the following equation:

$$\boldsymbol{p} = m\boldsymbol{v} \qquad \text{..........(1)}$$

where $\boldsymbol{p}$ is momentum, m is the mass of the object and $\boldsymbol{v}$ is its velocity. Since $\boldsymbol{v}$ is a vector, $\boldsymbol{p}$ is also a vector quantity. It inherits the direction of the velocity vector. The unit of momentum is kgm/s. Sometime momentum is also expressed in units $N\sec$. There is no special symbol or name for the units of momentum.

Characteristics

Momentum is found to have some properties of the *force* domain, and some of the *energy* domain. As such, it may quite genuinely be called a *hybrid* of *force* and *energy*. Talking in terms of calculus, we find that if we differentiate momentum with respect to time, we get *force* and if we integrate it with respect to velocity, we get *energy*. This is shown below:

$$\frac{d\boldsymbol{p}}{dt} = m\frac{d\boldsymbol{v}}{dt} = m\boldsymbol{a} = \boldsymbol{F} \qquad \int \boldsymbol{p}\,d\boldsymbol{v} = m\int \boldsymbol{v}\,d\boldsymbol{v} = (1/2)m\boldsymbol{v}^2 \qquad \text{..........(2)}$$

Like energy, momentum is a conserved quantity. Momentum of a system undergoing a transition, does not change. Thus the total momentum of a system (of particles) immediately after a process remains equal to the total momentum of the system immediately before the process. We have, however, a constraint (or pre-condition or pre-requisite): the velocities of the participating particles should be uniform before the start of the process and should be uniform after the end of the process. Whatever happens during the process, does not concern us. Thus for a system consisting of n participants:

$$\left(\sum_{n=1}^{n} \boldsymbol{p}_n\right)_{after} = \left(\sum_{n=1}^{n} \boldsymbol{p}_n\right)_{before} \qquad \text{..........(3)}$$

Collisions

The above mentioned characteristics make *momentum* an ideal tool for the study of collisions. Let us consider the collision of two particles, and then study the conservation of their momenta. If the particles have masses m_1 nd m_2, and are travelling with (uniform) velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ then their total momentum before collision will be:

$$m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$$

Let their respective velocities after collision be $\boldsymbol{v}_1'$ and $\boldsymbol{v}_2'$. The total momentum after collision will then be:

$$m_1\boldsymbol{v}_1' + m_2\boldsymbol{v}_2'$$

Momentum conservation principle leads to:

$$m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2 = m_1\boldsymbol{v}_1' + m_2\boldsymbol{v}_2' \qquad ....(4)$$

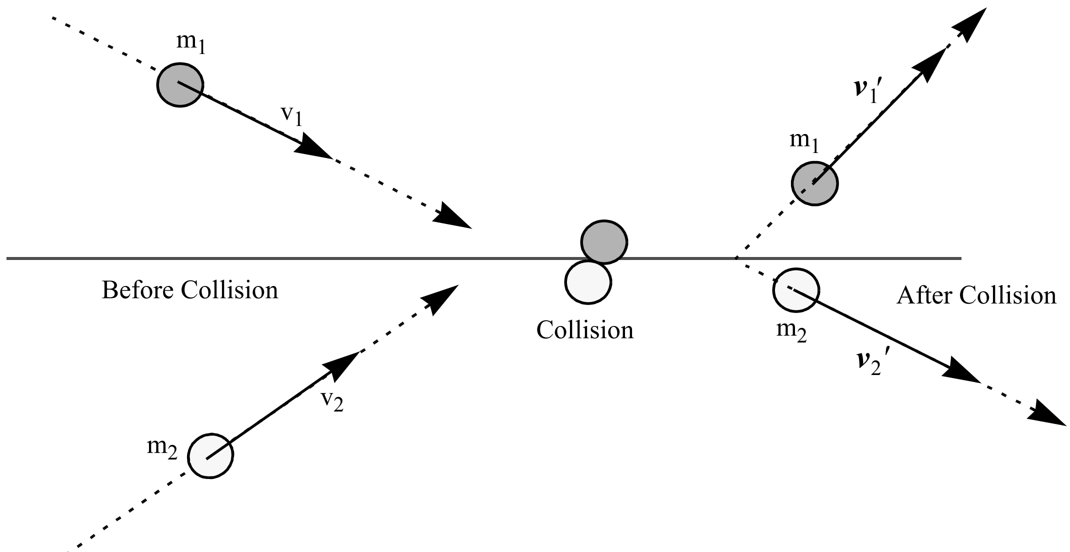

Fig (1) A Collision

Collisions are divided into two groups: (1) elastic (2) inelastic. Momentum remains a conserved quantity for both types of collisions. An elastic collision is defined as one in which the energy due to the motion of the particles (kinetic energy) does not change form. It doesn't get changed (for example) to potential energy. Again, no heat is generated in such a collision. This enables us to develop another useful tool for dealing with elastic collisions: the *conservation* of kinetic energy. This conservation is expressed by the following equation:

$$(1/2)m_1v_1^2 + (1/2)m_2v_2^2 = (1/2)m_1v_1'^2 + (1/2)m_2v_2'^2 \qquad ....(5)$$

Both elastic and inelastic collisions can be one-, two-, or three-dimensional. One can split up the momentum conservation equation into component equations. Following is the set of equations for momentum conservation for a two-dimensional collision.

$$m_1v_{1x} + m_2v_{2x} = m_1v_{1x}' + m_2v_{2x}' \qquad ....(6)$$

$$m_1v_{1y} + m_2v_{2y} = m_1v_{1y}' + m_2v_{2y}' \qquad ....(7)$$

Please note that we cannot split the energy conservation equation into component equations because energy is not a vector quantity. Eqns (6) and (7) apply to *all* collisions whereas Eqn (5) can only be used for elastic collisions.

Objective of the Experiment

To verify the laws of conservation of momentum and kinetic energy for a two-body one-dimensional elastic collision; the target body (particle) being initially at rest.

Setting Up

Setting up A One-Dimensional Elastic Collision

Let particle *A* (mass m_1, velocity $\boldsymbol{v}_1$) collide elastically, head-on with particle "B" (mass m_2,velocity $\boldsymbol{v}_2 = 0$). After collision, the two particles move with velocities $\boldsymbol{v}_1'$ and $\boldsymbol{v}_2'$. The collision being one-dimensional, all motion will be confined to the x-axis of a coordinate system.

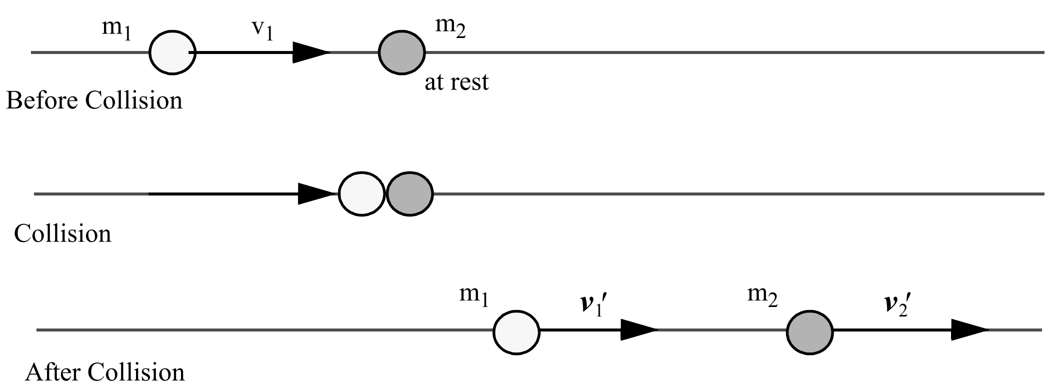

Fig (2) A One-Dimensional Collision

Setting $\boldsymbol{v}_2 = 0$ in Eqn (4) we get:

$$m_1\boldsymbol{v}_1 = m_1\boldsymbol{v}_1' + m_2\boldsymbol{v}_2' \qquad \text{..........(8)}$$

Re-arranging:

$$m_1(\boldsymbol{v}_1 - \boldsymbol{v}_1') = m_2\boldsymbol{v}_2' \qquad \text{..........(9)}$$

Likewise setting $\boldsymbol{v}_2 = 0$ in Eqn (5) we get:

$$(1/2)m_1v_1^2 = (1/2)m_1v_1'^2 + (1/2)m_2v_2'^2$$

Cancelling out $(1/2)$ and rearranging we get:

$$m_1v_1^2 - m_1v_1'^2 = m_2v_2'^2$$

Factorizing the left hand side, we get:

$$m_1(\boldsymbol{v}_1 - \boldsymbol{v}_1')(\boldsymbol{v}_1 + \boldsymbol{v}_1') = m_2v_2'^2 \qquad \text{..........(10)}$$

Dividing Eqn (10) by Eqn (9) we get:

$$\boldsymbol{v}_1 + \boldsymbol{v}_1' = \boldsymbol{v}_2' \quad \text{or} \quad \boldsymbol{v}_1' = \boldsymbol{v}_2' - \boldsymbol{v}_1 \qquad \text{..........(11)}$$

We insert this value of $\boldsymbol{v}_1'$ in Eqn (8), to get:

$$m_1 \boldsymbol{v}_1 = m_1(\boldsymbol{v}_2' - \boldsymbol{v}_1) + m_2 \boldsymbol{v}_2'$$

Separating out and collecting together terms in $\boldsymbol{v}_1$ and $\boldsymbol{v}_2'$ we get:

$$2m_1 \boldsymbol{v}_1 = (m_1 + m_2)\boldsymbol{v}_2'$$

Solving for $\boldsymbol{v}_2'$ we get:

$$\boldsymbol{v}_2' = \left(\frac{2m_1}{m_1 + m_2}\right)\boldsymbol{v}_1 \qquad \text{..........(12)}$$

A value of v_1' may be obtained by subtracting $\boldsymbol{v}_1$ from $\boldsymbol{v}_2'$.

Setting up the Experiment

We shall use two gliders for the experiment that will run on an air track. One glider will be treated as the *incident* (body) particle and will be given an initial velocity using an electrical launcher. The other will be the treated as the *target* (body) particle and will be held at rest half way on the track. The gliders are fitted with *flags* of known width. Photogate timers will be used to record the time that a flag will take to pass through the timer. Let the width of the flags be l. Let the time taken by the flag of the incident glider *before* collision be t_1 and that taken by the flag of the target glider *after* collision be t_2' . The velocities of the two gliders will then be found as:

$$v_1 = \frac{l}{t_1} \qquad v_2' = \frac{l}{t_2'}$$

Inserting these values in Eqn (12) and cancelling out l we get:

$$\frac{1}{t_2'} = \left(\frac{2m_1}{m_1 + m_2}\right)\left(\frac{1}{t_1}\right)$$

Taking reciprocals on each side:

$$t_2' = \left(\frac{m_1 + m_2}{2m_1}\right)(t_1) = \frac{m_1 t_1}{2m_1} + \frac{m_2 t_1}{2m_1} = \frac{t_1}{2} + \left(\frac{t_1}{2m_1}\right)(m_2)$$

The final equation is:

$$t_2' = \frac{t_1}{2} + \left(\frac{t_1}{2m_1}\right)(m_2) \qquad \text{..........(13)}$$

It matches the equation of a straight line:

$$y = b + mx$$

The comparison suggests that we treat m_2, the mass of the target particle (glider #2), as independent variable and the time that its flag takes to pass through the timer after collision t_2' , as dependent variable. We, therefore, need to arrange collisions of the incident particle (glider #1) with the target particle (glider #2) of variable mass then m_2 and t_2' will emerge as independent and dependent variables respectively. A graph of t_2' against m_2 will yield a straight line of slope $t_1/2$ and intercept $t_1/(2m_1)$. Both these are the properties of the incident particle (glider #1). If the properties of the incident particle, as found from the graph, match their actual values (as set for the experiment), we shall be convinced that the laws of one-dimensional elastic collision are valid.

If we divide the intercept by the slope we get m_1, the mass of the incident particle (glider #1). Again, the intercept itself is one-half of the time that the incident particle takes to pass through the photogate timer. Both these are constants of the collision, as set up for the experiment.

Transferring Momentum

It will be noticed that the proposed one dimensional collision does not take place directly. We, for example, do not let one glider collide with the other to make a direct metal to metal contact. Instead, a tightly strung rubber band is used. The kinetic energy of the incident particle is transformed into the elastic potential energy of the rubber band. The rubber band un-stretches itself on account of the restoring force and the elastic potential energy is transformed into the kinetic energy of the target particle. This indirect transfer of momentum was necessary because the metal to metal contact does not take place along the line of *centers-of-mass* of the two gliders! We wish to remind you that the law of conservation of momentum is valid *only* if collision occurs along the line of centers-of-mass of the colliding particles. The center of mass of a glider is located in the lower half of its body whereas the collision occurs at the upper half of its body. Try colliding an incident glider directly with another of identical mass (initially at rest). The incident glider **will not stop dead**, as is predicted by the theory!

Apparatus Required

(1) Linear air track
(2) Air source
(3) Two gliders
(4) Glider launching system (electric),
(5) Electronic timer
(6) Two photogates
(7) A digital balance capable of measuring up to one hundredth of a gram,
(8) Accessories: masses, digital vernier calipers, etc.

Procedure

(1) Familiarize yourself with (a) the air track, (b) the gliders, and (c) the electrical launching system.

(2) Measure the diameter of the cylindrical part of the flag using the digital vernier caliper. Make three measurements for three different rotational positions of the flag and calculate the average diameter. Convert to meter.

(3) Place the target glider in the middle of the track (100 cm mark). Switch on the air flow. Adjust the flow if necessary. Level the track such that the target glider remains at rest.

(4) Set the electronic timer in ***Elastic Collisions*** mode and select one memory. In this mode the timer will automatically measure the values of time recorded by the two photogates representing the velocities of the two gliders. Letter *A* will indicate photogate for incident glider and letter *B* will indicate the photogate for the target glider. Each value of time will be recorded to one-tenth of a millisecond.

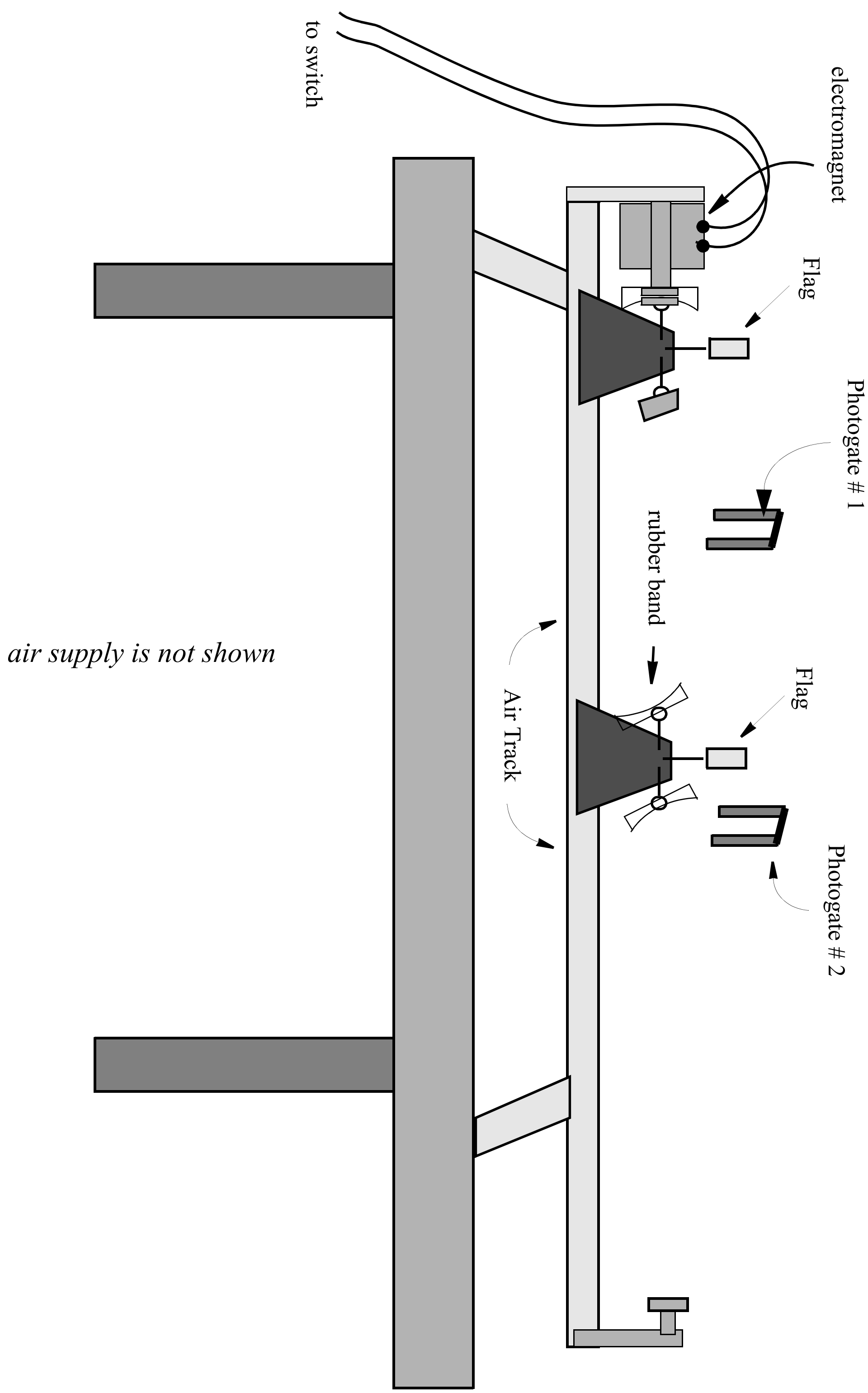

Fig (3). Air Track Set-Up for the Collision Experiment

(5) Place the edge of the target glider facing the launcher, at the 100 cm mark on the air track. Then set the position of photogate #2 such that it is about 3 cm away from the flag, in a direction away from the launcher. Adjust the height (z-axis), position (x-axis), and the width (y-axis) of the photogate window such that the cylindrical part of the flag will pass through the photogate comfortably.

(6) Hold the incident glider assembly such that it is about 1 cm away from the target glider assembly, toward the launcher. Then set the position of photogate #1 such that it is about 3 cm away from the flag of the incident glider, in a direction toward the launcher. Adjust the height (z-axis), position (x-axis), and the width (y-axis) of its window such that the cylindrical part of the flag will pass through the photogate easily.

(7) Using the digital balance find m_1 and m_2, the masses of the two gliders; and record in the data sheet. These masses will be used for the first trial.

(8) The launcher-circuit should not be kept energized for long periods of time. When ready to take a trial, bring the incident glider near the launcher, switch on the circuit and let the magnet hold the glider. In the process the rubber band will get stretched.

(9) Clear the timer.

(10) Launch the incident glider by just opening the electrical switch. The timer will record the time t_1, representing the initial velocity of the incident glider in Channel *A*, and t_2', representing the final velocity of the target glider in channel *B* of the timer.

Repeat three times to have a total of four runs for this trial and a total of four pairs of values of t_1 and t_2'. Please note that you will need to clear the timer for every trial.

(11) Increase the mass of the target glider in steps of 20 grams by placing two equal masses (to add up to the increment), one on each side of the glider. Repeat steps 9 and 10.

(12) Repeat step (11) until all ten trials have been completed. The last value of the mass of the target glider will be a little over 400 grams.

(13) The experiment ends. Switch off the air flow, the electronic timer, the power supply of the launchers, and the digital balance.

Calculations & Graphs

(1) Calculate the average of the four values of t_1 and t_2' and enter in the appropriate columns in the data table.

(2) Find the average of the 10 average values of t_1. Call it $t_{1,av}$. Enter this value in the *foot* of data table # 2, where "$t_{1,av}$ = " appears.

(3) Normalize all average t_2' values to $t_{1,av}$. To do this, divide the average t_2' value of a trial by its corresponding average t_1 value and multiply by $t_{1,av}$. Enter in the appropriate column in the data table. call these $t'_{2,av}$.

(4) Plot $t'_{2,av}$ values on the y-axis and the corresponding m_2 values on the x-axis, using a computer and the Cricketgraph program.

(5) The values of slope and intercept are printed out by the computer.

*You must instruct the computer to print out the values of slope and intercept with large number of decimal places. This is essential for getting decent **results**.*

(6) Divide the intercept by the slope. This will be the mass of the incident glider. Compare with the actual mass and find percent error.

(7) Multiply the intercept by *2*. The resulting number is the experimental value of time $t_{1,\,av}$. Divide D (column #5 of Data Table #1) by this $t_{1,\,av}$. This is the *experimental* value of the initial velocity of the incident glider, $\boldsymbol{v}_1$.

Calculate the *expected* value of $\boldsymbol{v}_1$ by dividing D (column #5 of Data Table #1) by $t_{1,\,av}$, as found and entered at the *foot* of Data Table (2).

Compare the two values of $\boldsymbol{v}_1$ and find percent error.

(8) Complete the report with ***Results***

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Data & Data Tables

Name.................................... **Date**..............................

Instructor.............................. **Lab Section**....................

Partner.......................... **Table #**..........................

Mass of incident glider $m_1 =$ (g)

Table 1: The Diameter of the Cylindrical Part of the Flag of the Incident Glider

$D_1\ (mm)$ (1)	$D_2\ (mm)$ (2)	$D_3\ (mm)$ (3)	$D_{av} = D\ (mm)$ (4)	$D\ (m)$ (5)

Table 2. Elastic Collision in One Dimension.

Serial # (1)		m_2 (g) (2)	t_1 $(\sec)$ (3)	t_2' $(\sec)$ (4)	average t_1, $(\sec)$ (5)	average t_2', $(\sec)$ (6)	normalized $t'_{2,av}$, $(\sec)$ (7)
1	1						
	2						
	3						
	4						
2	1						
	2						
	3						
	4						
3	1						
	2						
	3						
	4						
4	1						
	2						
	3						
	4						

Table 2. Elastic Collision in One Dimension.

Serial # (1)		m_2 (g) (2)	t_1 $(\sec)$ (3)	t_2' $(\sec)$ (4)	average t_1, $(\sec)$ (5)	average t_2', $(\sec)$ (6)	normalized $t'_{2,av}$, $(\sec)$ (7)
5	1						
	2						
	3						
	4						
6	1						
	2						
	3						
	4						
7	1						
	2						
	3						
	4						
8	1						
	2						
	3						
	4						
9	1						
	2						
	3						
	4						
10	1						
	2						
	3						
	4						
$t_{1,av}$ =							