Experiment # 13

Energy Conservation (1)

Principle

Definition

The traditional definition of energy is that *energy is capacity to do work*. Although very general, the definition does point out that there is an intimate relationship between work and energy. When work is done by a system, it expends its own energy and its energy reserves are depleted. When work is done on a system, the system gains energy and its energy contents increase. Work, on the other hand, is always done by a force. A force is said to have done work on a system only if it (the force) succeeded in displacing the object through a non-zero distance (displacement).The unit of energy (and that of work) is $kg\ m^2/s^2$ or *Joules*. We write J for Joules.

Energy is an abstract quantity and has been endowed with two unique features which make it a very special thing. Firstly, it can change form. Energy comes in several different forms and it is always possible to *transform* energy from one form to a radically different form. (Remember *transformer* toys; or the ogre who could transform, for example, a lion into a mouse?) Secondly we can neither create energy nor can we annihilate it. We say that the *energy of a confined system is conserved.* A confined system is one whose energy contents do not get altered by its neighbors (or surroundings). The net energy contents of this system must always remain the same; even though, internally, energy may get converted (or transformed) from one form to any other form, any number of times.

Forms of Energy

In Physics, *energy* is found mainly in the following forms: mechanical energy, electrical energy, thermal energy, light energy, sound energy, and nuclear energy. Other branches of science also have characteristic energies; e.g. chemistry has chemical energy. Mechanical energy is of three kinds: kinetic energy (due to motion), gravitational potential energy (due to elevated positions, with respect to the earth), and elastic potential energy (when in stressed positions). Every energy is related to a force that may be regarded as its *parent* force. Thus the parent force of kinetic energy is Newton's second law force; that of gravitational potential energy is the Gravitational force and the parent force of the elastic potential energy is the Hooke's law force.

Conservative & Non-conservative Forces

With respect to the work the forces do, they are divided into two types: *conservative* and *non-conservative*. If the work done by a force on a system in taking it from one energy state to another energy state is independent of the path, then the force is said to be conservative. If, on the other hand, the work done by the force depends on the path, the force is non-conservative. Of the above mentioned three forces, the last two are conservative. In any process if some mechanical energy cannot be accounted for, it must have been converted into a non-mechanical form. The force responsible for such work is said to be a non-conservative force. This always happens with friction. Whenever friction is present some heat is generated. This shows that some mechanical energy has gotten transformed into thermal energy.

As an example of work being independent of the path, we shall show that the work done by the weight force F_g in taking an object of mass m from A to C is the same whether we follow path AC or path AB followed by path BC.:

Path AC: On the inclined plane the partial weight force F_{gx} , is $mg\sin\theta$. The displacement through which the force does work is d. As the force and the displacement are both directed along AC, $\cos\phi = +1$. The work done is

$$W_{AC} = (mg\sin\theta)(d) = mgd\sin\theta \qquad ....(1)$$

Path AB: The weight force here is mg which does work through a displacement of length AB. As the force mg is directed downward, the force and the displacement $AB = h$ are in the same direction which makes $\cos\phi = +1$ and we get:

$$W_{AB} = (mg)(h)$$

From Trigonometry: $(AB) = h = d\sin\theta$

Therefore: $W_{AB} = mgd\sin\theta$

Path BC: As the mass m is being taken horizontally from B to C, the work done by the weight force mg is zero. This is because the weight force is at right angles to the displacement. and $\cos\phi = 0$. We get:

$$W_{BC} = 0$$

Total work done:

$$W_{AB} + W_{BC} = mgd\sin\theta + 0 = mgd\sin\theta \qquad ....(2)$$

Results (1) and (2) prove the premise.

$W_{AC} = W_{AB} + W_{BC}$

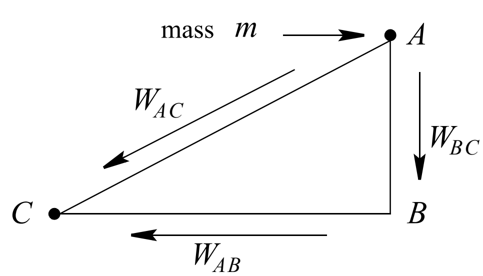


Fig (1) F_G is a Conservative Force

The Law of Conservation of Energy

The law is applicable to *closed* systems only. The word *closed* tell us that the energy of the system doesn't get altered or influenced by its neighbors (or surroundings). The terms *confined* and *isolated* are also frequently used to convey the same sense. The law states: *The net change of the energy contents of a closed system, in any process, is zero; provided that it is being acted upon by conservative forces only.* If one or more non-conservative forces are also acting on the system then the law states that the *net change of the energy contents of the system equals the work done by the non-conservative forces.*

Let W_c be the work done by conservative force(s) and W_{nc} be the work done by non-conservative forces. Let ΔE be the net change of the energy-contents of a closed system. For the two possibilities of the law, we may then write:

(a) When only conservative forces are present:

$$\Delta E = 0 \qquad \text{..........(3)}$$

(b) When non-conservative forces are *also* present:

$$\Delta E = W_{nc} \qquad \text{..........(4)}$$

It should be clearly understood that non-conservative forces do not *annihilate* energy! They only transform mechanical energy into other forms of energy; the most common being the thermal (or heat) energy. The forces that convert mechanical energy into thermal energy are known as *dissipative* forces; force of friction being one of them.

Finally, we may state that for the three types of mechanical energy, ΔE may consist of one or more of ΔKE, ΔPE_g, and ΔPE_{el}. In general, we may write:

$$\Delta E = \Delta KE + \Delta PE_g + \Delta PE_{el} \qquad \text{..........(5)}$$

Objectives of the Experiment

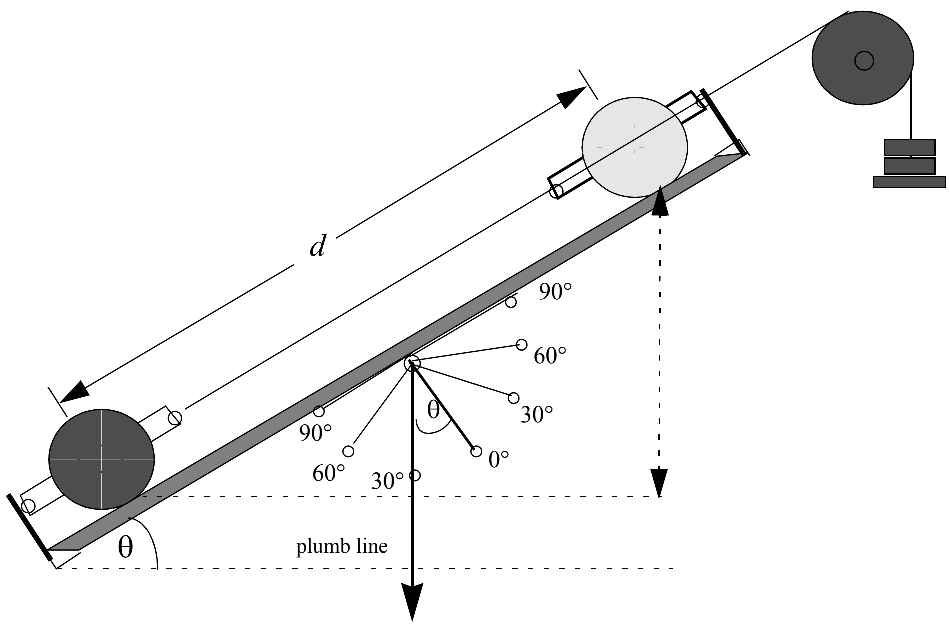
To verify the law of conservation of energy when non-conservative forces are also present.

Setting up

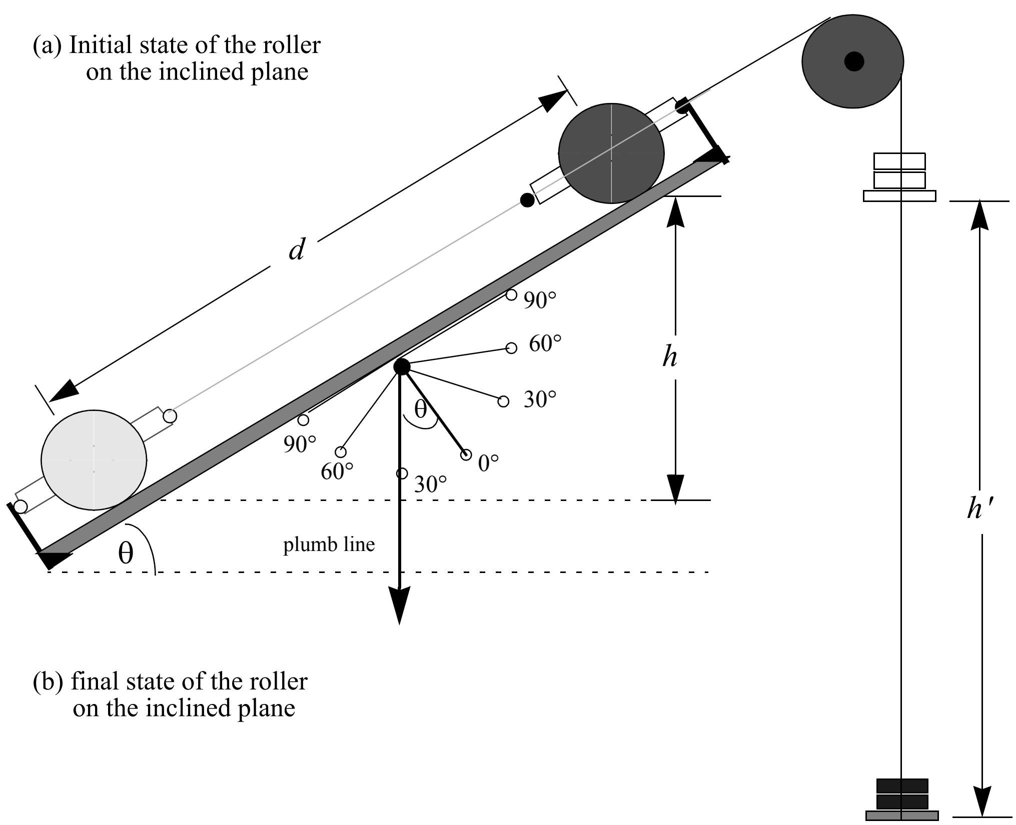
We select a system that does not involve elastic potential energy. We do so for the sake of simplicity and not because a system involving elastic potential energy will be unmanageable. In looking for simplicity however, we do not compromise on the quality of the work. It is important that the law should be satisfactorily studied and nothing important be left out.

One customarily selects a mass on wheels (Hall's car or a roller) that travel up and down an inclined plane. Such a system is shown in Fig (2). The motion of the mass on the inclined plane affords us (1) kinetic energy (because the object moves) and (2) gravitational potential energy; (because the mass gains or loses height). Some dissipative forces will act on the system, as will be shown later. In anticipation of the presence of the dissipative forces, we write the applicable energy conservation law as:

$$\Delta KE + \Delta PE_g = W_{nc} \qquad \text{..........(6)}$$



(a) Initial state of the roller on the inclined plane



(b) final state of the roller on the inclined plane

Fig (2) A Roller on an Inclined Plane

The up and down motion of the roller could be effected by pulling it manually. Such a pull will neither be of constant magnitude nor will it be guaranteed to remain parallel to the inclined plane. To overcome this shortcoming, we use a suspended mass to exert a mathematically uniform force of pull on the roller. The pulley (which must be used) may be so positioned that the cord remains parallel to the inclined plane.

Mathematical Analysis

The purpose of analysis is (a) to minimize the number of parameters for laboratory measurement, and (b) to shape or reshape Eqn (4) so as to render it comparable to the equation of a straight line. The parameters are:

(1) mass of roller: M

(2) mass of the suspended mass: m

(3) velocity of the roller: v_1

(4) velocity of the suspended mass v_2

(5) distance by which the roller travels along the inclined plane: d

(6) the height gained or lost by the roller as it moves through d : h

(7) the height by which the suspended mass m moves vertically up or down: h'

(8) the tension forces in the cord attached to M : F_{T1up} and F_{T1down}

(9) the tension forces in the cord attached to m : F_{T2up} and F_{T2down}

(10) the acceleration of the roller: $\boldsymbol{a}_1$

(11) the acceleration of the suspended mass: $\boldsymbol{a}_2$

(12) the dissipative force(s): $\boldsymbol{F}_{nc}$

(13) the weight force of the roller on the inclined plane: $\boldsymbol{F}_{gx}$

(14) the weight force of the suspended mass: F_g

(15) the angle of inclination of the inclined plane with the horizontal: θ.

These are a lot of parameters!

The Process of Elimination of Variables (1):

(1) We sacrifice some truth for the sake of mathematical manageability and assume that the pulley is massless and frictionless. This enables us to write:

$$F_{T1up} = F_{T2up} = F_{Tup} \quad \text{and} \quad F_{T1down} = F_{T2down} = F_{Tdown}$$

(2) We sacrifice some more truth and assume that the cord is inextensible or unstretchable. We then write: $\boldsymbol{a_1} = \boldsymbol{a_2} = \boldsymbol{a}$.

(3) We observe that h' must equal d! (Why?).

(4) We use our knowledge of trigonometry and write $h = d\sin\theta$. This eliminates the measurement of h.

(5) We observe that the theory involves ΔKE and not KE, just by itself. This is very exciting because we can arrange ΔKE to be zero without requiring KE to be zero. The way

to have $\Delta KE = 0$ is to let the roller move with *uniform* velocity. The roller will still be moving and will still have non-zero velocity v and non-zero kinetic energy KE; but because the initial and final kinetic energies will be the same, there will be no *change* in kinetic energy! This can easily be effected by selecting a suitable value of the suspended mass m. *This eliminates the need of knowing (and measuring) the velocity* $\boldsymbol{v}$.

(6) With $\boldsymbol{v}$ remaining uniform, the system will be in equilibrium. There will be no acceleration $\boldsymbol{a}$ either.

We now have 8 parameters.

The Process of Elimination of Variables (2):

Since ΔKE has been set to zero, Eqn (6) now reads:

$$\Delta PE_g = W_{nc} \qquad \text{..........(7)}$$

We shall now calculate the changes in the gravitational potential energy of our system, consisting of (i) the roller, and (ii) the suspended mass.

(a) Roller moves up the inclined plane with uniform speed.

The roller (M) gains potential energy, as it moves up through a vertical height h, but the suspended mass (m_{down}) loses potential energy as it moves downward through a vertical height h'. Remembering that $h = d\sin\theta$ and $h' = d$, we find that:

$$\Delta PE_g = Mgd\sin\theta - m_{down}gd$$

Inserting in Eqn (7), we get:

$$Mgd\sin\theta - m_{down}gd = W_{nc} \qquad \text{..........(8)}$$

(b) Roller moves down the inclined plane with uniform speed.

The roller now loses potential energy as it moves down through a vertical height h, and the suspended mass (m_{up}) gains potential energy as it moves upward through a vertical height h'. Remembering that $h = d\sin\theta$ and $h' = d$, we find that

$$\Delta PE_g = -Mgd\sin\theta + m_{up}gd$$

Inserting in Eqn (7), we get:

$$-Mgd\sin\theta + m_{up}gd = W_{nc} \qquad \text{..........(9)}$$

(c) The Cumulative Change in Potential Energy

From Eqns (8) and (9), equating the left hand sides, we get:

$$Mgd\sin\theta - m_{down}gd = -Mgd\sin\theta + m_{up}gd$$

Rearranging:

$$2(Mgd\sin\theta) = m_{up}gd + m_{down}gd$$

Cancelling out g and d from the two sides, we get:

$$2(M\sin\theta) = m_{up} + m_{down}$$

Letting

$$(m_{up} + m_{down})/2 = m_{av} \quad \text{..........(10)}$$

we get, as our final equation (and we switched sides too!)

$$m_{av} = M \sin\theta \quad \text{..........(11)}$$

This is the end of the simplification process. We eliminated many more parameters and are down to three parameters from fifteen. *Some simplification!*

Eqn (11) is our final equation. Not only that it is marvelously simple but it is also in the form of the equation of a straight line!

The steps of the experiment are now evident. We are guided by Eqn (11) to select a number of values of angle θ and for each θ, find m_{up} and m_{down} experimentally, for a *uniform* motion of the roller on the inclined plane. A graph of m_{av} against $\sin\theta$ will yield a straight line of slope M, the mass of the roller. This prediction is based on the validity of the law of conservation of energy (Eqn 4). Thus, if performed and the experiment did yield the value M for the mass of the roller, the law of energy conservation will stand verified! This indirect proof will be very satisfying as it demonstrates our skill in reducing the number of parameters from 15 to just 3 without putting any disfiguring constraints on the system!

Apparatus Required

(1) Inclined plane apparatus with magnetically supporting "wall" and the inclined plane with attached protractor and plumb line, to be supported magnetically on the "wall".

(2) Massless and frictionless pulley, to be magnetically supported on the "wall".

(3) Roller

(4) Two sets of masses

(5) Cord, mass holder and other accessories

Procedure

1) Set up the apparatus as demonstrated by the instructor.

2) Find the mass of the roller using the digital balance. Record its value as M.

3) Select the following angles: 15°, 20°, 25°.......... 70°.

4) For each angle, adjust the position of the pulley such that the cord is parallel to the inclined plane.

5) Place sufficient mass on the mass holder to enable the roller to move up with uniform speed. This mass, when added to the mass of the holder, is.m_{up} Determine its value using a digital balance and enter in the data table.The roller should move very slowly but steadily. One may give the roller a little push to get it started from rest.

6) For the same angle, next find m_{down} by adjusting the value of the suspended mass such that the roller moves down the inclined plane with uniform velocity. Find the total mass using the digital balance and record as m_{down}.

7) When all 12 trials have been completed, the experiment is also completed. Arrange all equipment and material neatly on the table.

Calculations and Graph

1) Find m_{av} for each trial, from the values of m_{up} and m_{down}, using Equation (18)

2) Find $\sin\theta$ for all 12 angles used in the experiment

3) Plot m_{av} on y-axis and $\sin\theta$ on x-axis. Draw a best fit straight line and find its slope. It should match M, as found in step (2) of procedure. Compare the two values and find percent error.

4) Complete the report with Results,

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Data & Data Tables

Name.................................... **Date**..............................

Instructor.............................. **Lab Section**...................

Partner.......................... **Table #**..........................

(a) Mass of the Roller M (g)

Please note that m_{down} and m_{up} must include mass of the mass holder and should be determined using a digital balance.

Table 1: Determination of Average Mass m_{av} for Different Angles of Inclination θ

#	θ (degrees)	m_{down} (g)	m_{up} (g)	m_{av} (g)
1	15°			
2	20°			
3	25°			
4	30°			
5	35°			
6	40°			
7	45°			
8	50°			
9	55°			
10	60°			
11	65°			
12	70°			

Additional information or data (if any):

An additional table, just in case.....

Mass of the Roller M (g)

Table 2: Determination of Average Mass m_{av} for Different Angles of Inclination θ

#	θ (degrees)	m_{down} (g)	m_{up} (g)	m_{av} (g)
1	15°			
2	20°			
3	25°			
4	30°			
5	35°			
6	40°			
7	45°			
8	50°			
9	55°			
10	60°			
11	65°			
12	70°			