Experiment # 12

Elasticity
Hooke's Law

Principle

Elasticity

Hooke's Law is all about the *third* basic property of material *things.* This property is *elasticity.* To just remind you, the first property is *inertia,* in lieu of the inertial mass that material things have. The second property is *attraction*, in lieu of the gravitational mass that the material things have. Interestingly enough, the third property, the subject of study of this experiment, is totally mass-independents.

Definition

All materials and substances get *strained* (change shape and size) when subjected to *stress* (synonymous with pressure). They remain so as long as the stress (pressure) is there. When the pressure is withdrawn, they returns to their normal, unstrained state.

All materials (and substances), no matter how rigid, have elasticity. They will *all* develop strain when subjected to sufficiently large stress (pressure). The role of *sufficiently large* pressure is not immediately obvious to all and sundry, and we tend to think that elasticity is a property of the so-called *elastic* objects only. Thus, for routine business, only things like rubber bands, springs and stretched surfaces (like those of drums, trampoline etc.) are elastic. Things like walls, concrete floors, road surfaces, on the other hand, are deemed inelastic and are commonly known as *rigid* bodies. Rest assured even the rigid objects become elastic in the face of humongous pressure. Consider people upstairs from us. When they hold a dancing session, our ceiling (their floor) vibrates. This may cause us not to share the same sentiments with them. Likewise when a loaded 18-wheeler passes by us, the so-called *rigid* road loses its rigidity

When exploiting this property for gainful purposes, we, naturally, choose those materials and substances that develop strain with a moderate amount of stress (pressure) We make *devices* from them. Some common devices are springs, rubber bands, bungee cords, shock-absorbers, surfaces of drums, tennis rackets, to name a few.

The **Modulus**

Elasticity of materials and substances (and not devices), is measured in terms of a *modulus*, called the *Modulus of Elasticity.* We write E for it. The modulus correlates the *stress* (pressure) to which materials and substances are subjected to, and the resulting *strain* that gets developed in them. It should be clearly understood that only *materials and substances* have the *modulus*. The devices don't. Devices have *elastic constants* that we shall explain later.

Let's write;

$$E = \frac{stress}{strain} \qquad \text{..........(1)}$$

Stress or *Pressure* is defined in physics as *force per unit area.* Thus

$$stress = F/A \qquad \text{..........(2)}$$

Strain, on the other hand is defined in physics as *change in shape relative to the original shape.* As objects can be 1-D, 2-D or 3-D, there will be different expressions for *strain*. Confining to a 1-D system, we shall define strain to be *change in length per unit original length* or

$$strain = \Delta l / l_o \qquad \text{..........(3)}$$

This sets

$$E = \frac{F/A}{\Delta l / l_o} \qquad \text{..........(4)}$$

<u>The law</u>

For materials and substances in the solid form, force is *the* practical thing to apply. Solving Eqn (4) for F, we get

$$F = \left(\frac{EA}{l_o}\right)(\Delta l) \qquad \text{..........(5)}$$

The three parameters, E, A, and l_o are all constants. Robert Hooke, who studied the phenomenon in the 17th century, was wise enough to suggest that the three constants be replaced by a single constant. For this single constant, he chose the letter K. As this particular elasticity is one dimensional, Hooke simplified Δl by writing x for it. With these changes, Eqn (5) became:

$$F = Kx \qquad \text{..........(6)}$$

Eqn (6) is known as *Hooke's Law.*

The constant K is known by several names, such as *force constant, Hooke's Law constant, elastic constant, spring constant*, *stiffness constant*, etc. The unit is *N/m*. We shall use the name *elastic constant*. The elastic constant K is what devices have. Each device will have one value of K. Devices made from the same material (of modulus E) will *all* have different values of K.

<u>Restoring Force</u>

When a force, (vide Eqn 6), is applied to an elastic device, the device develops an equal and opposite force, in accordance with Newton's third law. The reaction force gets applied to the agency that exerted the force on the device in the first place. When we stretch a rubber band, we feel a proportionate amount of pressure on our fingers. This is due to the reaction force. This reactionary force is called *restoring force* F', directed opposite to the applied force. We write:

$$F' = -Kx \qquad \text{..........(7)}$$

<u>*The* **Unstressed** *or the* **Mean** *Positions of Elastic Devices*</u>

When not subjected to a force, all devices stay in their so-called *unstressed* or *mean* position. For all 1-D devices, the stressing forces are always directed away from their unstressed positions and all restoring forces are always directed toward their unstressed position.

<u>Oscillations</u>

An elastic device may be set into simple harmonic oscillations. These oscillations are quite like those of a simple pendulum. Even though all elastic devices will oscillate or vibrate, we shall demonstrate the phenomenon for a coiled spring, in the vertical mode. A reasonable weight force is applied to the spring (by placing some mass on the mass holder) to stretch it by few centimeters, as shown in Fig (1b). This is the new mean position of the spring, to be called the *stretched mean position.* We then pull the mass holder some more, manually, downward, as shown in Fig (1c). When released gently, the spring oscillates about the *stretched mean position.* The extreme positions of oscillations are shown in Fig (1c) and (1d). The time of one oscillation is called the *time period* T, and is measured in seconds.

The amplitude of oscillation A, is shown by double headed arrows. Mathematical analysis shows that:

$$T = 2\pi\sqrt{\frac{m}{K}} \qquad \text{(sec)} \qquad \text{..........(8)}$$

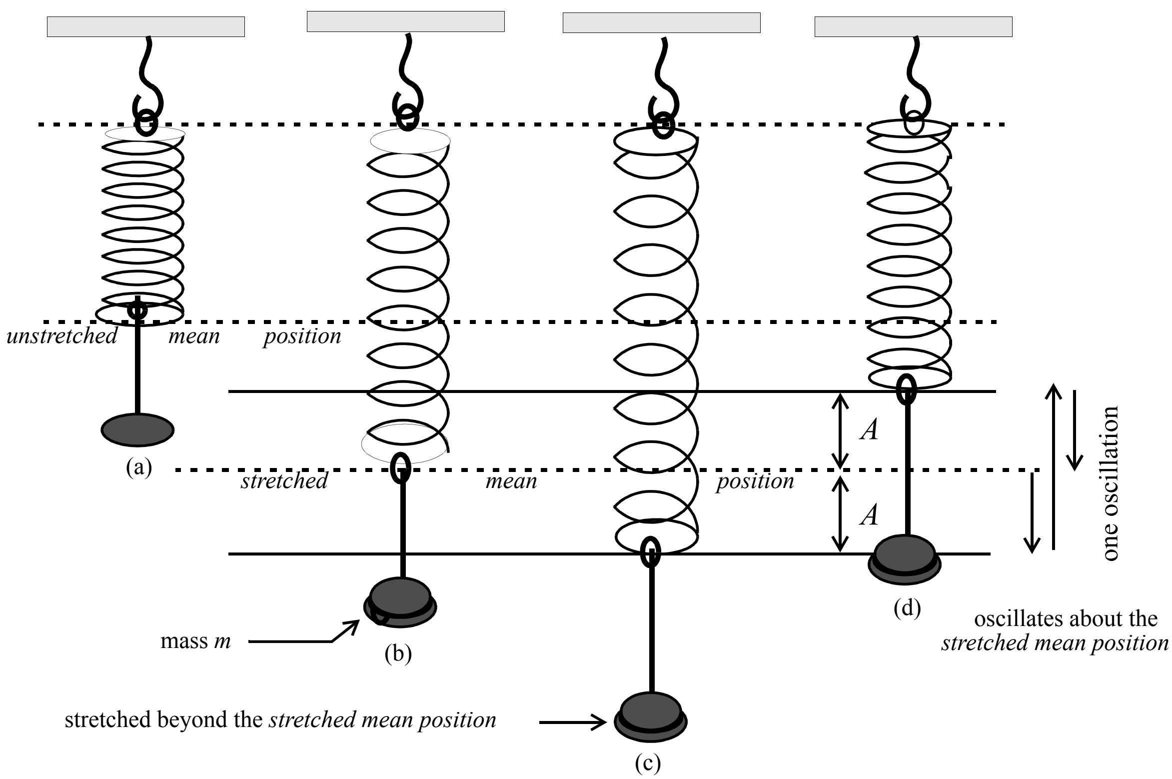

Fig (1) Oscillations of a coiled spring

One disadvantage of the vertical position is that the mass of the spring itself is active and adds to the mass placed on the holder. The effect is rather complicated. Consider dividing the spring into two halves. The mass of the lower half will exert a weight force on the upper half. But if we divide it into three parts then the mass of lower two-thirds will exert a weight force on the upper one-third of the spring. We may also say that the lower two-thirds pull on the upper one-third of the spring; and so on. Mathematical considerations lead us to an estimate: the effective mass of the spring that pulls on the spring is approximately one-third the mass of the spring. One should, therefore, add a third of the mass of the spring to the suspended masses. Writing m_{eff} for the fraction of the mass of the spring that pulls on the spring, we get:

$$T = 2\pi\sqrt{\frac{m + m_{eff}}{K}} \qquad \text{..........(9)}$$

We have an interesting situation here. If we ignore the mass of the spring (i.e. think of it as an ideal spring), we shall be entitled to the use Eqn (8); but if we do not ignore the said mass (i.e. prefer to use it as a real life spring), we shall be using Eqn (9).

Objectives of the Experiment

To determine the elastic constant K of the given spring in many different ways and compare them with one another.

Setting up

We are given a coiled spring as the elastic object for the experiment. It will be suspended from a vertical support and a mass hanger will be attached to it, at its lower end. From the earlier discussions, we find that the elastic constant K can be determined in three different ways. These are discussed below.

(1) Using the Direct Method

Direct method will amount to using the basic Hooke's Law equation, i.e. Eqn (6):

$$F = Kx \qquad \text{..........Eqn (6)}$$

This equation matches the equation of a straight line in the form $y = mx$, directly. The comparison prompts us to select different values of the elongation x and for each, find the corresponding value of the applicable force that would produce the said elongation. In our system of the vertically placed spring, the *applicable* force will be *weight force* F_g, obtained by placing masses on the mass holder. A graph of F_g Vs. x will yield a straight line. The slope of this straight line will be the required elastic constant K.

(2) Using the Oscillations Method, as Applied to an Ideal Spring

The oscillations method for ideal springs requires us to use Eqn (8). This equation does not match the equation of a straight line. In an effort to reshape it, we shall try taking logs. Separating the constants we get:

$$T = \left(\frac{2\pi}{\sqrt{K}}\right)(\sqrt{m})$$

Taking logs, we obtain:

$$\log T = \log\left(\frac{2\pi}{\sqrt{K}}\right) + \frac{1}{2}\log(m) \qquad \text{..........(10)}$$

Eqn (10) matches the equation of a straight line in the form $y = b + mx$. We are prompted to treat m as the independent variable and select different values for it. We are further prompted to find the time period of oscillations T for each m that we select.

The spring can be set into oscillations using the technique described above and the experiment can be performed. A plot of $\log T$ Vs. $\log m$ will yield a straight line graph with a slope of 0.50 and a y-axis intercept of $\log(2\pi/\sqrt{K})$. The value of the elastic constant K can be extracted from the y-axis intercept.

(3) Using the Oscillations Method, as Applied to a Real Life Spring

The oscillations method for real life springs requires us to use Eqn (9). This equation does not match the equation of a straight line either. Because of the distributed mass (under the radical sign) in the numerator, we cannot use the technique of taking logs. Following is the *brute force* method for forcing Eqn (5) to match the equation of a straight line.

(a) Square both sides to liquidate the *radical* sign:

$$T^2 = 4\pi^2\left(\frac{m + m_{eff}}{K}\right)$$

(b) Open the parentheses and separate the constants. Remembering that m_{eff} is also a constant, we get:

$$T^2 = \frac{4\pi^2 m_{eff}}{K} + \frac{4\pi^2}{K}(m) \qquad \text{..........(11)}$$

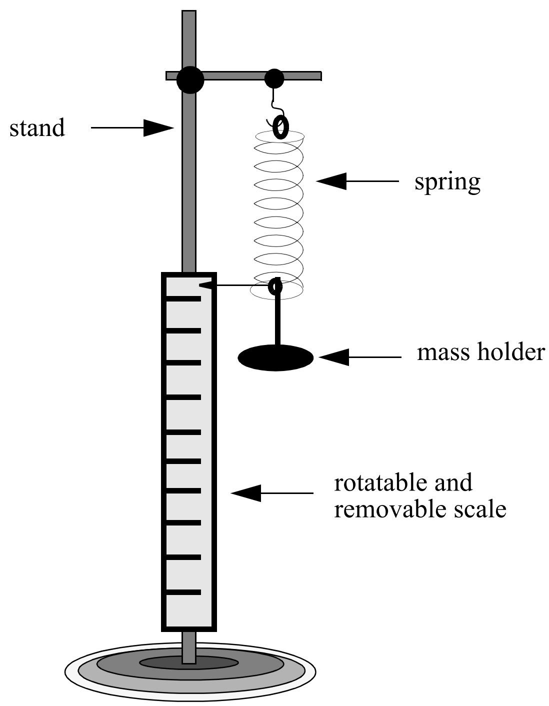

Fig (2) Hooke's Law Apparatus

Eqn (11) now matches the equation of a straight line $y = b + mx$. We are prompted to treat m as the independent variable and select different values for it. We are further prompted to find the time period of oscillations T for each m, experimentally.

It is obvious that no new data is required. We shall use the data obtained for the previous part to plot another graph (the third for this experiment) as per Eqn (11). Plotting T^2 against m, we expect to get a straight line with a slope of magnitude $(4\pi^2/K)$ and an intercept of magnitude

$(4\pi^2 m_{eff})/K$. We shall find a third value of the elastic constant K from the slope of this graph.

Can we find m_{eff} from this graph? Theoretically it is possible. Plug in the value of K as found here (or its average value) in the y-axis intercept and solve for m_{eff}. It should be understood however, that finding m_{eff} is not as easy as it appears here. This is because finding a very small mass (few grams) in a cluster of very large masses (several hundred grams) is not entirely logical. It is obvious that m_{eff} can very easily be *lost* in the acceptable margin of errors for the experiment.

The rationale for attempting to find m_{eff} is that we have all the necessary ingredients for calculating it. So why not try? If we get a sensible value, well and good. If not, we shall not be disappointed either.

Apparatus

(1) Hooke's Law apparatus, (spring with stand and scale)

(2) Mass-holder and set of masses,

(3) Electronic timer

(4) Photogate

(5) A digital balance capable of measuring to one hundredth of a gram.

Procedure

(A) Determining K Using the Direct Method:

(1) Find the mass of the spring m', using the digital balance and enter in the data sheet.

(2) Set up the apparatus, making sure that the spring and the weight holder are in the correct vertical position.

(3) Set the scale so that the base of the weight holder is at "zero" of the scale.

(4) Place enough mass on the mass holder such that the spring is elongated by $x_1 = 1.00$ cm, to the best of your visual judgement. You should observe the scale by bringing your eyes at the same level as the base of the mass holder.

(5) Determine the total mass: the mass placed on the mass- holder *and* the mass of the mass-holder itself, using the digital balance. Call it m_1 and record in the data sheet.

(6) Repeat steps (4) and (5) for elongations of the spring in the amounts: $x_2 = 2.00$ cm, $x_3 = 3.00$ cm,...... $x_{12} = 12.00$ cm. Let the corresponding masses be called m_1, m_2,..... m_{12}. Record in data sheet.

Note: Some digital balances can only handle 400 g at one time. If this be the case and if your mass is in excess of 400 g, break it up into parts, as necessary.

(B) Determining K, Using the Oscillations Method

(7) Remove the scale and fix the spring near the top of the stand.

(8) Set the electronic timer in the ***Hooke's Law*** mode (or the *pendulum* mode). Set it to measure the time periods of 15 successive oscillations. The timer also calculates the average time period which gets displayed when the *enter* key is depressed. Each value of time is measured up to one-tenth of a millisecond. All 15 values of the time period and the average value, are accessible individually and may be read and recorded (if so desired). Let the *average* time period be called T.

(9) Place masses: m_1, m_2,........... m_{12}, on the holder (one at a time) in amounts: 200 g, 240 g,.......... 640 g for 12 trials. Find the composite mass (mass of the mass-holder *plus* the mass placed upon it) in each case, using the digital balance.

(10) The *stretched mean position* of the loaded spring will be different for each mass. For each m adjust the position of the photogate window such that the pointer will cut the beam when at the *stretched mean position.* Having adjusted the height of the photogate, move it away to a side.

(11) Place the first prescribed mass on the mass-holder and set the spring in oscillations. As the spring begins to oscillate, the movement of the pointer will be somewhat erratic in the beginning but will settle down soon. Now bring the photogate and position it such that the pointer will interrupt the beam. The timer will measures the necessary values of time. When the timer has completed the necessary sequence of measurement, you may, stop the spring manually. It is recommended that you run through the measured values of time and see if they are consistent. If satisfied, record the average time period only and call it T_1 . There is no need to record all individual timings.

(12) Place the second prescribed mass on the mass-holder and repeat step 11. Record the time period as T_2 .

(13) Repeat step 12 for the rest of the prescribed masses and record values of time periods T_3, T_4,........ T_{12} .

(14) The experiment ends. Switch off the timer and disconnect its power supply. Arrange everything neatly on the table.

Calculations and Graph

CAUTION: Instruct the computer to print the equation of the straight line with at least five decimal digits, in all three graphs. Please use scientific notation.

(A) Determining K Using Direct Method:

1) Convert all x values to meters and all m values to Newtons (change to kilograms and then multiply by g, the acceleration due to gravity). Call these F_1 , F_2 ,......... F_{12}

2) Plot F values on y-axis and x values on the x-axis, using a computer. Instruct the computer to fit a straight line and display its equation. The slope, as read from the equation, is K_1, in N/m.

(B) Determining K treating the spring as an ideal spring

3) Convert all masses to kilograms

4) Find logs of (a) all T values, and (b) all m values.

5) Plot $\log T$ on y-axis and $\log m$ on x-axis, using a computer. Instruct the computer to fit a straight line and display its equation. Identify slope and intercept from this equation.

6) The *expected* value of the slope is 0.500. This may be compared with the *experimental* value of slope found in step (5), above. From the y-axis intercept, (also found in step 5), find the second value of the elastic constant K_2. Note that the y-axis intercept is related to K_2 by the following equation:

$$b = \log\left(\frac{2\pi}{\sqrt{K}}\right)$$

(C) Determining K, treating the spring as a real life spring.

7) Find T^2 for all T values.

8) Plot T^2 on y-axis and m on the x-axis, using a computer. Instruct the computer to fit a straight line and display its equation. Identify slope and intercept from this equation.

9) From the slope which equals $(4\pi^2/K)$, find the third value of the elastic constant, K_3.

10) Find average of all 3 values of K found so far. Call it K_{av}. Find percent deviation (same as percent difference) of all three K values with respect to K_{av}.

11) From the intercept which equals $(4\pi^2 m_{eff})/K$, find a value of m_{eff}, using K_{av} as K.

12) Multiply m' (as found in step 1 of Procedure) by 0.34. This is your expected value of m_{eff}. Enter in the appropriate section of the *Results* table. In the last column of the table in this section, circle either *yes* or *no,* using your judgement.

Note: If the intercept is negative, stop immediately and write that m_{eff} could not be found. Why?

Conclusions and Discussions

Write your conclusions from the experiment and discuss them. The Conclusion part of the report should be written carefully as it is important.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Data & Data Tables

Name...................................
Date.............................
Instructor.............................
Lab Section...................
Partner..........................
Table #..........................

Mass of the spring m' = (g) = kg)

Table 1: Determining K using Eqn (2)

Trial #	Elongation of the spring x (cm)	Elongation of the spring x (m)	Composite mass m (g)	Composite mass m (kg)	Force (N) multiply mass (kg) from previous column by g = 9.8 m/s^2
1	1.0				
2	2.0				
3	3.0				
4	4.0				
5	5.0				
6	6.0				
7	7.0				
8	8.0				
9	9.0				
10	10.0				
11	11.0				
12	12.0				

Additional information or data (if any):

Table 2: Determining K Using the Method of Oscillations

Trial #	Mass on mass-holder (g)	Total mass: m (mass on mass-holder) + (mass of mass-holder) (g)	Total mass m (kg)	Time Period T (sec)
1	200			
2	240			
3	280			
4	320			
5	360			
6	400			
7	440			
8	480			
9	520			
10	560			
11	600			
12	640			

Additional information or date (if any):

Name *Partner's name*

Results

Determining the Elastic Constant of the Given Spring

A. Using the Direct Method

K_1 N/m

B. Using the Oscillations Method, Treating Spring as an Ideal Spring

K_2 N/m

C. Using the Oscillations Method, Treating Spring as a Real Life Spring

K_3 N/m

D. Average value of K

K_{av} N/m

E. Deviations from Average

The Various K Values

	The K values	Average K value	% deviation
K_1			
K_2			
K_3			

F. The Authenticity Test

The Quadrature of Mass and Time Period

	Expected	Experimental	% error
power of m	0.500		

G. The Effective Mass of the Given Spring

The Effective Mass

	Expected	Experimental	comparable?
m_{eff}			yes / no