

## *Experiment # 11*

# Rolling Friction The Coefficient

### Principles

#### Definition & Basic Properties

Please refer to Experiment # 10, (Kinetic Friction, The Coefficient), for full discussion of the principles. In this experiment we shall study the role of *Static Friction* in the *rolling* motion of rollable objects.

#### Rolling Objects & Friction

When a rigid rollable object (ring, disk, cylinder, sphere) rolls on the rigid surface of another solid (such as table top, floor, inclined plane, rounded surface of a roller coaster, surface of a cylinder or sphere), only one point of the rolling object comes in a momentary contact with only one point of the surface upon which it rolls. The two points stay at rest with respect to one another for the tiny fraction of one second. As they move on, another pair of points, one from each solid, comes in momentary contact with one another at which moment the two stay at rest. Thus the friction between the two solids is of the *static* kind and not of the *dynamic* or *kinetic* kind even though the two-body system is in a dynamic state.

#### The Role of Static Friction: To Cause Objects to Roll

The friction that causes objects to roll rather than slide, is static friction. Static friction acts to facilitate rather than impede rolling motion. In fact it is correct to say that objects roll because of static friction. If there was no static friction, objects would rather slide than roll. Yet static friction does not do any work on the rolling object and the force of static friction is not part of any force equation.

#### The Almost Always **Left Out** Fact of Life

To almost all textbooks and to all introductory level physics students, rolling objects are immune to the ill effects of friction. Thus a rolling object must keep rolling indefinitely. But it is a fact of life (and even a child will tell you) that a marble, rolling on a hard (and flat) floor does not keep rolling indefinitely and does come to a stop, sooner or later.

### Friction of the Third Kind: Rolling Friction

Professor Eugene Hecht talks about *Rolling Friction* in his textbook *Physics Algebra/Trig*, 2nd Edition. *The minute deformation of real world objects when they come in contact with one another*, the Professor explains, *causes some friction, even when rolling*. He then introduces the equation:

$$F_r = (\mu_r)(F_N) \quad \text{.....(1)}$$

### The Role of Rolling Friction: To Cause Objects Not to Roll

Rolling friction, in accordance with Eqn (1), acts to obstructs rolling motion. In this role, it is akin to kinetic friction; only much weaker, magnitude-wise. The discussion and use of rolling friction are usually left out of textbooks.

### Demonstrating the Existence of Rolling Friction

As frictional forces impede motion, we expect objects to travel faster when going up and slower when coming down. The deceleration of the object (rolling or sliding) when going up is greater than its acceleration when coming down.

If we project a block of wood up an inclined plane, a little attention will cause us to notice that the upward motion was quicker while the downward motion took longer. For a rolling object this difference in time is so small that, most likely, it will go unnoticed.

### Objective of the Experiment

*To Determine the Coefficient of Rolling Friction for the given car or cart*

### Setting Up

#### Using an Inclined Plane

One of the ways in which the coefficient of rolling friction can be determined is to project the rolling object up an inclined plane. It will decelerate as it goes up, stop momentarily and then roll downward. It will accelerate as it travels down the inclined plane. This technique will also demonstrate convincingly, the little known fact that the *upward motion is quicker and the downward motion is slower*

Consider an inclined plane, inclined to the horizontal at some angle  $\theta$ . As the object rolls upward, it is pulled down by both, the (partial) weight force and the force of friction, as shown in Fig (1a) below. As there is no up-directed force, the net downward force is:

$$F_{net} = F_{gx}' + F_{fr}' \quad \text{.....(2)}$$

$$F_{net} = mg \sin \theta + \mu mg \cos \theta \quad \dots\dots(3)$$

Setting  $F_{net}$  equal to  $ma_{up}$  (Newton's Second Law of Motion, we get:

$$|a_{up}| = g \sin \theta + \mu g \cos \theta \quad \dots\dots(4)$$

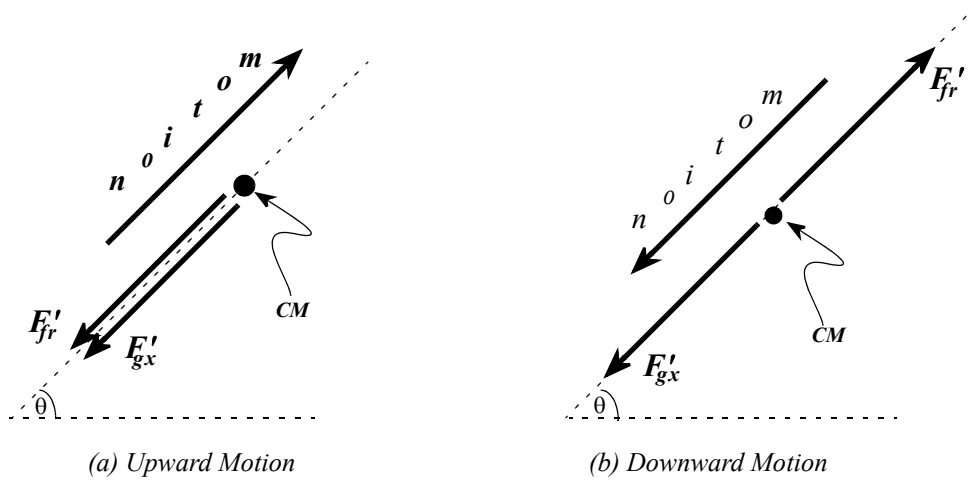


Fig (1) Force Diagram For the (a) Upward Motion, and (b) Downward Motion

As the object rolls downward the two forces act opposite to one another. The partial weight force pulls the object down but the force of friction impedes its downward motion by acting opposite the direction of motion. This is shown in Fig (1b).

$$F_{net} = F_{gx}' - F_{fr}' \quad \dots\dots(5)$$

$$F_{net} = mg \sin \theta - \mu mg \cos \theta \quad \dots\dots(6)$$

Setting  $F_{net}$  equal to  $ma_{up}$  (Newton's Second Law of Motion), we get:

$$|a_{down}| = g \sin \theta - \mu g \cos \theta \quad \dots\dots(7)$$

**Testing the Theory**

In what follows, we shall leave out the negative sign of  $a_{up}$  and use its magnitude only.  $a_{down}$  is always positive.

From Equations: (4) and (7), by adding, we get

$$(a_{up} + a_{down}) = 2g \sin \theta \quad \dots\dots(8)$$

Eqn (8) indicates that a graph of  $\frac{(a_{up} + a_{down})}{2}$  against  $\sin \theta$  will yield a straight line of slope equal to  $g$

Such a graph, if it yields a value of “g” with an accuracy of better than 99%, should be a convincing proof of the validity of the experiment as a whole and should validate other results, obtained from the same data.

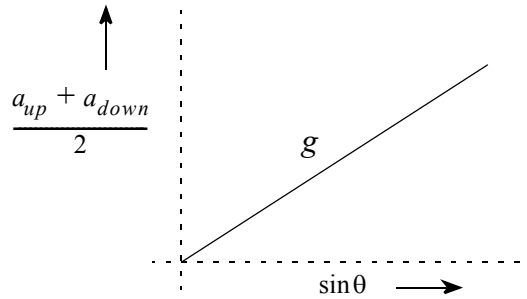


Fig (3) The “Acid Test”: Do We? or Don’t We?

Finding the Coefficient  $\mu_R$

From the same two equations, by subtracting, we get:

$$(a_{up} - a_{down}) = 2\mu g \cos\theta \tag{9}$$

Eqn (9) indicates that a graph of  $(a_{up} - a_{down}) / 2g$  against  $\cos\theta$  will yield a straight line of slope equal to  $\mu_R$ .

It turns out, however, that the graph does not produce a straight line. This is partly because  $\mu_R$  has a very small value and partly because the force of friction may differ from one trial to the next in an irregular manner.

Additionally, we find that there is a great disparity in the magnitudes plotted on x- and y-axes. It should be noted that  $(a_{up} - a_{down})$  is, in itself, a very small quantity and then we divide it by  $2g$ . Thus the numbers plotted on y-axis are very much smaller than those plotted on the x-axis.

In an attempt to reduce the great disparity in the magnitudes of numbers plotted on x- and y-axes, we proceed as follows:

Divide both sides of Eqn (4) by  $g \sin\theta$  to get:

$$\frac{a_{up}}{g \sin\theta} = 1 + \mu \frac{\cos\theta}{\sin\theta} \tag{10}$$

Next divide both sides of Eqn (7) by  $g \sin\theta$  to get:

$$\frac{a_{down}}{g \sin\theta} = 1 - \mu \frac{\cos\theta}{\sin\theta} \tag{11}$$

Subtracting Eqn (11) from Eqn (10), we get:

$$\text{Or} \quad \frac{a_{up}}{g \sin \theta} - \frac{a_{down}}{g \sin \theta} = 2\mu \frac{\cos \theta}{\sin \theta}$$

$$\frac{(a_{up} - a_{down})}{2g \sin \theta} = \mu \cot \theta \quad \dots\dots\dots(12)$$

The trick seems to work. We get a reasonable straight line with an acceptable value of  $r^2$ , **and**, a reasonable value of  $\mu_R$ !

### The Role of Smart Timer & the Acceleration Plate

This combination is sensitive enough to detect minute differences in the accelerations (and speeds) not only for sliding objects travelling up and down an inclined plane, but for rolling objects as well.

What is more creditable is the fact that the timer is sensitive enough to small changes in angles of inclination. In order to obtain (say) ten data points for plotting a graph, we need not choose angles in  $4^\circ$  or  $5^\circ$  steps! A change of elevation of one part in one-hundred is effectively resolved by the timer.

The smart timer and the acceleration plate combination permits students to determine coefficient of rolling friction in an introductory level physics laboratory, using a simple set-up.

### Apparatus

- (i) Track,
- (ii) Hall's Car,
- (iii) Smart Timer,
- (iv) Variable Gate Photogate,
- (v) Acceleration Plate,
- (vi) Miscellaneous articles, such as meter stick, ruler, small block of wood, etc.

### procedure

1. Set up the apparatus as shown.
2. Select height  $h$  as 6 cm . 7 cm ,..... 20 cm for 15 trials.
3. Set the timer in **Gate** mode. Set *memories* to 16.
4. For each angle, press the *start* key on the timer and then project the cart upward **gently** so that it passes through the photogate. As it comes to a stop, hold it to stop it from going down. Take care to minimize vibrations.
5. Press the *calc* key on the timer and check for coefficient  $x_0$  to be zero and  $r^2$  to be 1.0000. If the two values are respectively *zero* and *one*, record the value of half-acceleration.

If  $r^2$  is not equal to 1.0000, repeat the trial. Make sure that the car travels in a straight line and doesn't wobble. If  $r^2$  is consistently different from 1.0000, we may accept a value of 0.9999. If the coefficient  $x_o$  is consistently not zero but  $r^2$  is indeed 1.0000 (or 0.9999), we may accept some very small values for  $x_o$ , such as 0.0001, 0.0002 or 0.0003.

6. Repeat 5 times. Different values of half-acceleration will be found. These should be close together. We shall select the **largest** value. Call it  $a_{\text{up}}/2$ . Ignore the negative sign.
7. Now let the cart roll down on its own from some distance up on the track above the photogate. After it has passed through the photogate, gently stop it. If not stopped promptly, it will fall off the track. Use the criterion of step 5, to accept the result of the trial.
8. Repeat 5 times. Different values of acceleration will be found. These should be close together. We shall select the **smallest** value. Call it  $a_{\text{down}}/2$
9. Raise the track by one centimeter and repeat the above.

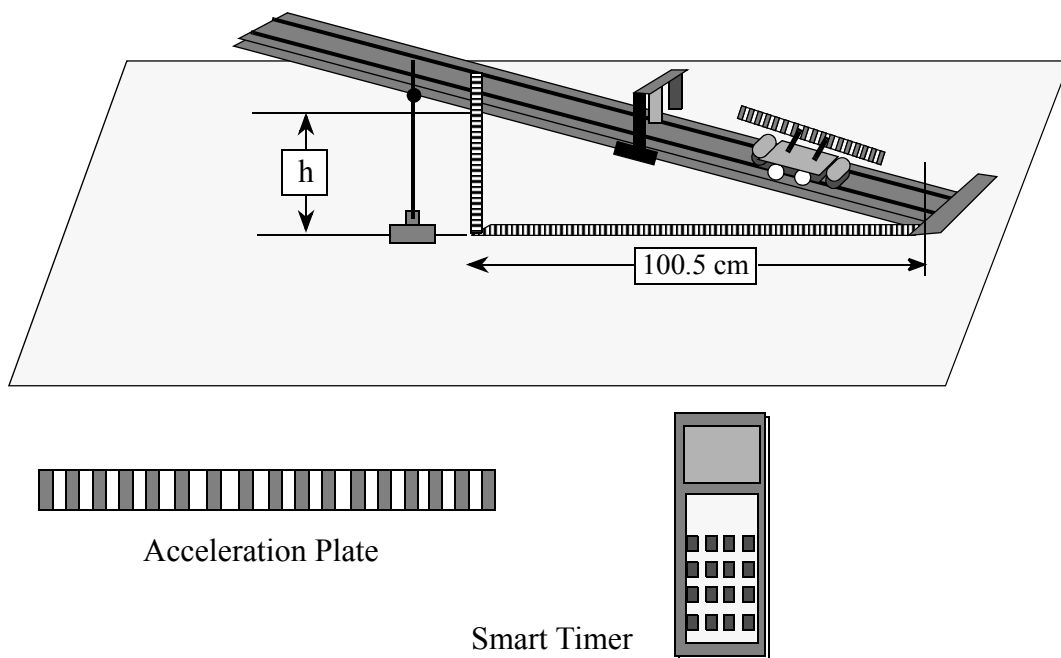


Fig (4) Setting up the Apparatus

### Calculations & Graphs

1. Calculate  $\angle\theta$  as  $\arctan\left(\frac{h \text{ cm}}{100}\right)$ . Keep 5 decimal places. Enter in the data table.
2. Calculate  $\sin\theta$ ,  $g\cos\theta$ , and  $\cot\theta$  for all 12 angles (using computer).
3. Multiply the selected values of half-accelerations by 2 and enter in proper columns in Table 2.
4. Calculate  $\left(\frac{a_{\text{up}}}{2} + \frac{a_{\text{down}}}{2}\right)$  for all 15 angles. Plot on y-axis against  $\sin\theta$  (x-axis) Instruct the computer to fit a straight line and print its equation with five decimal digits. Do not forget  $r^2$ . You should get a slope of magnitude 9.80 with an error margin of one-half of one percent *or less!*
5. Calculate  $\left(\frac{a_{\text{up}}}{2} - \frac{a_{\text{down}}}{2}\right)$  for all 15 angles. Divide all these values by  $g\sin\theta$  and plot on y-axis. Plot corresponding  $\cot\theta$  values on the x-axis. Instruct the computer to fit a straight line and print its equation with five decimal digits. Do not forget  $r^2$
6. Compile *Results*.

### Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

### What Did You Learn in this Experiment?

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given





### Data & Data Tables

Name.....

Date.....

Instructor.....

Lab Section.....

Partner.....

Table #.....

The adjacent side is 100 cm.

Ignore the negative sign of  $a_{\text{up}}/2$ . Enter magnitudes only in all data and calculation tables

**Table 1. Angles & Half-Accelerations**

#	$h$ (cm)	$\angle\theta$	$a_{\text{up}}/2$ (m/s <sup>2</sup> )	$a_{\text{down}}/2$ (m/s <sup>2</sup> )
1			1	1
			2	2
			3	3
			4	4
			5	5
2			1	1
			2	2
			3	3
			4	4
			5	
3			1	1
			2	2
			3	3
			4	4
			5	5
4			1	1
			2	2
			3	3
			4	4
			5	5
5			1	1
			2	2
			3	3
			4	4
			5	5

**Table 1. Angles & Half-Accelerations**

#	$h$ (cm)	$\angle\theta$	$a_{\text{up}}/2$ (m/s <sup>2</sup> )	$a_{\text{down}}/2$ (m/s <sup>2</sup> )
6			1 2 3 4 5	1 2 3 4 5
7			1 2 3 4 5	1 2 3 4 5
8			1 2 3 4 5	1 2 3 4 5
9			1 2 3 4 5	1 2 3 4 5
10			1 2 3 4 5	1 2 3 4 5
11			1 2 3 4 5	1 2 3 4 5
12			1 2 3 4 5	1 2 3 4 5

**Table 1. Angles & Half-Accelerations**

#	$h$ (cm)	$\angle\theta$	$a_{\text{up}}/2$ (m/s <sup>2</sup> )	$a_{\text{down}}/2$ (m/s <sup>2</sup> )
13			1 2 3 4 5	1 2 3 4 5
14			1 2 3 4 5	1 2 3 4 5
15			1 2 3 4 5	1 2 3 4 5

