Experiment # 9

Newton's Second Law of Motion
Atwood's Machine in x-Mode - Work-Energy Theorem

Principle

The Second Law

The principles have been adequately explained in Experiment #8 and need not be repeated here. We refer you to Experiment #8 for necessary details. We shall reproduce the equation for Newton's Second Law here for ready reference.

$$\boldsymbol{F}_{net} = M\boldsymbol{a} \qquad \text{..........(1)}$$

The unit of force is *kg m/s²*. A compact unit of force is *Newton*. One Newton (written as *N*) is that force that produces an acceleration of 1.00 m/s^2 in an object of mass 1.00 kg.

Objective of the Experiment

To verify Newton's Second Law of Motion and the Work-Energy Theorem

Setting up

It is interesting to know that one doesn't need Newton's second law to determine the acceleration of an object. The kinematic equations, as we know, provide us with ample means to determine the acceleration. Kinematic equations use velocities of material objects, their displacements, and the time of travel, to determine the accelerations of that object. There is no mention of force and no mention of mass either. Newton's second law, on the other hand, finds acceleration from forces and masses alone! These two radically different ways of finding the acceleration afford us an excellent opportunity for the verification of the law: determine the acceleration using kinematics in the laboratory and see if it is in agreement with the value calculated from the Second Law. The reason we choose kinematics for the laboratory is entirely because it is easier to handle.

Verification in the z-mode

The Atwood's Machine (Historical Background)

The conventional approach to the verification of Newton's second law of motion is to use what is known as an *Atwood's machine*; shown below:

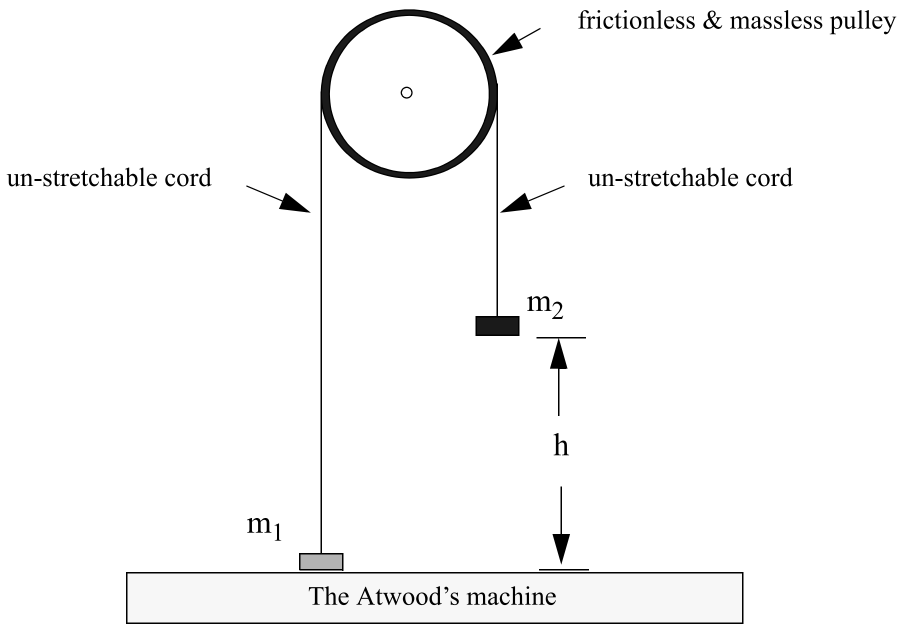

Fig (1) The Atwood's Machine

It consists of a pulley and two unequal masses. The masses are attached to the two ends of a cord which runs over the pulley. The larger mass moves downward, thereby pulling the smaller mass upward. One measures the time of fall of the larger mass as it falls through a preselected height. This enables one to determine the acceleration of the system experimentally, independently of any law of motion.

In an Atwood's machine the accelerated motion takes place (by design) in the z-mode, where there is no friction and the effect of air resistance is minimal. This greatly simplifies mathematical analysis. The system is analyzed using Newton's Second Law of Motion and an expression for $\boldsymbol{F}_{net}$, acting on the system, is found. This expression then *predicts* the acceleration of the system. At the same time, the experimental value of acceleration is determined using kinematics in the z-mode. One then looks for a match between the experimental and the predicted values of the said acceleration.

Shortcomings of Atwood's Machine

The method is rather crude and the results do not reflect a truly satisfying verification. This is mainly due to our inherent inability to synchronize a timer with the start and stop of the process. For every person there is a *reaction* time that elapses between the person ***sensing the need for action*** and actually ***acting***. This time varies from person to person. The reaction time may be as short as 0.3 second for one person and may be as large as one second for another. Again, the same person may be quicker (than average) at one time and slower at another. Because the acceleration of the system is proportional (inversely) to the ***square*** of time, a small error in time will propagate and produce large error in the magnitude of acceleration! The use of a digital photogate timer does not afford much help here because of its lack of adaptability to the Atwood's machine system.

The rotational inertia of the pulley is another disturbing factor. For the two suspended masses not to collide in mid air, the pulley has to be large and large pulleys have to be strong to withstand the weight of several hundred grams. Large enough pulleys, therefore, will not have a vanishingly small mass. The inertia of the pulley will have to be taken into account.

Verification in x-Mode

Like the z-mode, if friction-free environment were present in the x-mode, an accelerated motion here would be equally welcome. With the advent of linear air tracks, it is now possible to have friction-free motion in the x-mode.

Consider a glider that glides on a linear air track, such as the one shown in Fig (2). One can safely ignore friction and assume that the effect of air resistance is minimal. Cords are tied to each end of the glider and passed over pulleys. These pulleys need not be large and strong. The other ends of the cords are tied to mass holders that carry unequal masses. The larger mass pulls the glider and the glider acquires an accelerated motion in the x-mode.

One photogate timer is required. The glider is moved a convenient distance to the left of the photogate timer and released gently from rest. As the flag of the glider passes through the photogate windows, the timer records the time that the flag took to pass through the window. This is all we need. The glider is stopped manually, quickly. Remember the motion is accelerated; if not stopped in time, the glider will hit the end bumper of the air track and may cause some damage. Similarly, the mass-holders will hit (respectively) the floor and the left side pulley, causing the masses to fly around in the room wildly.

Mathematical Analysis

Having set up an x-mode accelerated system (the glider), we proceed with the mathematical analysis. The system consists of (a) glider with hooks and flag, (b) two masses with mass holders, (c) cords, and (d) pulleys.

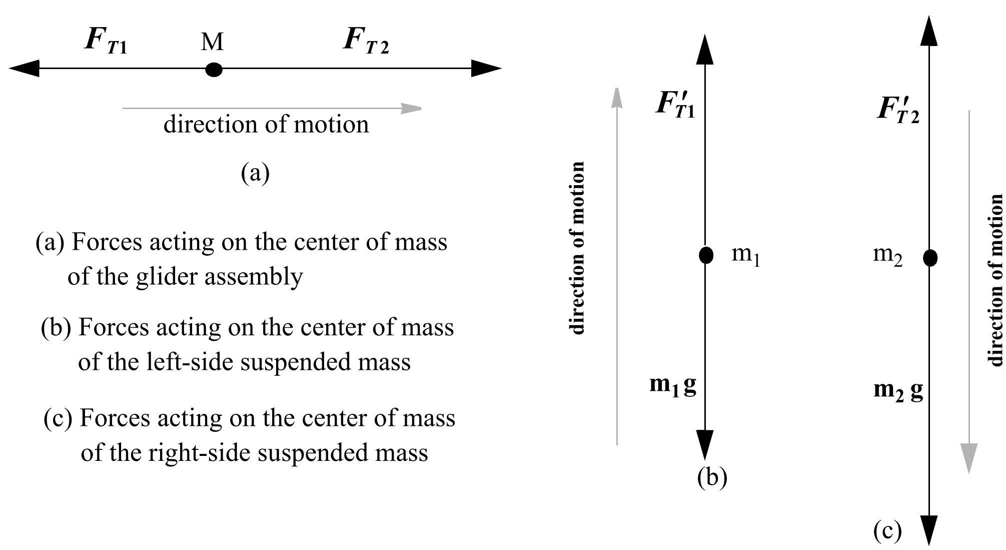

Fig (3) The Force Diagrams

Step one is to sacrifice some truth to gain mathematical manageability and assume that the cords are massless and that the pulleys are not only massless but frictionless as well. The pulleys, used in this experiment, are really light-weight and are set on ball bearings. The cords that we use are specially made for laboratory work but we must keep in mind that they are not really unstretchable. The simplifying assumptions leave us with three moving parts: the glider and the two masses. Assuming that each is a point mass, we draw force diagrams for each. These are shown in Fig (3). Let the mass of the glider assembly (i.e. glider, the hooks and the flag) be called M; that of the left side mass and the mass holder be named m_1 and that of the right side mass and holder be named m_2.

(a) From the force diagram of M, we deduce (in accordance with Newton's second law):

$$\boldsymbol{F_{T2}} - \boldsymbol{F_{T1}} = M\boldsymbol{a} \qquad \text{..........(2)}$$

(b) From the force diagram for m_1 we find:

$$\boldsymbol{F'_{T1}} - m_1\boldsymbol{g} = m_1\boldsymbol{a_1} \qquad \text{..........(3)}$$

(c) From the force diagram for m_2, we similarly find:

$$m_2\boldsymbol{g} - \boldsymbol{F'_{T2}} = m_2\,\boldsymbol{a_2} \qquad \text{..........(4)}$$

Because the pulleys are massless and frictionless,

$$\boldsymbol{F'_{T1}} = \boldsymbol{F_{T1}} \quad \text{and} \quad \boldsymbol{F'_{T2}} = \boldsymbol{F_{T2}} \qquad \text{..........(5)}$$

Because the cord is in-extensible,

$$\boldsymbol{a_1} = \boldsymbol{a_2} = \boldsymbol{a} \qquad \text{..........(6)}$$

This leaves us with:

$$\boldsymbol{F_{T1}} = m_1\boldsymbol{g} + m_1\boldsymbol{a} \quad \text{and} \quad \boldsymbol{F_{T2}} = m_2\boldsymbol{g} - m_2 a \qquad \text{..........(7)}$$

$$\boldsymbol{F_{T2}} - \boldsymbol{F_{T1}} = (m_2 - m_1)\boldsymbol{g} - (m_1 + m_2)\boldsymbol{a} \qquad \text{..........(8)}$$

Plug in this value of $\boldsymbol{F_{T2}} - \boldsymbol{F_{T1}}$ in equation (6),we get:

$$(m_2 - m_1)\boldsymbol{g} - (m_1 + m_2)\,\boldsymbol{a} = M\boldsymbol{a} \qquad \text{..........(9)}$$

Re-arranging,

$$\boldsymbol{a} = \frac{(m_2 - m_1)g}{(m_1 + m_2 + M)} \qquad \text{..........(10)}$$

This is our final equation for calculating the acceleration of the glider, using the Second Law. As for the Kinematics, we simply recall the time independent kinematic equation:

$$\boldsymbol{d} = \frac{1}{2a}(v^2 - v_o^2)$$

$$\boldsymbol{a} = \frac{1}{2d}(v^2 - v_o^2) \qquad \text{..........(11)}$$

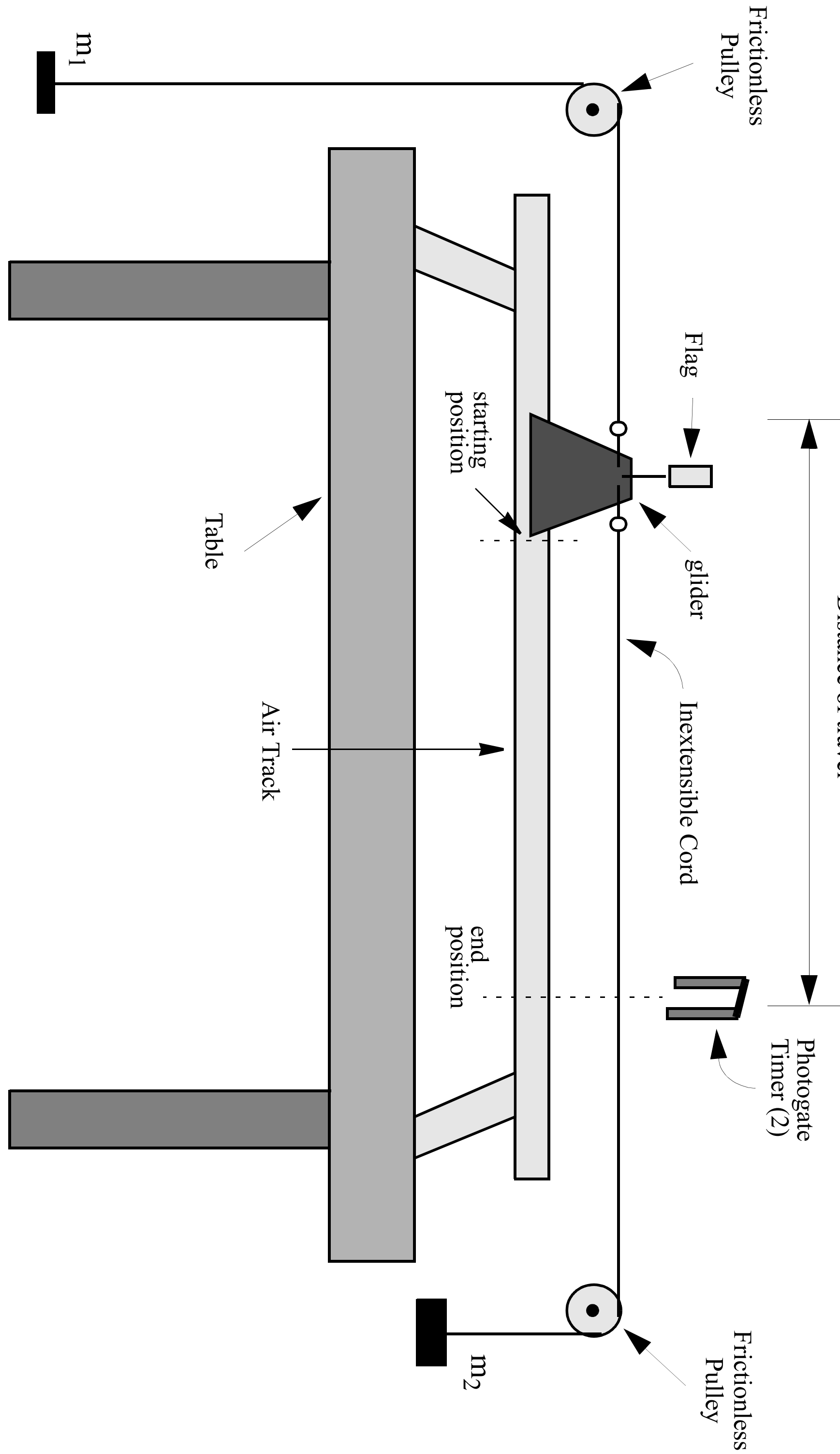

Fig (3) Horizontally Accelerated System

Verifying the Law By Comparing a_{kin} *and* a_{dyn}

As we let our glider start from rest, v_0 will be zero and Eqn (11) will reduce to:

$$a = \frac{v^2}{2\boldsymbol{d}} \qquad \text{..........(12)}$$

From the recorded values of the time that the flag takes to move past the photogate window, we find the final velocity of the glider. $\boldsymbol{v}$. Knowing the displacement d we calculate the acceleration of the system, using Eqn (12). This is the *kinematic* value of $\boldsymbol{a}$. Its value, in accordance with the Second Law, is found from Equation (10) and hence is its *dynamic* value. An agreement in the two values will establish the validity of the Second Law.

This procedure should be repeated several times to achieve a satisfactory verification. This can be done by selecting a number of values of m_1 and m_2. A plot of kinematic values of a against the dynamic values should yield a straight line of slope unity. There are a number of ways in which the values of m_1 and m_2 can be selected.

Classical Atwood's machine experimenters chose their masses (i) by keeping the sum of the two masses constant, and (ii) by keeping their difference constant. In order to understand the wisdom of their choice, we proceed as in the next section.

Verifying the Law By Plotting A Different Graph

Equating the right hand sides of Eqns (10) and (12) we get:

$$\frac{v^2}{2\boldsymbol{d}} = \frac{(m_2 - m_1)g}{(m_1 + m_2 + M)}$$

$$v^2 = \frac{(m_2 - m_1)2dg}{(m_1 + m_2 + M)}$$

$$\frac{1}{v^2} = \frac{M + m_1 + m_2}{(m_2 - m_1)2dg} \qquad \text{..........(13)}$$

$$\frac{1}{v^2} = \frac{M + m_2}{(m_2 - m_1)2dg} + \frac{m_1}{(m_2 - m_1)2dg} \qquad \text{..........(14)}$$

It is obvious that the only way for us to match Eqn (14) t with the equation of a straight line, is to express one of the masses in terms of the other mass. This explains the wisdom of the classical experimenters.

Let us choose to keep the difference of the two masses constant. To this end we choose m_2 to be m grams greater than m_1. Thus $m_2 = m_1 + m$, where m is constant, leaving m_1 and m_2 to remain variables. This choice makes

$$m_1 + m_2 = 2m_1 + m \qquad \text{and} \qquad m_2 - m_1 = m$$

Inserting these values in Eqn (13), we get

$$\frac{1}{v^2} = \frac{M + 2m_1 + m}{(2d)(mg)}$$

$$\frac{1}{v^2} = \frac{M+m}{(2d)(mg)} + \frac{m_1}{(mg)(d)} \qquad \text{..........(15)}$$

Plotting $1/v^2$ on y-axis against m_1 (on x-axis), we should get a straight line of slope

$$\frac{1}{(mg)(d)} \qquad \text{..........(16)}$$

and a y-axis intercept of

$$\frac{M+m}{(2d)(mg)} \qquad \text{..........(17)}$$

The intercept is easily and quite obviously explained in terms of work. Even though we have not studied work at this time, it will be totally unwise to miss out a golden opportunity of proving principles of work and energy. The expression $(mg)(d)$ represents the work done by the weight force of the mass-difference of the two masses, on the system. The weight force of the larger mass does positive work while that of the smaller mass, does negative work. The net work done, therefore, is the work done by the weight force of their mass-difference. Tension forces, while pulling and pushing the glider assembly (mass M) do equal and opposite work. Their work, therefore, doesn't count.

The reciprocal of the slope, Eqn (16), gives us the "total work done on the whole system". The unit of work is *Joule* and we write *J* for it.

To interpret the intercept, we set m_1 to zero. Then the only mass whose weight force will act on the system will be m. We shall get:

$$\frac{1}{v^2} = \frac{M+m}{(2d)(mg)} = \frac{M+m}{2(mgd)}$$

Rearranging, we get:

$$(1/2)(M+m)(v^2) = mgd \qquad \text{..........(18)}$$

Eqn (18) is a wonderful and unexpected verification of the Work-Energy Theorem that also we have not studied yet. According to this theorem total work W_{net} done on a system by *all* the forces, equals the change in kinetic energy of the whole system. Realizing that the tension force F_T does not do any work, as the unneighborly cord passes over an unneighborly pulley, the only force that is doing work is the weight force of the (only) suspended mass m. This force does positive work through the distance d. Thus the net work done is $(mg)d$, the right hand side of Eqn (18). The left hand side is the kinetic energy of the whole system (includes both masses). You must understand that the mass of the glider assembly M is not the least bit involved in work but it certainly wants its share in the outcome of the work, the kinetic energy.

From all this enlightening discussion we learn that the (final) velocity v of the system (starting from rest), at the end of the d meters of travel, when the only suspended mass is m, will be given by the radical of the reciprocal of the y-axis intercept:

$$v_f = \sqrt{\frac{2(mgd)}{M+m}} \qquad \text{..........(19)}$$

It is not only interesting but highly educative to observe that no matter what masses we choose, the net work done on the system remains constant, at $W_{net} = mgd$ and that the kinetic energy of the system remains constant as well. If m_1 increases, v decreases accordingly.

Apparatus Required

(1) Linear air track
(2) Air source
(3) Glider with flag
(4) Electronic time
(5) Two photogates
(6) Accessories: masses, mass holder, un-stretchable cord, vernier calipers, etc.

Procedure

The apparatus is set on the tables. Please check the following;

(a) The air track is horizontal. This may be checked by setting the glider at rest while the air is flowing. If the glider stays at rest or does not move significantly then the track is assumed to be horizontal.

(b) To the best of your visual judgement, the two cords are horizontal.

(c) The photogate window is set such that the infrared beam is interrupted by the cylindrical part of the flag. This will eliminate any error due to the flat part of the flag not being parallel to the track.

Following steps are recommended:

It has been noticed that the results are very sensitive to the values of the masses. Hence it is suggested that the masses m_1 and m_2 be determined always, using a digital balance only. The balance should be capable of reading up to one-hundredth of one gram.

(1) Measure the diameter of the cylindrical part of the flag using the digital vernier callipers. Make three measurements for three different rotational positions of the flag and calculate the average diameter.

(2) Find the mass of the glider assembly (the glider, the two hooks and the flag), using a digital balance.

(3) Unless otherwise instructed, select a distance of 80 cm in advance of the photogates. This is d. Select the starting position of the glider as the 50 cm mark (or the 150 cm mark) on the air track. To set the position of the photogate, (i) align the right hand side edge of the glider (or its left hand edge) at the 130 cm mark (or the 70 cm mark) on the air track. (ii) move the photogate window to approach the flag from the right hand side, (or the left hand side) and freeze it the instant the timer begins measuring time.

(4) Set the electronic timer to the ***gate*** mode. In this mode the timer will record the time the flag takes to pass through the photogate window. The time will be measured to one-tenth of a millisecond.

Following instructions apply to an air track that has air supply at the left hand side. If you are on a table where the air supply is at the right hand side, switch **left hand side** *to* **right hand side** *and vice versa.*

(5) Select a mass of 20 g and find its actual value using the digital balance. Enter in the data sheet for Trial #1 in the column for m_2. Enter "0" in the column for m_1. Tie this 20 g mass to the cord on the right hand side of the glider, by passing the end of the cord through the central hole of the 20 g mass. No mass holder is to be used. This will be the "first trial' value of m_2. m_1 will not be used at this time.

(6) Unless otherwise instructed, let m_2 be 20 grams larger than m_1.

(7) For the second and subsequent trials, choose the following masses for m_1, and place them on the left hand side mass-holder: 10g, 20g,........150g.

(8) The masses placed on the right hand side mass holder (m_2), will always be 20 g larger than those placed on the left hand side mass holder.

(9) When a pair of masses, have been placed on respective mass holders, find the total mass of each of m_1 and m_2, using the digital balance and enter in the data sheet.

(10) When a pair of masses have been placed on the mass-holders, check that, for the initial and final positions of the glider, the masses remain suspended in air (i.e. do not hit the floor or the pulley). With larger masses, the cord will be found to stretch. slightly Hence it is important to check it out for every trial.

(11) Clear the timer.

(12) Carefully set the glider such that its right most edge is exactly at the 50 cm (or 150 cm) mark and gently release the glider. As the glider moves past the photogate, the timer measures the necessary value of time.

(13) As soon as the glider has moved past the photogate window, stop it by pressing the glider against the track. This should be done gently but firmly. Also make sure that your finger does not get into the window. You should also be careful not to disturb the positions of the photogate.

(14) Read and record the values of t

(15 Place the next pair of prescribed masses on the two mass-holders and repeat steps 9 through 14, for the next 15 trials.

(16 The experiment ends; switch off the timer and the air flow. Rearrange all apparatus neatly on the table.

Calculations and Graph

(1) The diameter of the flag is D cm. Divide D by t to get the final velocity v of the glider. Calculate these values for all 16 trials.

(2) Find the average of all 16 values of $m = m_2 - m_1$ as entered in the data sheet. Keep five significant decimal digits.

(3) Calculate the acceleration a using Equation (12) for each trial. Call these a_{kin} .

(4) Calculate a using Equation (10) for each trial. Call these a_{dyn} .

(5) Plot a_{kin} against a_{dyn} and draw a best fit straight line. Find its slope and compare with its expected value (unity). Find percent error.

(6) Next, plot $1/v^2$ on y-axis against m_1 (x-axis). ask the computer to fit a straight line and print out its equation to five decimal digits. Do not forget r^2

(7) Find the reciprocal of the slope. This is the *experimental* value of *the net work done on the system.* The unit is *J.*

(8) Calculate the *expected* value of *the net work done on the system* using Eqn (16) and then taking its reciprocal. Or simply calculate mgd

(9) Compare the experimental and the expected values of *the net work done on the system* and find percent error.

(10) Find the reciprocal of the y-axis intercept and then take its square root (radical). This is the *experimental* value of *the velocity of the system when* m_1 *is zero*

(11) Find the *expected* value of *the velocity of the system when* m_1 *is zero* using Eqn (19), using the average value of m, as found in step (2) above.

(12) Compare the experimental and the expected values of *the velocity of the system when* m_1 *is zero* and find percent error.

(13 A *Results* table is provided at the end, for your convenience.

Conclusions and Discussions

Write your conclusions from the experiment and discuss them. The Conclusion part of the report should be written carefully as it is important.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Data & Data Tables

Name.................................... **Date**..............................

Instructor.............................. **Lab Section**....................

Partner...........................T **able #**..........................

(a) Mass of the glider assembly $M =$ $(g) =$ (kg)

(b) Distance of travel: $d =$ (m)

(c) the mass difference: m kg

Table 1: Measuring the Diameter of the Cylindrical Part of the Flag

$D_1, (mm)$	$D_2\ (mm)$	$D_3\ (mm)$	$D_{av} = D\ (mm)$	$D\ (m)$

Table 1: Data for Kinematic Determination of Acceleration

Trial #	m_1 kg	m_2 kg	m_1 (kg)	m_2 (kg)	$m = m_2 - m_1$ kg	t (sec)
	nominal values		actual values			
1	0	0.020 (no mass holder)				
2	0.010	0.010 + 0.020				
3						
4						
5						
6						
7						
8						

Table 1: Data for Kinematic Determination of Acceleration

Trial #	m_1 kg	m_2 kg	m_1 (kg)	m_2 (kg)	$m = m_2 - m_1$ kg	t $(\sec)$
	nominal values		actual values			
9						
10						
11						
12						
13						
14						
15						
16						
Average value of m						

Additional observations or data (if any) may be recorded on the back side of this page.

Name

Partner's name

Date

Results

Verifying Newton's Second Law

(A) Direct Method: Comparing a_{kin} and a_{dyn} Directly

A Direct Comparison of a_{kin} and a_{dyn}

		Expected Value	Experimental Value	% error
1	Ratio of a_{kin} and a_{dyn}			

(B) Indirect Method: The Hybrid System
Verifying the Work-Energy Theorem

The Work - Energy Theorem

		Expected Value	Experimental Value	% error
1	The net work done on the system			
2	velocity of the system when m_1 is zero			