Experiment # 8

Newton's Second Law of Motion
Direct Verification

Principle

The Second Law

Newton's Second Law of motion, in essence, defines *net force* as an agency that imparts acceleration to material objects. A force acting on an object must produce acceleration. It is impossible to have a force and no acceleration. Conversely, if an object is accelerating, a *net force* must necessarily be acting upon it. In addition, a force does nothing else but impart acceleration and one can say that the sole aim in life of a *net force* is to impart acceleration. If an object is under the influence of a force and is apparently not accelerating (a book placed on a table; for example; or a car travelling at a constant speed), then this force must have been *frozen* (made inactive) by an equal and opposite force, thereby making the *net force* disappear into the oblivion.

The term *net force* acknowledges the possibility of several forces acting on an object simultaneously. These forces are combined to form the *net force* and it is the single *net force* that acts on the object.

Newton studied the effects of *net forces* of constant magnitudes on objects of different masses and found that each mass experienced a different acceleration. It was found that the larger the mass, the smaller was the acceleration and vice versa; or

$$\boldsymbol{a} \propto \frac{1}{M} \qquad \text{..........(1)}$$

As for the effect of forces of different magnitudes on a given mass, it was found that the greater the *net force,* the greater was the acceleration of the mass, and vice versa; or

$$\boldsymbol{a} \propto \boldsymbol{F}_{net} \qquad \text{..........(2)}$$

Combining the two studies, a law was formulated as:

$$\boldsymbol{a} = \frac{\boldsymbol{F}_{net}}{M} \qquad \text{..........(3)}$$

Or:

$$\boldsymbol{F}_{net} = M\boldsymbol{a} \qquad \text{..........(4)}$$

These equations represent a law rather than an equation. One will determine $\boldsymbol{F}_{net}$ t from the given data on forces and then *equate* this $\boldsymbol{F}_{net}$ to "$m\boldsymbol{a}$". The unit of force is "kg-m/s^2". We introduce a compact unit of force: "Newton". One Newton (written as "N") is that *net force* that produces an acceleration of 1.00 m/s^2 in an object of mass 1.00 kg.

Objective of the Experiment

To verify Newton's Second Law of Motion

Setting up

Basics

We may write Eqn (3) in two different forms, one highlighting the dependence of acceleration on net force and the other, its inverse dependence on mass. These are

$$a = (\boldsymbol{F}_{net}) \frac{1}{M} \qquad \text{..........(5)}$$

$$a = \left(\frac{1}{M}\right) \boldsymbol{F}_{net} \qquad \text{..........(6)}$$

In order to study Eqn (5), we need (i) several objects, each of a different mass, (ii) a single force of constant magnitude, and (iii) a device to monitor the accelerations imparted to those masses. To study Eqn (6), we need (i) one object of fixed mass, (ii) several forces, each of different magnitude, and (iii) a device to monitor the accelerations produced by those forces

An object that can move freely in x-mode, is a glider on a linear air track. As it moves on a cushion of air, no frictional forces act upon it. It is, therefore, possible to subject it to a single force to impart it an acceleration. This single force will then be the net force $\boldsymbol{F}_{net}$. A glider has a fixed mass. But it is possible to place additional masses upon it. Thus, it can easily serve both (i) as *several objects, each of a different mass,* or (ii) as *one object of fixed mass.*

The most common force, the force of push or pull applied by a person, is unfortunately, not a uniform force. No one can ever apply a mathematically uniform force for any reasonable length of time. The solution is to let the earth apply the force of pull for us. Earth's force of pull, $\boldsymbol{F}_g = m\boldsymbol{g}$ is an infinitely uniform force. We may use a suspended mass m for this purpose. As we can freely choose values of m, it can easily serve both (i) as *a single force of constant magnitude,* and (ii) as *several forces, each of different magnitude*.

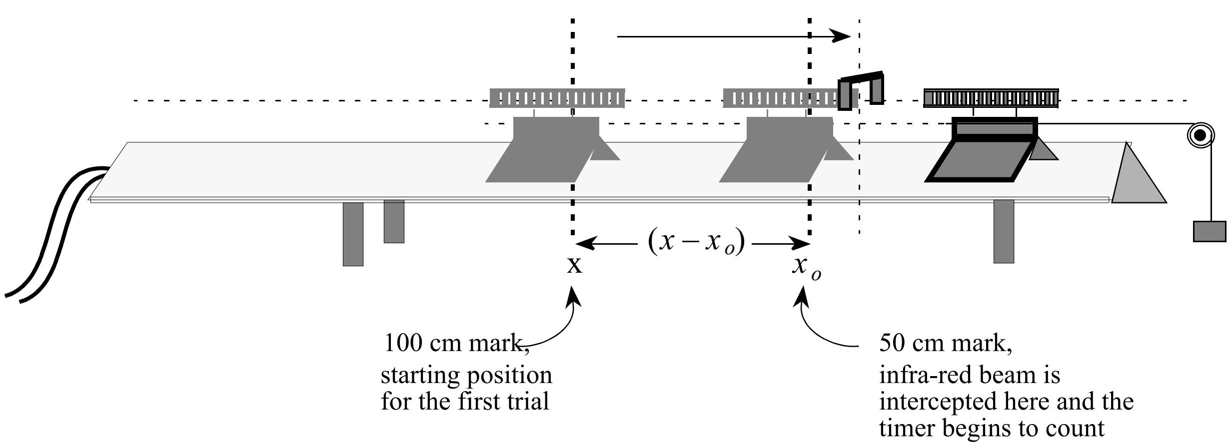

Fig (1) The Glider and the Suspended Mass

Component parts of the apparatus are shown in Fig (2).

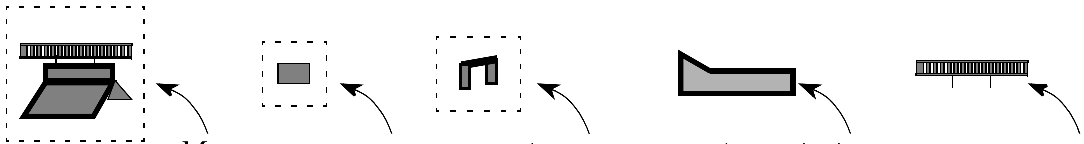

Fig (2) Component Parts

The Chaos & Its Cure

Earth's force of pull is a z-mode thing and our glider is an x-mode system. As z-mode forces are totally useless in x-mode, we need to *transfer* the force from z-mode to x-mode. To do this, we need a cord and a pulley. Earth's force of pull will produce tension force F_T in the cord in the z-mode. The pulley will change the domain of this force from z-mode to x-mode. Thus it will be the tension force F_T that will get applied to the glider and not the earth's force of pull $m\boldsymbol{g}$.

Will F_T be equal to $m\boldsymbol{g}$ and will it be equally uniform?

Equally uniform yes but not equal in magnitude. Technically speaking the tension force in x-mode will be equal to the tension force in z-mode *only* if the pulley were to be massless and frictionless. There will then be no *losses* in the pulley and all of the pull in z-mode will get transmitted to the pull in x-mode.

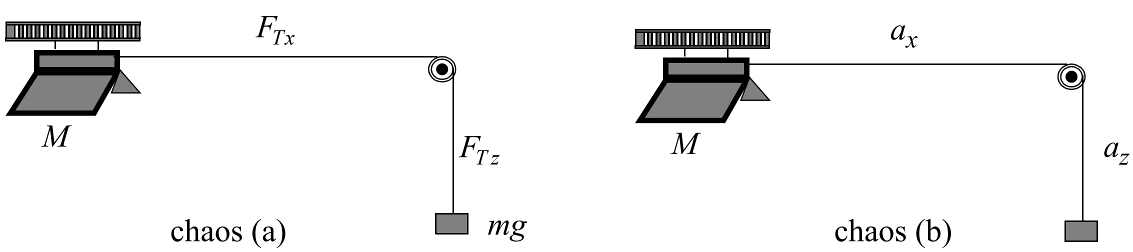

Fig (2) The Chaos

Thus for the pulley being massless and frictionless, we may write:

$$F_{Tz} = F_{Tx} = F_T \qquad \text{..........(7)}$$

The chaos does not end here. We have no guarantee that the acceleration of mass M will be the same as that of mass m. To this end we make another assumption. If the cord itself be massless and in-extensible, then the two sides of the cord will have equal accelerations.

Thus for the cord being massless and in-extensible, we may write:

$$\boldsymbol{a}_z = \boldsymbol{a}_x = \boldsymbol{a} \qquad \text{..........(8)}$$

Force Diagrams & The Elimination of F_T

As there are two moving objects in our system, (M and m) we need draw two force diagrams. These are shown below, in Fig (3).

force diagram for mass M force diagram for suspended mass m

Fig (3) The Force Diagrams

Applying Newton's second law to each object and remembering that they both have the same acceleration, we get:

$$F_T = M\boldsymbol{a} \qquad ....(9)$$

$$m\boldsymbol{g} - F_T = m\boldsymbol{a} \qquad ....(10)$$

Adding $$m\boldsymbol{g} = (M+m)\boldsymbol{a}$$

Solving for $\boldsymbol{a}$, we get:

$$\boldsymbol{a} = \frac{m\boldsymbol{g}}{(M+m)} \qquad ....(11)$$

This is the magnitude of acceleration of the system, as predicted by Newton's Second Law of Motion.

How do we verify the law, if we have already used it?

An interesting (and thoroughly valid) procedure of verification is to see if the predictions of the theory, match the experimental findings. If the experimental findings are made independently of the theory and yet the two match, the theory must be correct. Timer 2000 measures acceleration purely from kinematic principles and is totally independent of Newton's laws of motion. We, therefore, have an excellent basis for the verification of the theory.

For the verification of Eqns (5) & (6), we rearrange Eqn (11):

$$\frac{\boldsymbol{a}}{\boldsymbol{g}} = \frac{m}{(M+m)}$$

Inverting both sides:

$$\frac{g}{a} = \frac{M+m}{m} = 1 + \frac{M}{m} \qquad ....(12)$$

Equation (12) can be rearranged in two different ways. One of these will be used to verify Eqn (3) and the other to verify Eqn (4).

Rearrangement #1

Write Eqn (12) as:

$$\frac{g}{a} = 1 + \left(\frac{1}{m}\right)(M) \qquad ....(13)$$

This rearrangement is suitable for the verification of the second law in the form of Eqn (5). Here m, representing $\boldsymbol{F}_{net}$, is being kept constant and the mass M of the object (the glider) is being varied. Comparing Eqn (13), with the equation of a straight line:

$$y = b + mx \qquad \text{..........(14)}$$

We find:

(i) Eqn (13) is indeed in the form of the equation of a straight line and when plotted, a straight line *will* be obtained,

(ii) M, the mass of the object (glider) is the independent variable and, as such, we shall choose several different values for it.,

(iii) the acceleration $\boldsymbol{a}$ is the dependent variable. We shall find the values of $\boldsymbol{a}$ corresponding to each values of M that we choose, experimentally.

(iv) The slope of the straight line will (upon taking reciprocal) be the suspended mass m, that represents the preselected value of $\boldsymbol{F}_{net}$. If the value found from graph matches its actual value, we shall be convinced that acceleration imparted to the object is indeed inversely proportional to its mass

(v) The intercept tells us that should the mass of the object (the glider) become zero, the suspended mass will have a free fall. Acceleration of the system will then be equal to g and the ratio g/a will become unity. The intercept will, therefore, serve as a *test of authenticity* for our investigations.

Rearrangement #2

Write Eqn (12) as:

$$\frac{g}{a} = 1 + (M)\left(\frac{1}{m}\right) \qquad \text{..........(15)}$$

This rearrangement is suitable for the verification of the second law in the form of Eqn (6), Here M, the mass of the object (the glider) is being kept constant and the net force $\boldsymbol{F}_{net}$, (in the form of the suspended mass m) is being varied. Comparing Eqn (15) with the equation of a straight line (Eqn 14), we find:

(i) Eqn (15) is indeed in the form of the equation of a straight line and when plotted, a straight line *will* be obtained,

(ii) m, the suspended mass, representing $\boldsymbol{F}_{net}$, is the independent variable and, as such, we shall choose many different values for it.

(iii) the acceleration $\boldsymbol{a}$ is the dependent variable. We shall find the values of $\boldsymbol{a}$ corresponding to each values of m that we choose, experimentally.

(iv) The slope of the straight line will be the mass M of the glider (our object). If the value found from graph matches the predetermined value of the glider, we shall be convinced that the acceleration imparted to the object is indeed directly proportional to the net force applied to the object.

(v) The intercept tells us that should the tension force F_T (represented by the suspended mass m) become so large such that $1/m$ tends to zero, then the acceleration imparted by this force will tend to equal the acceleration due to gravity. The ratio g/a will become unity. The intercept will, therefore, serve as a *test of authenticity* for our investigations.

Apparatus Required

(1) Linear air track
(2) Air source
(3) Glider with Windows Plate attached to it
(4) Electronic timer
(5) Photogate
(6) Accessories: pulley, masses, mass holder, unstretchable cord, digital balance, 6" ruler,

Procedure

1. Place the glider on the air track and, with the air supply on, level the track around its mid-point. The glider when placed at rest, should stay motionless. If moved, by giving it a slight push, the glider should run smoothly on the air cushion.
2. Set up the photogate on its stand and adjust height such that the windows plate cuts the beam comfortably. The beam is neither too high nor too low.
3. Set up the electronic timer and connect the power cord and the photogate cord. Set the timer for the 0.5 cm windows plate
4. With the air supply off, set the glider assembly such that the left-most edge (or the right-most edge) is at 130 cm (or 70 cm) mark.
5. Position the photogate such that it gets activated when the glider assembly is at the designated 130 cm (or 70 cm) mark.
6. The system is ready for collecting the data.

Verifying the Law by Keeping F_{net} Constant

7. Use a mass of 50 g as the suspended mass. The total mass whose weight force will pull the glider in the form of tension force F_T , will be 50 g plus the mass of the mass holder. This will be called m . Find the exact value of m using the digital balance and enter in the data sheet.
8. Find the mass of the glider assembly, M using the digital balance and enter in the data sheet (Table 1) for the first trial.
9. Switch on the air supply (and let it remain on for the duration of the experiment).
10. Hold the glider gently on the air track such that its left-most edge (or the right-most edge) is 50 cm away from the photogate. This will be the 80 cm (or 120 cm) mark on the air track. Press the start key on the timer. Release the glider gently and let it pass through the photogate. Immediately after it has cleared the photogate, stop it gently but firmly so that it does not hit the bumper and the suspended mass does not hit the floor.
11. Press the *calc* key on the timer. Check the value of r^2 , the coefficient of determination. If it is 1.0000, then proceed further. If not, then repeat the trial. If r^2 is indeed 1.0000, then check the

value of the first coefficient of the polynomial, i.e. x_o. It should technically be zero. If it is not zero, repeat the trial. If it is consistently greater than zero, then such small values as 0.0001 to 0.0003 may be unhappily accepted. When r^2 is 1.0000, and the value of x_o is acceptable, record the values of *half-acceleration* from the timer into the data sheet.

12. For the second trial increase M by 10 g, by placing one 5 g mass, on each of the two sides of the glider. Find the new value of M using the digital balance and record in the data sheet for trial #2. Repeat steps (10), and (11).
13. For subsequent trials, keep increasing the mass of M in steps of 10 g and for each M, repeat steps (10), and (11). Stop after completing 16 trials.
14. This part of the experiment is now complete.

Verifying the Law by Keeping M Constant

15.)The glider M will now be treated as an "object of fixed mass". The value is known from step (8) above.
16. Select values of the suspended mass, m as 10g, 15 g,.... 85 g., *plus* the mass of the mass holder. Determine each mass using the digital balance and enter in the data sheet.
17. For each m repeat steps (10), and (11). There will be a total of 16 trials for this part also.
18. The experiment ends. Switch off, disconnect and place everything neatly on the table.

Calculations& Graph

Verifying the Law by Keeping F_{net} Constant

1. Divide $g/2$ by each of the 16 *half-acceleration* values in your data sheet, to get the g/a values for the 16 trials of the experiment.
2. Plot g/a on y-axis and M values on the x-axis. Instruct the computer to fit a straight line and print out the equation with five decimal digits. Do not forget r^2.
3. Take the reciprocal of the slope. It should match the value of m, as found in step (7) of procedure. Find percent error.
4. The y-axis intercept is expected to be unity. Compare and find percent error.

Verifying the Law by Keeping M Constant

5. Divide $g/2$ by each of the 16 *half-acceleration* values in your data sheet, to get the g/a values for the 16 trials of the experiment.
6. Take reciprocal of all values of m to get $1/m$ values.

7. Plot g/a on y-axis and $1/m$ on the x-axis. Instruct the computer to fit a straight line and print out the equation with five decimal digits. Do not forget r^2.
8. The slope of the line is expected to be M. as found in step (8) of procedure. Compare and find percent error.
9. The y-axis intercept is expected to be unity. Compare and find percent error.
10. The experiment ends. Switch off the timer.

Interpreting Slopes

The slope in the first part of the experiment represents, *Ratio of* g *to* a *when the mass of the object,* M *is zero*

The slope in the second part of the experiment represents, *Ratio of* g *to* a *when the acceleration imparted by* F_T *equals* g.

Conclusions and Discussions

Write your conclusions from the experiment and discuss them. The Conclusion part of the report should be written carefully as it is important.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Note:

As we are only measuring *half accelerations* in this experiment, precise position of the starting point of the glider and the precise position of the photogate are totally unimportant and could easily be done away with. One could, in principle, place the photogate anywhere, as long as it was some reasonable distance away from the starting position of the glider. *And* the starting position of the glider itself, could be arbitrarily chosen to be anywhere on the track and did not have to be the same for all trails of any part of the experiment.

Some order and some discipline, however, is always recommended for scientific projects.

Data & Data Tables

Name.................................... **Date**..............................
Instructor.............................. **Lab Section**....................
Partner........................... **Table #**..........................

Mass of the "suspended mass plus the mass of the mass holder": g or kg

Table (1) Keeping the Force Constant

	Mass "M" (g)	$a/2$	r^2	x_o		Mass "M" (g)	$a/2$	r^2	x_o
1.					9				
2.					10				
3.					11				
4.					12				
5.					13				
6.					14				
7.					15				
8.					16				

(B) Keeping the mass of the object "M" constant and varying the force, represented by the suspended mass "m"

Mass of the glider "M": : kg

Table (1) Keeping the Mass of the Object Constant

	Mass "m" (g)	$a/2$	r^2	x_o		Mass "m" (g)	$a/2$	r^2	x_o
1.					9				
2.					10				
3.					11				
4.					12				
5.					13				
6.					14				
7.					15				
8.					16				