Experiment # 7

Projectile Motion
Time of Flight

Principle

Definition

An object that moves, totally unsupported, simultaneously in x- and z-modes, is called a *projectile*. Objects that are thrown, kicked or fired in air, undergo unsupported motion not only in the horizontal direction (x-mode) but also in the vertical direction (z-mode). Such objects are called *projectiles*. Balls used in practically all kinds of ball games, bullets fired from guns, divers jumping from diving boards, frisbees are some everyday examples.

Scientifically speaking, projectile motion is a two-dimensional (2-D) motion. in the vertical (x-z) plane. The path of a projectile is called its *trajectory* and the time for which it remains in air is called its *time of flight* (t_{fl}). The maximum height (z-mode) to which the projectile rises, is called its *maximum height* (h_{max}) and the maximum horizontal distance (x-mode) that it travels, is designated as its *range* R.

Characteristics of 2-D Motion

Undergoing a two dimensional motion implies that the projectile will have to have two patterns of motion, one in each of the two modes. One would expect these patterns to be that of uniform motion because uniform motion is the natural outcome of the act of throwing. The expectation holds for the x-mode but not for the z-mode, for the simple reason that z-mode has a built-in acceleration. This is *acceleration due to gravity,* g*,* glued to the z-mode by the earth's pull.

We have an interesting scenario here. The projectile moves with uniform speed in the x-mode, and, at the same time, has accelerated motion, in the z-mode. Uniform and nonuniform in the same breath! As the modes themselves cannot and do not communicate with one another the projectile keeps obeying their dictates till such *time* that its trajectory comes to an end. This *time* is the time of flight, t_{fl}.

Coming to think of it, t_{fl} puts an end to the activities in both modes *at the same time.* The duration of activity in each mode is, therefore, just t_{fl}. This suggests that we can inter-relate the two (otherwise independent) modes to one another through t_{fl}. The recipe to do so will be to calculate or find or determine t_{fl} whichever way we can and then share it with the modes. This will then enable us to solve projectile problems.

Mathematical Tools

(a) The x-mode:

As the motion is uniform, we have only one formula:

$$v = d/t \qquad \text{..........(1)}$$

(b) The z-mode:

The full range of applicable formulae is: given in Table # 1, below:

Table # 1 Kinematic Formulae for z-Mode

Full Length Formula	Short Form Formula	Deviate Format
$v_z = v_{oz} + gt$ ....(2)	$v_z = gt$ ....(3)	$h =$
$h = v_{oz}t + (1/2)gt^2$ ....(4)	$h = (1/2)gt^2$ ....(5)	$v_z =$
$h = (1/2)(v_z + v_{oz})t$.........(6)	$h = (1/2)(v_z)t$ ....(7)	$g =$
$h = (v_z^2 - v_{oz}^2)/(2g)$.........(8)	$h = (v_z^2)/(2g)$ ....(9)	$t =$
		$v_{oz} =$

Solving Problems

To solve a problem for projectiles, follow the following three-step format: (1) form two *bins,* one each for the two modes of motion, (2) dump the given data in the respective bins, (3) find the value of the parameter *time* using the data in whichever bin it can be calculated. Carry this *time* to the other bin and solve the problem. Following is a useful format:

Table # 2 Format For Projectiles

x - mode		z - mode	
$v = d/t$		$h = v_{oz} + (1/2)gt^2$	
$v_{ox} =$ $d =$ $t_{hor} =$	$v_x =$ $R =$ $t_{fl} =$	$v_{oz} =$ $h =$ $t_{vert} =$ $g = -9.8\ m/s^2$	$v_z =$ $h_{max} =$ $t_{fl} =$
g and v_{oz} (or v_z) are in the same direction when g is causing v_{oz} (or v_z) to increase. g and v_{oz} (or v_z) are in opposite directions when g is causing v_{oz} (or v_z) to decrease.			

A symmetrical trajectory is one where the projectile starts and lands on the same level and there is no net gain or loss of height. In other words, the vertical displacement of the projectile is zero. But other forms are also abundantly found where the projectile starts at one level and ends at a different level (lower or higher than the first). We may call these as "asymmetric" trajectories. For symmetrical trajectories, some simplified formulae are available but for the asymmetric ones, one must make use of the format given above.

An interesting aspect of the projectile motion is that it is independent of the mass of the projectile! Mass doesn't show up in any formula. Thus projectiles of all masses will have identical trajectories under identical conditions and will have identical flight times.

Objectives of the Experiment

To study the motion of a projectile and to show that the parameter "time" has identical values in the x- and the z-modes; and to show that the projectile motion is independent of the mass of the projectile.

Setting up

A really simple projectile would be a rigid sphere (e.g. a marble) that rolls off the edge of a table. The system is simple because the initial z-mode velocity v_{oz} of the marble, in this case, will be zero. The marble will not fall vertically down the edge of the table. Depending upon its speed prior to fall-off, it will impact the floor a certain distance away from the foot of the table. Thus by the time it hits the floor, it would have travelled some horizontal (x-mode) distance. Such an arrangement may be used in a laboratory for carrying out the objectives of this experiment.

To impart an initial velocity to the marble in the x-mode (the surface of the table), we may use a ramp. Letting the marble slide down from different heights on this ramp, the marble will develop different initial velocities. To make sure that the velocity with which the marble leaves the table is really horizontal, the last few inches of the ramp must be absolutely flat. Depending on this initial x-mode velocity, the marble will travel a proportionate distance horizontally, before it hits the floor (the touch down). This will be the horizontal range, d. If we measure this distance from the foot of the table, we shall find:

$$d \propto v_{ox} \qquad \text{..........(10)}$$

This inference comes purely from experimental observation. Introduce a constant of proportionality, "k" and write:

$$d = k\, v_{ox} \qquad \text{..........(11)}$$

Balancing the dimensions (M,L,T) on the two sides, we find that the constant k must have the dimensions of time t. Replacing k by t, we get:

$$d = t\, v_{ox} \qquad \text{..........(12)}$$

This was expected. It is just a rearrangement of the formula for uniform velocity of an object in x-mode.

We, however, need to know the magnitude of v_{ox}, the velocity imparted to the marble by the ramp. This can be accomplished by using a photogate timer in the gate mode. The timer will record the time that the marble takes to move past the infra-red beam. Knowing the diameter of the marble, one can find the velocity:

$$v_{ox} = \frac{diameter\ of\ the\ marble}{time\ of\ getting\ past\ the\ window} = \frac{D}{t_{pass}} \qquad \text{..........(13)}$$

Equation (12) is in the form of equation of a straight line: $y = mx + b$ with $b = 0$. It prompts us to select a number of velocities v_{ox} for the marble and measure their corresponding ranges d. A plot of v_{ox} against d will yield the time t as the slope of the best fit straight line. This is the life-time of the projectile in the x-mode. Call it t_{hor}.

The life-time of the projectile in the z-mode may be found from kinematics. The marble undergoes a *free fall*. The height of the table is fixed; the value of acceleration due to gravity is always 9.80 m/s^2, and its initial vertical velocity in z-mode is zero. (Remember we made the last few centimeters of the ramp absolutely flat! That was clever!) With $v_{oz} = 0$ and treating v_z as the absentee parameter, we are eligible to use the short form equation, Eqn (5). All this formula requires, is h, the height of the edge of the ramp from the floor! Eqn (5) is:

$$h = (\tfrac{1}{2})\, g t^2 \quad \text{or} \quad t = \sqrt{\frac{2h}{g}}$$

$$t = \sqrt{\frac{h}{4.9}} = t_{vert} \qquad \text{..........(14)}$$

We expect that the values of time found from the afore mentioned graph and that calculated from Equation (14) will match to within reasonable experimental limits. As this is the objective of the experiment, we look for a suitable lab arrangement. One is shown below. The photogate timer is installed at the edge of the table. A sheet of paper is suitably placed on the floor and the spots of fall are recorded with the help of a carbon paper.

As the height of the table is fixed, the value of t_{vert} is also fixed. This forces t_{hor} to have exactly the same value as t_{vert}. From a previous experiment we learned that t_{vert} is independent of the mass of the falling object. As such, t_{hor} cannot have different values for projectiles of different masses either. This may be verified by selecting projectiles of different masses using the same ramp mounted on the same table. When selecting balls of different sizes, one must make sure that the infrared beam of the photogate passes through the *mid-line* of the ball. One may use a transparent plastic ruler to adjust the position of the photogate window to match the *mid-line* of the ball with the infra red beam. If such an adjustment is not made carefully, the velocity of the ball, as it leaves the ramp, will be incorrect. Such an adjustment is illustrated in Fig (1). In case of doubt, one should consult the instructor.

Apparatus Required

(1) A ramp with a plumb line and a table clamp,

(2) Stand with clamp to hold the photogate in position,

(3) Electronic timer

(4) Photogate

(5) Sheet of paper, carbon paper, cellotape for sticking the paper to the floor.

(6) Set of three balls of different masses (but of the same diameter).

(7) Vernier calipers.

Procedure

(A) Determining t_{hor}

1) Set up the ramp.

2) Find the diameters of the three given balls using the digital vernier caliper. Measure the diameter of each ball three times for three different orientations of the ball. Find average diameters, convert to meters, and call them D_1, D_2, and D_3.

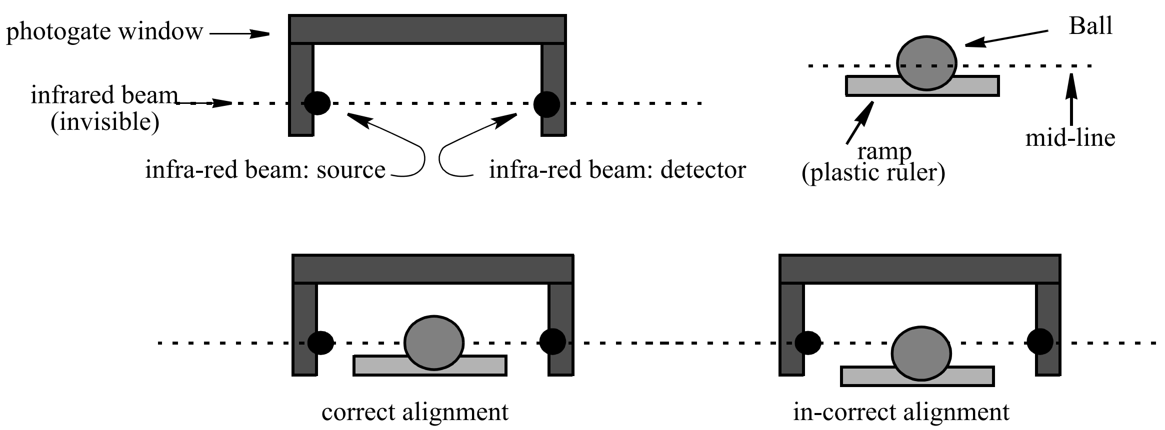

Fig (1) Aligning the Infra-red Beam

3) Set the electronic timer in the ***Projectile Motion I*** mode and select 12 memories. If you are using timer 2000, then select the *Gate* mode. The timer will automatically choose Block #1 with 16 memories. The timer will record all 12 value of t_{pass} for the 12 trials by shifting to the next memory automatically. When all trials for one ball are completed, use the *up* and *down* keys to reach these timings and record them in the data sheet.

(4) Set up a sheet of paper appropriately on the floor and fix its position (at one-fourth of the width of the paper) for the first ball, using cellotape. You will be *relocating* the paper for the second and the third balls, by one-fourth of the width of the paper, each time.

(5) Make a mark on the paper directly under the plumb line. Call it A. This point represents the end of the ramp and the beginning of the range of the marble. All horizontal ranges of the marble will be measured with respect to this point.

(6) Hold the ball lightly at the end of the ramp and measure the height from the floor to the lower edge of the ball (*not* from its mid-line). This is the height "h" of the trajectory. Convert to meters. Make sure that you hold the meter stick in a vertical position and *not* in a slanting posture.

7) Align the infra-red beam with the mid-line of the ball, as carefully as you can. One needs to be patient here.

8) Make a couple of trial runs of the ball from the ramp to familiarize yourself with the projectile and its trajectory. It will also give you some idea where the ball will land on the paper. You should then place the given carbon paper around the expected position of the landing of the ball, when the actual experiment begins.

9) Select the top-most position on the ramp for the first trial. Hold the ball here, press the *reset* or the *start* key on the timer and then release the ball gently. While holding the ball make sure that you are not pressing the ball against the ramp. This will decrease the height of the ramp.

10) Repeat step (9) for 11 more trials but *do not* press the *reset* or the *start* key, a second time. The timer takes care of that automatically. For each subsequent trial, select release positions of the ball on the ramp at 1/2 inch intervals from the top.

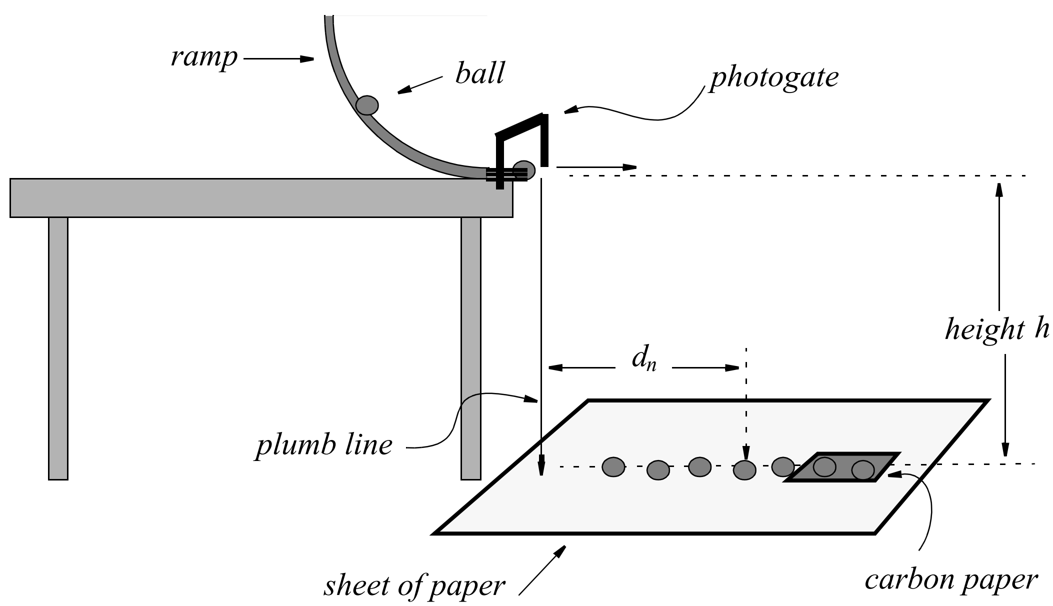

Fig (2) The Experiment

(B) Establishing Mass Independence

11) Repeat steps (9) and (10) for two more projectiles with masses substantially different from the one used above. In each case carry out the alignment of the mid-line with the infrared beam, very carefully. If, however, the balls are of the same size, you do not need to make fresh alignment.

12) The experimental work ends. Switch off timer and disconnect the adapter. Arrange all apparatus neatly on the table.

Calculations and Graphs

1) Calculate t_{vert} from Equation (14) using the height of the table h, as found in step (6) of procedure.
2) Calculate the 12 values of v_{ox} as $v_{ox} = D/t_{pass}$. Thus:

$$v_{ox\,1} = D/t_1 \qquad v_{ox\,2} = D/t_2 \qquad \ldots\ldots\ldots\ldots \qquad v_{ox\,12} = D/t_{12}$$

3) Measure the distances of all 12 spots from the point *A* (marked under the plumb line). Call these $d_1, d_2, d_3 \ldots\ldots d_{12}$. The distance should be measured from *A* to the center of the spot in a straight line.
4) Plot d values (from step 3) on y-axis and the v_{ox} values (from step 2) on the x-axis, using a computer. The computer will fit a straight line and find its slope. The slope is t_{hor}. The equation of the straight line should be printed with 5 significant decimal digits.
5) Compare the two values of time: t_{hor} and t_{vert}. We expect them to match with one another (within, of course, the experimental limits). Find percent error.
6) Repeat the above for the other two projectiles.

   It is possible (and recommended) to plot all three graphs on the same paper, by using the facility *overlay* graph, in the Cricketgraph program.
7) Enter the results in the *Results* section.

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given.

Data & Data Tables

Name.................................... **Date**..............................
Instructor.............................. **Lab Section**....................
Partner........................... **Table #**..........................

Table 1: Masses & Diameters of Balls

Ball #	Mass (g) of Ball or its Size	Diameter (mm)			Average Diameter	
		First Trial	Second Trial	Third Trial	(mm)	(m)
1						
2						
3						

Height of the edge of the ramp from floor: cm = m

Table 2: Times & x-mode Displacements

#1	t_{pass} (sec)	range d (m)	#2	t_{pass} (sec)	range d (m)	#3	t_{pass} (sec)	range d (m)
1			1			1		
2			2			2		
3			3			3		
4			4			4		
5			5			5		
6			6			6		
7			7			7		
8			8			8		
9			9			9		
10			10			10		
11			11			11		
12			12			12		

Additional information or data (if any): (use space on the other side of this sheet)

Additional Table, just in case............

Table 3: Masses & Diameters of Balls

Ball #	Mass (g) of Ball or its Size	Diameter (mm)			Average Diameter	
		First Trial	Second Trial	Third Trial	(mm)	(m)
1						
2						
3						

Height of the edge of the ramp from floor: $cm =$ m

Table 4: Times & x-mode Displacements

#1	t_{pass} (sec)	range d (m)	#2	t_{pass} (sec)	range d (m)	#3	t_{pass} (sec)	range d (m)
1			1			1		
2			2			2		
3			3			3		
4			4			4		
5			5			5		
6			6			6		
7			7			7		
8			8			8		
9			9			9		
10			10			10		
11			11			11		
12			12			12		