# Experiment # 6

# Vectors

## Principle

### Definition & Elementary Properties

Entities in physics are of two types: (i) those whose magnitudes are *senseless* (no sense of where they are, in space), and (ii) those whose magnitudes are *senseful* (are conscious of their orientation in space). These are respectively called *scalers* and *vectors*. Entities like distance, speed, time, energy are scalers, while entities like force, weight, torques are vectors. To differentiate vectors from scalers, we write vectors with bold letters. Hence we shall (for example) write **d** for displacement, **a** for acceleration, and **F** for force.

### Geometrical Representation

A vector is represented geometrically by an arrow. The length of the arrow, from its tip to its toe, represents the magnitude of the vector. The tip of the arrow is aligned with the vector's orientation in space. Fig (1) shows 8 vectors; each with a different magnitude and a different direction.

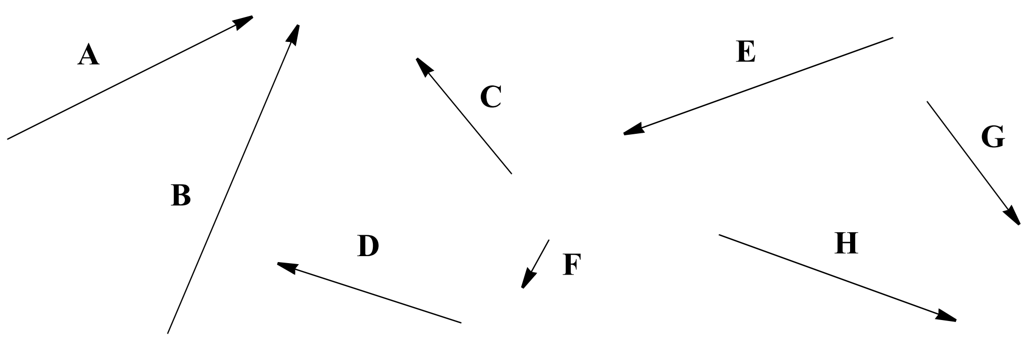


*Fig (1). Vectors: Geometrical Representation*

### Handling vectors

On occasions we need to relocate a vector.To preserve the magnitudes and direction of the vector, we can *only* move it parallel to itself. This is called *parallel transport.* We relocate vectors for many reasons, the simplest being to bring them to a reference frame. Fig (2a) shows all vectors of Fig (1) relocated to a single reference frame.

### Combining Vectors

Vectors in physics always act on an object. When many vectors act on an object, then each tends to move the object along its own direction. Obviously the object cannot move in many directions all at once. So we find a single vector. (the resultant vector) that truly represents all participating vectors. The resultant vector takes over, discards the participating vectors and acts alone on the object, causing it to move in a single direction.

The process of finding the resultant vector ***R*** is called *combining* vectors. Vectors may be combined either geometrically or analytically. Both processes have their own merits.

## The Geometrical Technique.

Vectors are relocated, one at a time, to align the *toe* of the next vector with the *tip* of the previous vector. We say that the vectors are tiptoed. Fig (2b) shows all 8 vectors of Fig (2a) tip-toed by selecting them in the order ***A, B,.... H***. going counterclockwise, in Fig (2a).

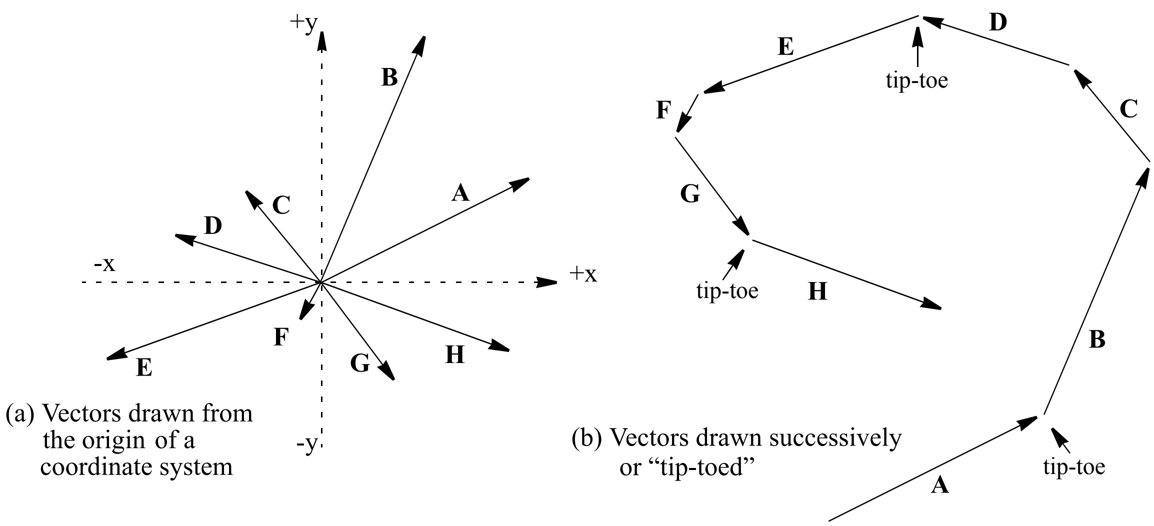


*Fig (2) Vectors of Fig (1); Repositioned.*

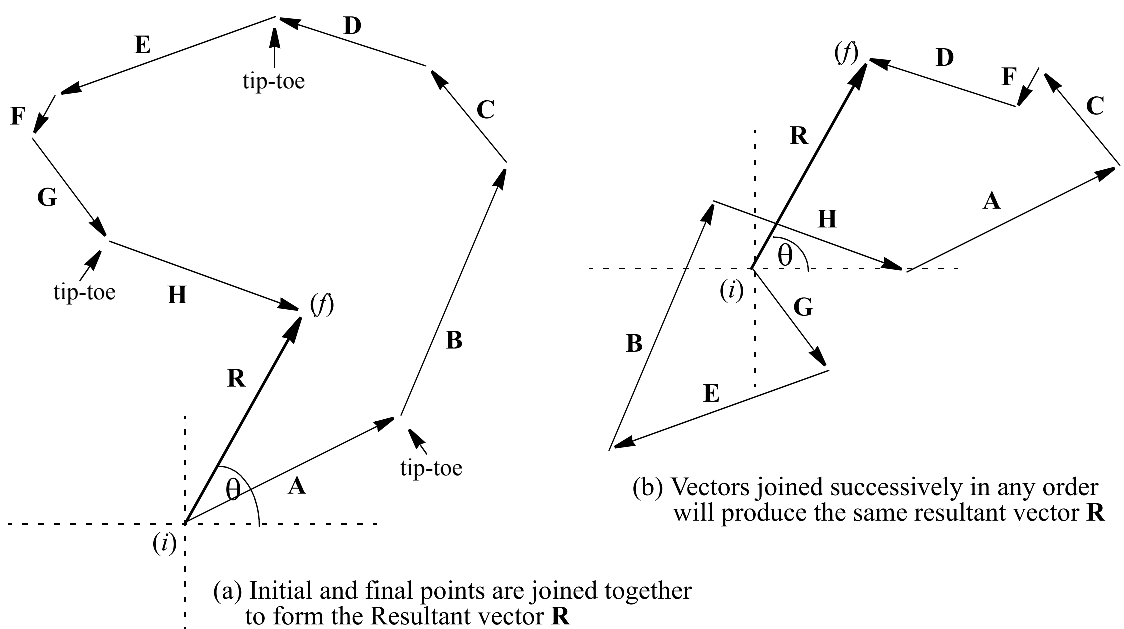


*Fig (3) Combining Vectors to Form "Resultant" Vector* ***R***

When all participating vectors have ben tiptoed, we draw a new vector, the resultant vector $\boldsymbol{R}$, by joining the toe of the first vector with the tip of the last vector. This is shown in Fig (3a). To show that vectors may be tiptoed in any order, the process has been repeated in (Fig3b) by choosing vectors in a different order. It is heartening to see that the two resultant vectors are identical.

## *Analytical Technique.*

We *resolve* a vector into its c*omponents*. Resolving vectors into components can be best understood by considering them as the hypotenuses of (imaginary) right angled triangles. Each *hypotenuse* grows an *adjacent* side parallel to the x-axis and an *opposite* side parallel to the y-axis. All adjacent sides are added / subtracted scalerly to get the resultant x-component, $R_x$. All opposite sides produce a resultant y-component $R_y$. Being perpendicular to one another, $R_x$ and $R_y$ are combined using Pythagorean theorem to produce the resultant vector $R$ (magnitude and direction). In Fig (4) all 8 vectors of Fig (1) have been resolved into their components. The resultant vector thus found is identical to the ones found geometrically.

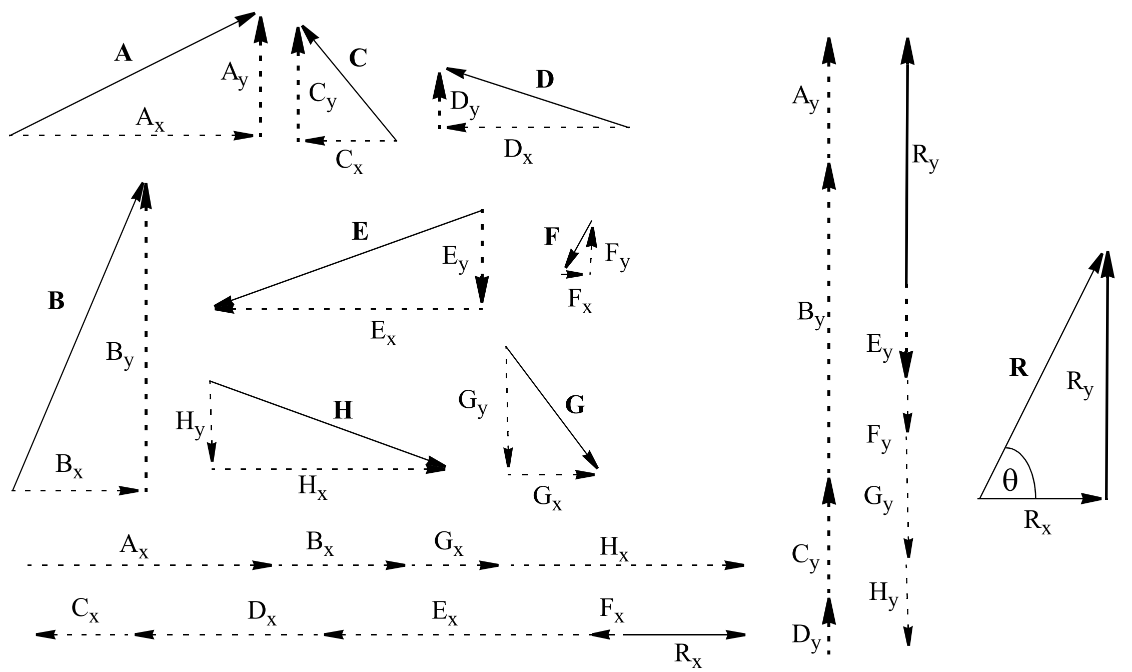


*Fig (4) Vectors as Hypotenuses of Right Angled Triangles; Analytical Combination.*

The adjacent and the opposite sides are obtained using trigonometry. The x-components are:

$$A_x = A\cos\theta_a; \quad B_x = B\cos\theta_b; \quad ............; \quad H_x = H\cos\theta_h; \quad R_x = \Sigma A_x \qquad ..........(1)$$

And the y-component are:

$$A_y = A\sin\theta_a; \quad B_y = B\sin\theta_b; \quad ...........; \quad H_y = H\sin\theta_h; \quad R_y = \Sigma A_y \qquad ..........(2)$$

Applying Pythagorean theorem,

$$R = \sqrt{R_x^2 + R_y^2} \; ; \quad \theta_R = \tan^{-1}\left(\frac{R_y}{R_x}\right) \qquad ..........(3)$$

To reconstruct a vector from its components: $A_x = A\cos\theta$ and $A_y = A\sin\theta$ we square them and add. We get:

$$A_x^2 + A_y^2 = A^2\cos^2\theta + A^2\sin^2\theta = A^2(\cos^2\theta + \sin^2\theta) = A^2 \quad \text{..........(4)}$$

Or

$$A^2 = A_x^2 + A_y^2 \quad \text{..........(5)}$$

This is the Pythagorean relation between a vector and its components.

## *Subtraction of Vectors*

There is no concept of *subtraction* in vector algebra. One can, however, form the *negative* of the vector to be subtracted and then combine in the usual manner. This is done by reversing the direction of the vector but leaving its magnitude intact. It must be clearly understood and remembered that the magnitude of the negative vector is *not* negative.

## *Combining Two Vectors*

If only two vectors are to be combined, we shall recommend (for *cost effectiveness*) the use of *cosine law* for calculating the magnitude of the resultant vector and the use of *sine law* for its direction. One would tiptoe the two vectors and determines the angle between them. This can always be done using the angles of the vectors, which are always provided.

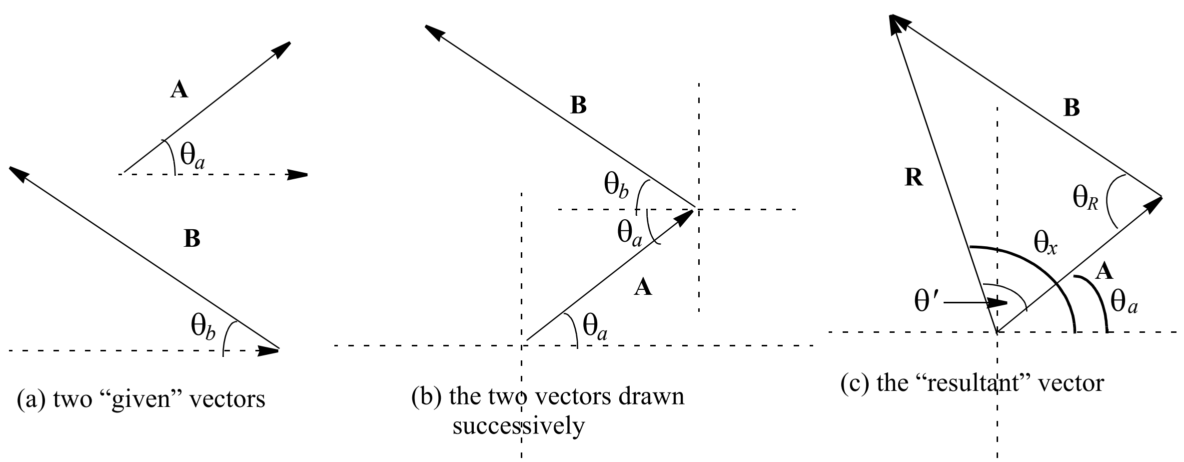


*Fig (5) Using Cosine & Sine laws for Combining Two Vectors*

The resultant vector $R$, is given by:

$$R^2 = A^2 + B^2 - 2AB\cos\theta_R \quad \text{..........(6)}$$

In the above example $\theta_R = \theta_a + \theta_b$. We find $\theta'$ by using the sine law:

$$\frac{\sin\theta'}{B} = \frac{\sin\theta_R}{R}$$

The angle of the resultant vector $R$ with respect to the x-axis, $\theta_x$, is (as shown) given by:

$$\theta_x = \theta' + \theta_a \quad \text{..........(7)}$$

## The Equilibriant Vector

An equilibriant vector is the negative vector of the resultant vector. We write $\boldsymbol{E}$ for it. Thus:

$$\boldsymbol{E} = -\boldsymbol{R} \qquad \text{or} \qquad \boldsymbol{R} = -\boldsymbol{E} \qquad ..........(8)$$

If the resultant vector of a given set of vectors be replaced by the equilibriant vector, the net effect of all vectors will become zero. Consider:

$$\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} = \boldsymbol{R}$$

As $\boldsymbol{R} = -\boldsymbol{E}$, we may write

$$\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} = -\boldsymbol{E}$$

Or

$$\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} + \boldsymbol{E} = 0 \qquad ..........(9)$$

We may say that we neutralized the resultant, and that in doing so we killed all the activity. The object is now in a state of equilibrium. It will either be in the state of rest or in the state of uniform motion.

## One Vector Acting on the Object

If only one vector, vector $\boldsymbol{A}$, is acting on an object then vector $\boldsymbol{A}$ will itself be the resultant vector. The object will not be in equilibrium.

To bring the object to an equilibrium position, we shall need the services of an equilibriant vector. An equilibriant vector is the negative of the resultant vector but as vector $\boldsymbol{A}$ itself is the resultant vector, the equilibriant vector will be the vector $-A$. For equilibrium, both vector $\boldsymbol{A}$ and vector $-A$ will have to act on the object simultaneously. We get:

$$A + (-A) = 0 \qquad ..........(10)$$

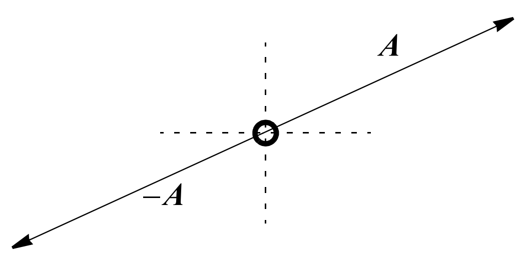


*Fig (6) A single Vector and its Resultant*

## Two Vectors Acting on an Object

If two vectors (say vectors $\boldsymbol{A}$ and $\boldsymbol{B}$) are acting on an object and vector $\boldsymbol{R}$ is their resultant vector, then:

$$\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{R}$$

This is shown in Fig (7c), where we have imported vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ from Fig (5), as shown in Fig (7a) and (7b). Let vector $\boldsymbol{E}$ be the equilibrium vector of vector $\boldsymbol{R}$

As $\boldsymbol{R} = -\boldsymbol{E}$, we get:

$$\boldsymbol{A} + \boldsymbol{B} = -\boldsymbol{E}$$

Or

$$\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{E} = 0 \qquad ..........(11)$$

This is shown in Fig (7d). The combined effect of the three vectors (vectors $\boldsymbol{A}$, $B$ and $\boldsymbol{E}$) on the object is zero and the object is in equilibrium. The three vectors necessarily make a triangle, as shown in Fig (7e). The triangles in Fig (5c) and (7e) are identical except that in one case the third side is a resultant vector while that in the other, is an equilibriant vector. This does not change the geometry and all equations given for the triangle in Fig (5c) are applicable to the triangle in Fig (7e).

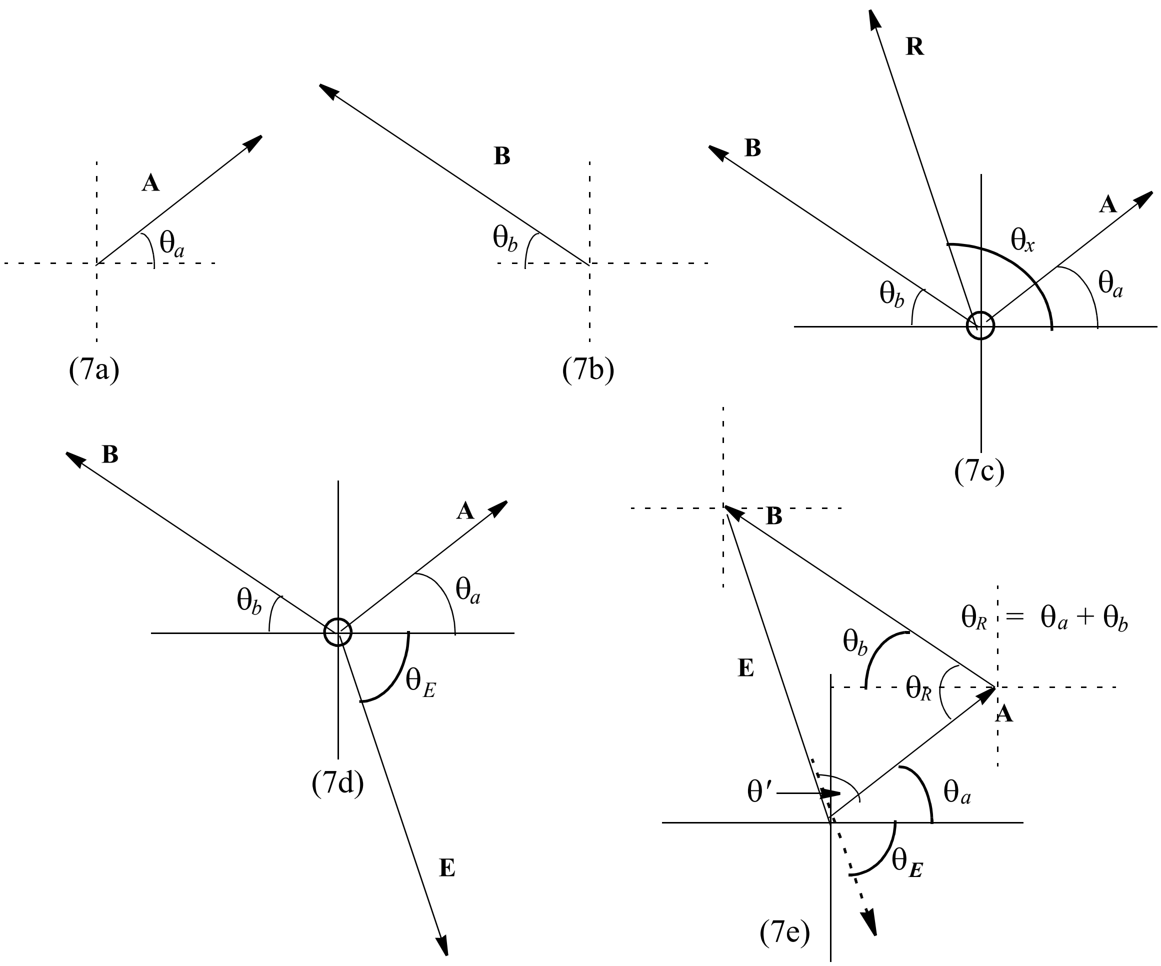


*Fig (7) The Resultant vector **R** and the Equilibriant vector **E** of two (given) vectors **A** and **B***

Recall Eqn (6). It reads:

$$R^2 = A^2 + B^2 - 2AB\cos\theta_R \qquad \text{..........Eqn (6)}$$

As $\boldsymbol{R} = -\boldsymbol{E}$, $R^2 = E^2$ and we get:

$$E^2 = A^2 + B^2 - 2AB\cos\theta_R \qquad \text{..........(12)}$$

where $\theta_R = \theta_a + \theta_b$

Equilibriant vectors should be viewed as *freezing agents* that serves to *freeze* the objects on which they act, or *immobilize* them.

## *Squares of Vectors*

An interesting property of vectors is that when squared, they lose all sense of direction and become scalers. This characteristic can be very useful at times. Among other things, it erases the distinction between a vector and its negative vector. Both (**A**)(**A**) and (-**A**)(-**A**) become equal to ($A^2$). A good example of its gainfulness is to note that squares of resultant and equilibriant vectors become one and the same thing: $R^2 = (-\boldsymbol{E})^2 = E^2$. Here is an interesting guideline:

When in trouble, use squares of vectors and not vectors themselves.

## ***Objectives of the Experiment***

*To study (1) the Component Rule for vectors, and*

*(2) the Resultant Rule for the combination of two vectors.*

## **Setting up**

The most practical vector is the force vector. An apparatus designed for the study of vectors is called a *Force Table.* The *object* to be subjected to forces, is a *ring*. A peg is erected at the center of the table and, for its equilibrium position, the ring should lie with the peg at its center, as shown in Fig (8a).

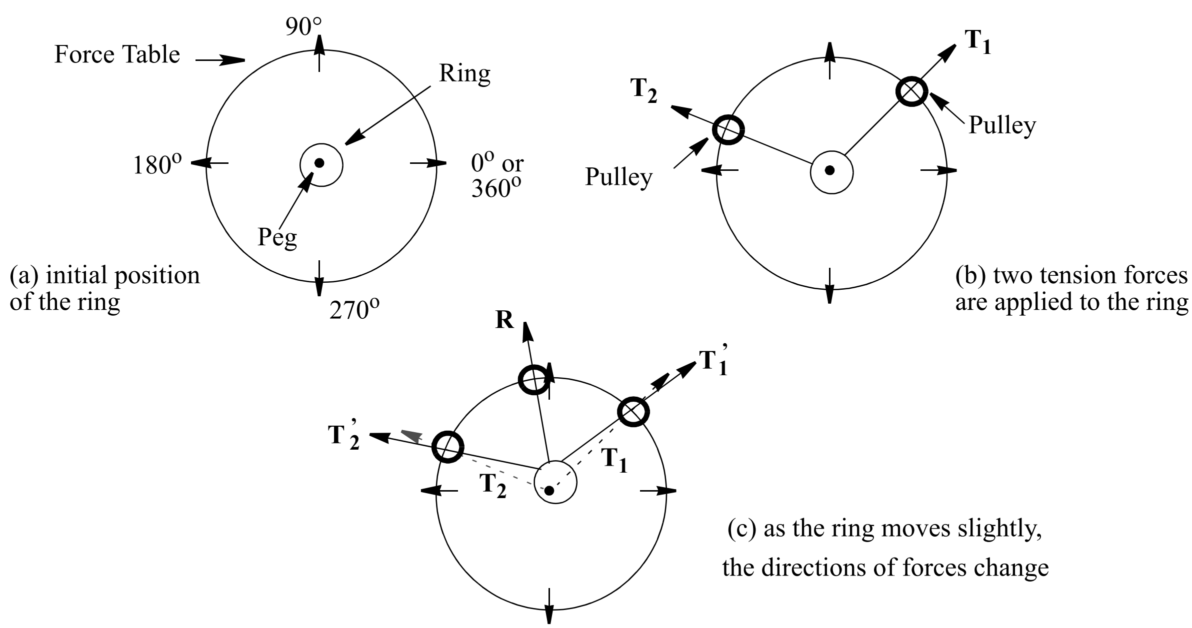


*Fig (8) Forces on a Force Table keep changing directions as the object moves*

One can tie a number of cords to the ring and attach a suspended mass (of suitable magnitude) to each cord. A *massless* and *frictionless* pulley is used to convert the direction of force from z-mode to x-mode. Each suspended mass acts as a weight force and produces a tension force in the respective cord. The ring is thus subjected to a number of tension forces. This is shown in Fig (8b). A net force $\boldsymbol{F}_{net}$ develops, acts on the ring and imparts it an acceleration $\boldsymbol{a}$. It is customary to write $\boldsymbol{R}$ for $\boldsymbol{F}_{\text{net}}$. The magnitude and direction of $\boldsymbol{R}$ may be calculated using the components method.

The acceleration will cause the object to not only move but to do so with non uniform acceleration. This is because the directions of tension forces keep changing from one instant to another. This causes $R$ to change which causes $F_{net}$ to change which causes the acceleration to change. Hence non uniform acceleration. We end up with an unmanageable dynamic system. This is shown in Fig (8c).

The Force Table is *not* designed to handle dynamic systems. So if we have to use the force table, it is necessary to convert our objectives to suit a static system, without losing their basic intensions. This can be done rather intuitively by using the principles of (i) equilibrium of vectors, and (ii) squares of vectors.

## *(a) The Component Rule*

Consider Eqn (10). It uses a single vector $\boldsymbol{A}$ and its equilibriant $-\boldsymbol{A}$. It can be written as:

$$\boldsymbol{A} = -(-\boldsymbol{A})$$

Squaring both sides

$$[\boldsymbol{A}]^2 = [-(-\boldsymbol{A})]^2$$
$$= (-\boldsymbol{A})^2$$
$$= (-A_x)^2 + (-A_y)^2$$

We get:

$$A^2 = (-A_x)^2 + (-A_y)^2 \qquad \text{..........(13)}$$

where $-A_x$ and $-A_y$ can, very conveniently, be considered to be the components of the equilibriant vector $-\boldsymbol{A}$. This makes Eqn. (13) a static equation, quite suitable for use with the Force Table. The technology leads us to an experimental procedure for fulfilling the requirements of the objective. Eqn. (13) is in the form of equation of a straight line so no further mathematical treatment is needed. An added advantage (of great significance from the experiment point of view) is that we managed to shrug off the *angles* in the process! *Squares of vectors are scalers, you know!*

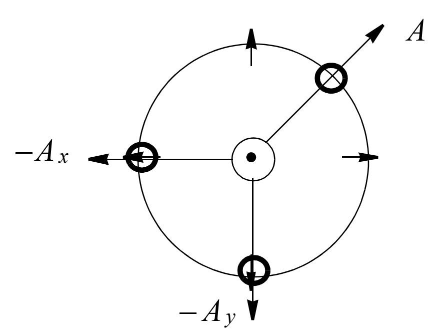


*Fig. (9) Study of the Component Rule*

A comparison with the equation of a straight line: $y = b + mx$, guides us to choose a number of values of $A_y$, the independent variable, and for each $A_y$, set the *object* in equilibrium by adjusting the magnitude and direction of $A$, where $A$ is the dependent variable. Then plot squares of values of $A$ on y-axis, against the squares of values of $A_y$ on the x-axis. A straight line of slope unity and intercept of magnitude $A_x^2$ will emerge. Interpreting the slope and intercept in physics, should be a good exercise for students. The arrangement is shown in Fig. (9) above.

We never talked about the negative signs of $A_x$ and $A_y$ in Eqn (13). Rest assured, their only purpose is to tell us where to locate $A_x$ and $A_y$. This is shown in Fig (9).

## *(b) The Resultant Rule*

For combining (and finding the resultant) of two vectors, we shall use the geometrical method, as described in the Principles section. The method gives us a triangle, to which cosine and sine laws can be applied. For the cosine law, we shall use Eqn (12):

$$E^2 = A^2 + B^2 - 2AB\cos\theta_R \qquad \text{............Eqn (12)}$$

On the Force Table, however, vectors cannot be tiptoed. They can only be attached to the objects and hence must be drawn with their toes at the origin. It is easy to see that the angle between the two vectors on the Force Table is the supplementary angle of the angle $\theta_R$. This is shown in Fig (10), the modified version of Fig (7c).

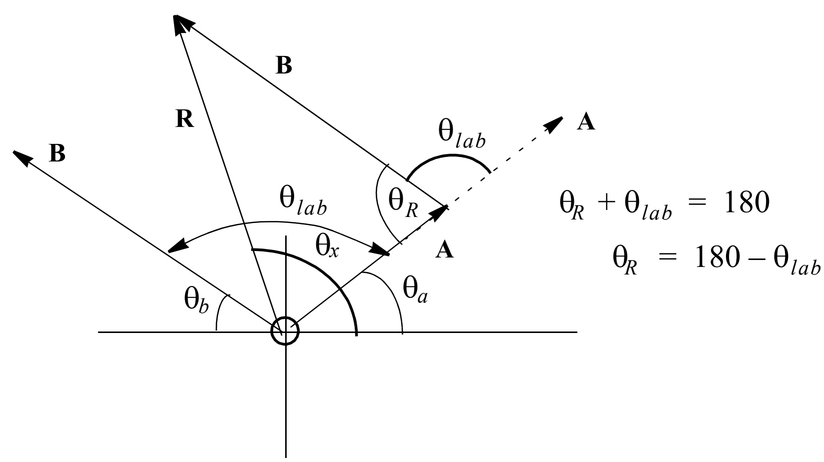


*Fig (10) Angle Between Vectors on the Force Table*

From Trigonometry, we learn that $\cos(180-\theta) = -\cos\theta$. Thus

$$\cos\theta_R = \cos(180-\theta_{lab}) = -\cos\theta_{lab}$$

Inserting this value of $\cos\theta_R$ in Eqn (12) we get:

$$E^2 = A^2 + B^2 + 2AB\cos\theta_{lab} \qquad \text{..........(14)}$$

The last equation, even though in the form of equation of a straight line (slope $2AB$ and intercept $A^2+B^2$) can be simplified further, (without any loss of generality) by choosing equal magnitudes for the two vectors. Setting $\boldsymbol{A} = \boldsymbol{B}$ and calling it $A$, we get:

$$E^2 = 2A^2 + 2A^2\cos\theta_{\text{lab}} \qquad \text{..........(15)}$$

Eqn (15) is our final equation for studying the resultant vector of two given vectors, using the geometrical method. It matches the equation of a straight line. It is an interesting equation with equal values for slope and intercept. The independent variable is the angle $\theta_{lab}$, for which we shall choose a dozen values. For each of these angles, we shall set our object in equilibrium by adjusting the magnitude and direction of vector $\boldsymbol{E}$ which is the dependent variable in Eqn (15). A graph of $E^2$ against $\cos\theta_{lab}$ will yield the expected straight line. The slope and the intercept will each yield a value for the magnitude of vector $\boldsymbol{A}$. We shall expect it to match the value we selected and used in the experiment.

## Apparatus Required

(1) The Force Table, complete with cords, pulley and other accessories

(2) Mass holders,

(3) set of masses

(4) Digital balance

## Procedure

### (A) Study of the Components Rule:

(1) Select a mass of 100 g for $A_x$. The actual mass will be 100g *plus* the mass of the mass-holder. Determine the entire mass using the digital balance. Set the tension force representing $-A_x$ at 180°.

(2) Select following 12 values for $A_y$: 20 g, 40 g,.....240 g; and place on the mass holder. Determine the total mass using the digital balance. Set the tension force representing $-A_y$ at 270°.

(3) For each $A_y$ find vector $\boldsymbol{A}$ (position and magnitude) for which the ring will be in equilibrium. Record the total mass by using the digital balance. Please note that each $\boldsymbol{A}$ will be at a different angle. We shall not need these angles. We may however, record them for verification (if needed). Please note that $-A_x$ and $-A_y$ are components of the vector $-A$ and that $(-A)^2 = A^2$.

### (B) Study of the Resultants Rule:

(4) Select vectors $A = B = 100$ g *plus* the mass of the mass holder. Determine the entire masses of these vectors using the digital balance. Enter in the data sheet. Also find the average value and enter in the data sheet. This is the “expected” value of vector “A”.

(5) Select 11 values of the angle $\theta_{lab}$. Let these be: 15°, 30°, 45°,..........165°. For each angle, determine the equilibriant vector $\boldsymbol{E}$**,** for which the ring will be in equilibrium. Record the total mass by using the digital balance. Please note that each $\boldsymbol{E}$ will be at a different angle. We shall not need these angles. We may however, record them for verification (if needed). Please also note that $E^2 = R^2$.

(6) That’s the end of experiment. Arrange all apparatus neatly on the table.

## ***Calculations and Graphs***

### *(A) The Components Rule:*

1. Calculate $A_y^2$ and $A^2$ for each trial and plot on x- and y-axes respectively.
2. Plot the graph using the computer. Instruct the computer to fit a straight line and print out the equation of the straight line, together with the $r^2$ value. The equation should have five decimal digits.
3. Take the radical of the intercept. This should match $A_x$. Compare and find percent error.
4. The expected value of the slope is $1.00$. Compare and find percent error.

   The *slope* is to be treated as a *test of authenticity.* It tells us that *should vector* $A_y$ *be zero, the ratio of vector* $A$ *to vector* $A_x$ *will always be unity.* In the Description column of Results, you should write, *Ratio of* $A$ *to* $A_x$ *when* $A_y$ *is zero.*

### *(B) The Resultants Rule:*

1. Calculate $\cos\theta_{lab}$ and $E^2$ for each trial and plot on x- and y-axes respectively
2. Draw the best fit straight line using a computer and fit a straight line to the data. Ask the computer to print the equation of the straight line together with the $r^2$ value. The equation should have five decimal digits.
3. Divide the slope by two and then take its radical. Do the same thing with the intercept. Each should individually match vector $A$ or $B$. Find the average value. This is the *experimental value* of vector *A*. Compare with the *expected* value and find percent error.

(4) Enter results in a table.

## ***Conclusions and Discussions***

Write your conclusions from the experiment and discuss them.

## ***What Did You Learn in this Experiment?.***

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

## Data & Data Tables

**Name**................................... **Date**..............................
**Instructor**............................. **Lab Section**...................
**Partner**.......................... **Table #**..........................

(a) Mass of mass holder for $A_x + 100\ g =$ $g$ (using digital balance)

**Table 1: Study of the Components Rule**

| # | Mass to be placed on pan $A_y$ $(g)$ | Total mass **of $A_y$**, using digital balance $(g)$ | Total mass of $A$ using digital balance $(g)$ | Angle θ for $A$ |
|---|---|---|---|---|
| 1 | 20 | | | |
| 2 | 40 | | | |
| 3 | 60 | | | |
| 4 | 80 | | | |
| 5 | 100 | | | |
| 6 | 120 | | | |
| 7 | 140 | | | |
| 8 | 160 | | | |
| 9 | 180 | | | |
| 10 | 200 | | | |
| 11 | 220 | | | |
| 12 | 240 | | | |

Additional data or observations (if any):

(a) Total mass of the first vector, determined using the digital balance: *g*
(b) Total mass of the second vector, determined using the digital balance: *g*
(c) Average mass (the 'expected" value of vector "A"): *g*

**Table 2: Study of the Resultant Rule**

| Trial # | Angle $\theta_{lab}$ (degrees) | Total mass of ***E*** (using digital balance) ($g$) | Angle $\theta_{table}$ for ***E*** |
|---|---|---|---|
| 1. | $15^o$ | | |
| 2. | $30^o$ | | |
| 3. | $45^o$ | | |
| 4. | $60^o$ | | |
| 5. | $75^o$ | | |
| 6. | $90^o$ | | |
| 7. | $105^o$ | | |
| 8. | $120^o$ | | |
| 9. | $135^o$ | | |
| 10. | $150^o$ | | |
| 11. | $165^o$ | | |

Additional data or observations (if any):