Experiment # 5

Kinematic Equations in z-Mode

Principles

Introduction

The terminology *z-mode* refers to motion in the vertical direction, straight up or straight down or both. Objects lie in a vertical plane but as long as they are in z-mode, they move strictly up-down and do not swerve sideways.

Motion in z-mode is different from motion in x-mode. This is because z-mode has a built-in acceleration, called *acceleration due to gravity*. We write *g* for it and its magnitude is 9.8 m/s^2. Motion in z-mode is always accelerated (exception: terminal velocities). If an object falls (freely), its speed of fall will keep becoming larger and larger at the rate of 9.8 m/s, every second. In other words, it will accelerate at 9.8 m/s^2. Again, if it happens to be moving upward (freely), its speed of going up will keep on becoming smaller and smaller at the rate of 9.8 m/s, every second; i.e., it will decelerate at 9.8 m/s^2.

Source of "Acceleration due to Gravity"

Gravity is due to the force of gravitation. The law governing this force is known as *Newton's Law of Universal Gravitation*. The law has jurisdiction over the entire universe. According to this law, every material object exerts a force of attraction on every other material object. For two objects, the force is mutual, though opposite in direction. It is found that the force is directly proportional to the product of the masses of the objects involved, and inversely proportional to the square of the distance between them. The distance is measured from the center of mass of one object to the center of mass of the other. Thus:

$$F_G \propto m_1 m_2 \qquad F_G \propto 1/r^2 \qquad \text{..........(1)}$$

Combining and introducing a constant of proportionality G,

$$F_G = \frac{G m_1 m_2}{r^2} \qquad G = 6.67 \times 10^{-11} \qquad \text{..........(2)}$$

Determining the Value of g for Earth

When an object falls (an apple from a tree, for example), it is actually being attracted by the *entire* earth, toward the center of the earth. For objects on the surface of earth (or sufficiently

Note: The value of G as given here was found experimentally. Its units are "Nm^2/kg^2".

close to it), the center-to-center distance will be the radius of earth: 6.38×10^{6} m. The mass of the earth is: 5.98×10^{24} kg. If the mass of the object be m kg, then:

$$F_G = \frac{6.67\times10^{-11}\times5.98\times10^{24}\times m}{(6.38\times10^{6})^2} = (9.7989)(m) \qquad \text{..........(3)}$$

Since this is the only force acting on the object, it is indeed F_{net} and as such we write:

$$F_{net} = (9.7989)(m) \qquad \text{..........(4)}$$

Applying Newton's second law: $F_{net} = ma$ we get:

$$(9.7989)(m) = (m)(a) \qquad a = 9.7989 = g \qquad \text{..........(5)}$$

Thus we find that the value of g is not just a random number. It is based on a valid scientific theory.

The Format

The format of motion in z-mode is exactly the same as that of motion in x-mode. The format is a set of relationships interconnecting the four parameters: velocity v, acceleration for z-mode g, displacement in z-mode h, and time t. These relations are:

$$v = v_o + gt \qquad \text{..........(6)}$$

$$h = v_o t + \tfrac{1}{2}gt^2 = v_o t + 4.9\,t^2 \qquad \text{..........(7)}$$

$$h = \tfrac{1}{2}(v + v_o)t \qquad \text{..........(8)}$$

$$h = \frac{(v^2 - v_o^2)}{2g} \qquad \text{..........(9)}$$

Eqns: 6, 7, 8, and 9 are the set of four kinematic equations in z-mode.

If the height of fall h, is measured with respect to some arbitrary reference position, h_o, we may express h as $(h - h_o)$ and write;

$$h = (h - h_o) \qquad \text{..........(10)}$$

It should be noted that the algebraic sign of the acceleration vector g is determined relative to the velocity vector and not relative to some reference frame. Velocity vector v and acceleration vector g have the same algebraic sign if g is causing the velocity to increase (downward motion of an object). And the two will have opposite signs if g is causing the velocity to decrease (upward motion of an object). It turns out that g always has a negative sign! Again, the value of g = 9.8 m/s^2, is only for a free flight (up or down). If the motion is not free (Atwood's machine, for example) the acceleration in z-mode will be different from 9.8 m/s^2.

It should further be noted that if an object is *dropped* or *released from rest*, it has no initial velocity. The object will have an initial velocity *only* when it is thrown! In cases where there is no initial velocity, i.e. $v_o = 0$, or no final velocity i.e. $v = 0$, the kinematic equations become much simpler, as shown below:

$$v = gt \qquad \text{..........(11)}$$

$$h = (1/2)gt^2 = 4.9\,t^2 \qquad \text{..........(12)}$$

$$h = (1/2)vt \qquad \text{..........(13)}$$

$$h = v^2/(2g) \qquad \text{..........(14)}$$

It will always be possible to use $(h - h_o)$ *for* h *in the above "short form" equations.*

The velocity appearing in Eqn (11) through (14) is the "surviving" velocity of the object (the one that is not zero). The necessary algebraic signs have been taken into consideration and one need use the *magnitudes* of variables only. *All algebraic signs must be dropped* when using the above set of *short form* equations.

Objective of the Experiment

(i) To determine the value of g*, the acceleration due to gravity, in z-mode for a freely falling object directly, using the time dependent method.*

(ii) To determine the value of g*, the acceleration due to gravity, in z-mode for a freely falling object indirectly, using the time independent method.*

Setting up

Course of Action

One way of determining the value of g precisely will be to drop a ball from some height above the top of a multistory building with windows and observers (with coordinated timers) positioned on each floor. As the ball passes the windows, the observers measure the time the ball takes to pass through their respective windows. The information can be used to compute g.

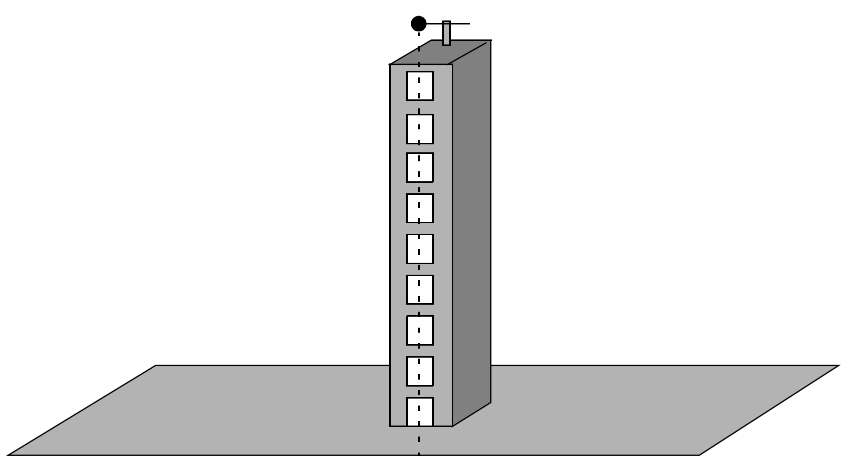

Fig (1) Dropping the Ball from the Top of a Multistory Building

The *other* will be to fix the ball in one place and have the multistory building dropped from some height above the ball. As the falling windows pass through the line of vision of the ball, one by one, we record the respective timings that the windows take to cross the ball.

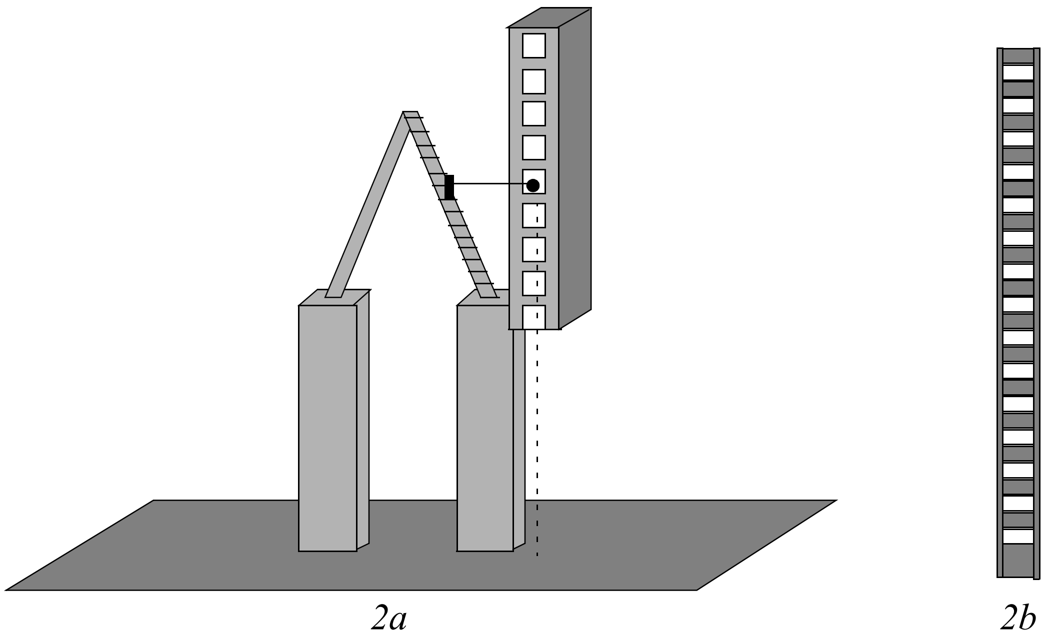

Fig (2) (a) Having the Building Dropped from Some Height above the Ball (b) The Replica

We choose the "other"!

Not Exactly

We built a 15-storey building for the experiment. Well, it is not exactly a building, it is a sort of replica of the building. May be it is not exactly a replica but it does have 15 stories and 15 windows. To tell you the truth, what we made is not exactly 15 stories and 15 windows but it does have a set of 15 rectangular holes to represent the windows and a corresponding set of 16 non-holes to represent the non-window parts of the building. But the good thing is that now we can drop the *building* ourselves and need hire no building droppers.

The Experiment

The windows plate shown in Fig (2b) can be dropped from different heights above a photogate connected to a timer. Any timer, however, will not work. We shall use a timer which is programmed to fit a second order polynomial, such as Timer 2000. As the plate falls, the base of the plate will acquire a certain velocity v_o before it interrupts the infrared beam of the photogate. As the *windows* pass through the photogate, one by one, the beam is interrupted sequentially by the solid parts of the plate (the non-windows). In the *acceleration* mode, timer 2000 will record the time of fall on a progressive basis: the first value of time t_1 will be the time of fall of the first pair of a window and a non-window. The second value of time t_2, will be the time of fall of the first two pairs of windows and non-windows. The last value of time t_{16}, will be the time of fall of all sixteen pairs of windows and non-windows. This strategy of recording time, enables us to use the equation:

$$h = h_o + v_o t + \tfrac{1}{2} g t^2 \qquad \text{..........(15)}$$

directly. This is because the above equation gives us the *displacement* in t seconds. Thus if we set $t = 1$ in the above equation, we shall get the height through which the object falls in one second. For $t = 2$ seconds, the equation will give us the total height of fall in two full seconds. For $t = 3$ seconds, one would get the total height of fall in 3 full seconds; and so on.

Once the windows plate has cleared the photogate, we shall press the *calc* key on the timer. The timer will fit a second order polynomial curve to the data and display the three coefficients together with the value of r^2, the coefficient of determination. The first coefficient (top left corner) is half-acceleration. The second coefficient (top right corner) is the velocity the windows plate had, the instant the leading edge of the plate interrupted the infrared beam. Additionally, we shall get some information in regard to the offset height represented by h_o, in Eqn (15). This will be the third coefficient (bottom left corner).

Technically, the first part of the *Objectives* will be achieved by a single run of the windows plate past the photogate and it will be of our least concern to bother *from what height above the photogate, was the plate dropped?* However, from the point of view of learning more about (i) the kinematic equations, and (ii) the techniques of experimentation, we *will* bother about *height above the photogate.* If we repeat the procedure *several* times by letting the plate fall (from rest) from different heights, we shall get the following data;

(1) *several* values of half-acceleration of the falling plate, that should all be the same within reasonable experimental error, and

(2) *several* values of the initial speed of the windows plate, that should all be different! Each will depend on the height above the photogate, where from the plate was dropped.

From (1) above, we may calculate the mean value of acceleration (after multiplying each by 2), and its standard deviation (using a calculator or a computer). From (2) above, we get an opportunity to find yet another value of acceleration using the time independent kinematic equation, Eqn. # 14. It will then be interesting to cross-match the values of acceleration and comment on the validity of kinematic equations involved.

The heights of fall can be measured arbitrarily with respect to any reference point (the top of the table, perhaps). If we then plot v^2 against $2h$, we shall get the full acceleration g of the glider (and not half-acceleration) from the slope of the straight line.

Our equation will be as follows:

$$(h - h_o) = \frac{v_o^2}{2g}$$

Re-arranging:

$$v_o^2 = -2gh_o + (g)(2h) \qquad \text{..........(16)}$$

Comparing Eqn (16) with the equation of a straight line:

$$y = b + mx$$

we find that now we shall get a y-axis intercept. The comparison also tells us that the intercept will be negative. We shall, however, ignore the negative sign. If we divide the intercept by the slope and further divide the quotient by 2, we should get the value of h_o! This value will be estimated (an exact determination may not be possible). If the value of h_o from graph matches the estimated value, we shall have faith in the results of our experiment, in the technique that we developed, and in the equations that we used.

Because of some technical difficulties, the instructor may ask you not to estimate h_o.

The experiment is unbelievably simple, as will be evident from the set up that we shall use. The plate, however, needs to be dropped gently (so that it does not wobble). We have an excellent arrangement for this too.

Apparatus Required

(1) The Falling Windows Plate,

(2) The advanced electronic timer: Timer 2000 **,**

(3) The Photogate **,**

(4) Laboratory jack, necessary clamps, rods, stand and a cardboard box with some cushioning material (crumpled newspaper, for example)

Procedure

1. Set up the apparatus as shown in Fig (4). The clamp that holds the falling plate is mounted on a *jack* as shown. Place the *receiver* box under the photogate so that the plate falls in it. The box should be filled with crumpled newspaper or foam.

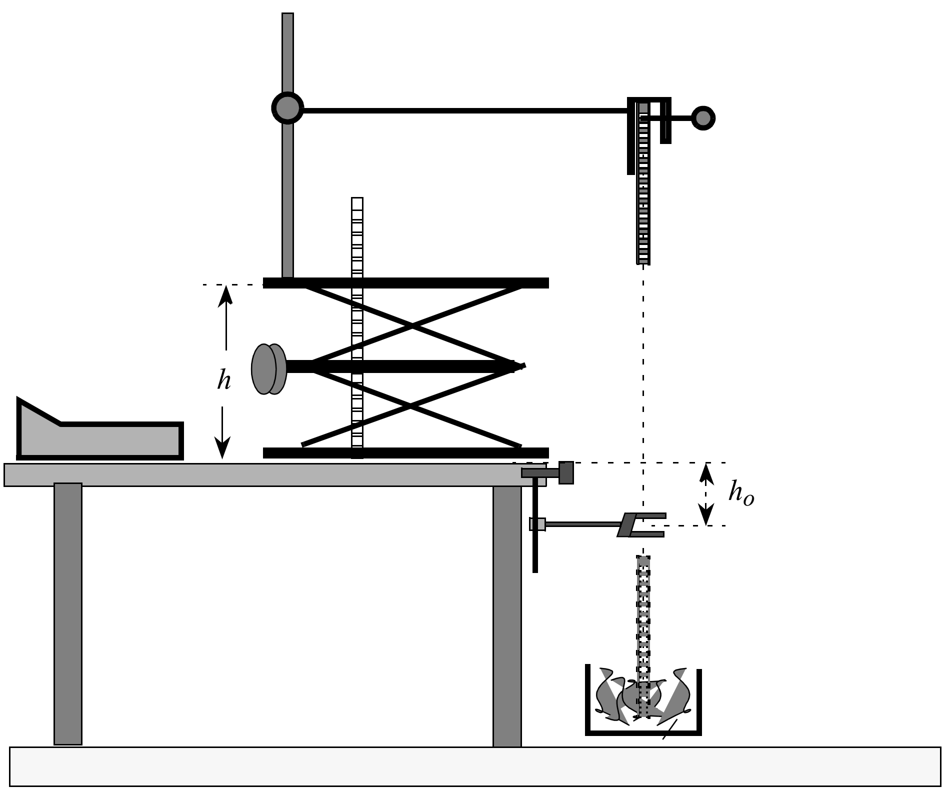

Fig (4) The Experimental Set-up

2. The clamp that holds the photogate is either mounted on the table itself, as shown, or held on a stand. In either case, it remains fixed in one position. Estimate the distance marked as h_o in the diagram and enter in the data table.
3. Set up Timer 2000 and connect the photogate to it.
4. Set up a ruler on a block of wood and place it by the side of the laboratory jack We shall read the position of the top surface of the jack on the ruler.
5. Lower the jack to nearly its lowest position such that the position of the top of the plate can be read as a convenient round number (such as 8 cm or 11 cm).

   It is very important to *read* the scale on the ruler while looking at it *along* the top surface of the jack. You must lower your head so that your eyes are in line with the top surface of the jack. If this instruction is not taken seriously, results will be disastrous.
6. Clamp the windows plate such that its wider end is in contact with the top of the clamp (on its inside). Make sure that the plate is placed symmetrically in the clamp and that the screw will hold the plate at its center. Clamp gently, just enough to hold it. Do not tighten the screw.
7. Record the position of the top surface of the jack, as read directly on the ruler, in the data sheet.
8. Press the *start* key on the timer and then gently release the windows plate. The plate will pass through the photogate and fall in the box.
9. Press the *calc* key on the timer. After making sure that the value of coefficient representing offset height is zero and the value of the coefficient of determination r^2 is 1.000, record the values of (i) half-acceleration $g/2$, and (ii) initial velocity v_o, in the data sheet. If r^2 is not 1.000, and/or h_o is not zero, repeat the trial.
10. Clamp the windows plate and raise the jack by 1.00 cm. Repeat steps (8) and (9).
11. repeat step (10) until you have completed 16 trials.
12. The experiment ends. Switch off, disconnect and place everything neatly on the table.

Calculations and Graphs

Pre graph Calculations (Should be carried out on the computer itself)

(i) Find the average of all 16 values of half-accelerations and then multiply by 2 to get the value for the acceleration due to gravity. Call it g_1.

(ii) Multiply all h values by 2 to get $2h$

(iii) Calculate the square of all 16 initial velocities v_o to get v_o^2

Graph

Plot a graph of v_o^2, (on y-axis), against $2h$, using the “cricketgraph” (or other suitable) program on a computer. Instruct the computer to fit a straight line and calculate the value of r^2. The straight line equation and r^2 should contain five significant decimal places. Record the values of slope and the y-axis intercept from the computer print out into your lab journal.

Post graph Calculations (cannot be done on the computer).

(i) The slope of the straight line is in fact acceleration of the system. Call it g_2.

(ii) Compare the two values of acceleration (g_1 and g_2) and find percent difference, using the equation:

$$\frac{|g_1 - g_2|}{(g_1 + g_2)} \times 200 \qquad \text{..........(13)}$$

(iii) Find the average of the two g values that you found from your experiment. Find percent error with the universally accepted value of g: 9.8 m/s^2.

(iv) Now for the y-axis intercept. Divide the intercept by the slope and then divide again by 2. This is h_o. It should match the value h_o. Find percent difference.

Results

(i) Enter the values of acceleration found (i) using time dependent method, and (ii) time independent method. (iii) Give their percent difference.

(ii) Enter the value of g_{av} (as the experimental value), g = 9.8 m/s^2 (as the expected value) and the percent error.

(iii) Enter the values of actual offset distance x_o and its experimental value. Also give the percent difference.

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Data & Data Tables

Name................................... **Date**..............................
Instructor.............................. **Lab Section**....................
Partner.......................... **Table #**..........................

Estimated value of h_o : cm = m

Table (1) Accelerations & Initial Velocities

#	Heights (as read on the ruler)	Timer 2000			
		half-acceleration	initial velocity	offset distance	r^2 value
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
average half-acceleration					

Additional Table (to be used if instructed by the instructor)

Table (2) Accelerations & Initial Velocities

#	Heights (as read on the ruler)	Timer 2000			
		half-acceleration	initial velocity	offset distance	r^2 value
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
average half-acceleration					