Experiment # 4a

Acceleration
Picket Fence I

Principles

Refer to the experiment "Kinematics in x-Mode" for necessary discussion of "principles". We shall reproduce the time dependent kinematic equation here, for ready reference:

$$d = v_o t + (1/2)at^2 \qquad \text{..........(1)}$$

For v_o or v to be zero, Eqn. (1) will reduce to:

$$d = (1/2)at^2 \qquad \text{..........(2)}$$

Objective of the Experiment

To determine the acceleration of the given system produced by the given force.

Setting Up

The force prescribed for this experiment is the "weight force" of a suspended mass. Weight force is immensely stable and is by far the most constant and uniform force. To avoid complications due to the presence of friction, we shall choose to perform the experiment on a linear air track. As the air track uses gliders, our "system" will be a glider. Such a system is shown in Fig (1).

Assuming linearity and frictionlessness, we eliminate all dissipative forces from our system. The use of a suspended mass requires the introduction of a pulley and a cord. These two innocent looking objects cause some concern to us for reasons of (i) the finite mass of the pulley, and (ii) the elasticity of the cord. The perturbations caused by these may be sizable. We try and choose a pulley that has negligible mass and is reasonably frictionless. Similarly we choose a cord that will not get stretched while the weight force is active. After taking these precautionary measures, we shall sacrifice some truth and assume that the pulley has no mass, is frictionless, and that the cord is un-stretchable.

We shall next let the weight force act on the glider and impart an acceleration to it. As the glider is set into motion, we shall measure the time of its travel using an electronic timer, Kinematic equations should then help us determine the acceleration of the glider. The given weight force will be used to theoretically calculate the acceleration.

We shall expect the two accelerations to match one another.

Mathematical Analysis: (a) Determining "Acceleration"

If the system starts from rest, we shall be able to use Eqn (2). Since an electronic timer is capable of precise measurement of time, we choose the parameters: *time of travel* t, and *distance of travel* d, and make use of the the time dependent kinematic equation.

An interesting way of measuring time and distance, is to plant a *flag* of known length, l, on the glider, and let the timer measure the time t that the flag will take to pass through the timer's photogate. It should be noted, however, that the velocity of the glider will not be constant during this time; because the glider is being continuously accelerated by the suspended mass!

Let the glider start its journey from a position on the air track, a distance d away from the photogate's infrared beam. As the glider begins moving from rest, the leading edge of the flag will travel a total distance d in time t_1 before hitting the beam and interrupting it. The trailing edge of the flag will reach the beam a little later. By the time it hits the beam and interrupts it, it would have travelled a distance $(d+l)$ in time t_2. Using Eqn (7) from Experiment #3, we may write the following for the leading edge of the flag:

$$d = (1/2)at_1^2 \qquad t_1 = \sqrt{\frac{2d}{a}}$$

For the trailing edge then we shall write:

$$(d+l) = (1/2)at_2^2 \qquad t_2 = \sqrt{\frac{2(d+l)}{a}}$$

If we place the timer in "gate" mode, it will measures the time interval for which the beam is interrupted. The leading edge of the flag will start the "timing" process and its trailing edge will stop it. Thus the timer will measure the time interval from the instant the leading edge of the flag started the interruption process, to the instant the trailing edge of the flag restored the beam. This time interval is found by subtracting t_1 from t_2. We get:

$$\Delta t = (t_2 - t_1) = \sqrt{\frac{2(d+l)}{a}} - \sqrt{\frac{2d}{a}}$$

Re-arranging:

$$\Delta t = \sqrt{\frac{2}{a}}[\sqrt{(d+l)} - \sqrt{d}]$$

$$(\Delta t)^2 = \left(\frac{2}{a}\right)[\sqrt{(d+l)} - \sqrt{d}]^2 \qquad \text{..........(3)}$$

Eqn (3) shows that for a given d and l, the measurement of Δt can lead us to a determination of the acceleration a, of the system.

In order to plot a graph, one needs a fair number of data points: ten or more, to be modest To this end, one may use ten or more flags of different lengths l and experimentally determine the time intervals Δt that each will take to pass through the photogate beam. Of course the distance d will be kept constant. This approach, however, is problematic because each of the ten (or so) flags will have a mass of its own, and for each new mass, the glider will have a new acceleration!

An intuitive way of overcoming this problem is to use a single flag of adjustable length but of constant mass. Such a *constant mass* flag can be made essentially, of two pins, whose *distance apart* will constitute the flag length. This *distance apart* can be accomplished by drilling a number of tiny holes in the glider at regular intervals. For ten flags, we need eleven holes. The first pin stays in one of the end holes, always. The second pin moves from hole to hole thereby varying the distance apart of the pins, (the flag length l).As the same two pins stay on the glider all the time, the mass of the glider stays constant. Such a glider is given the name *picket fence glider*. It has eleven equidistant, precision-drilled holes.

The 2-pin flag will not interrupt the beam continuously. The timer, therefore, cannot be used in *gate* mode. But this is no big deal. We switch from the *gate* mode to the *pulse* mode. In this mode, the leading edges of the two pins will, respectively, start and stop the timing process

Mathematical Analysis: (b) Calculating "Force"

To find the net force that provides the acceleration, we shall draw two force diagrams; one each for the glider and the suspended mass. These are shown in Fig (1).

From the force diagram for the glider we get:

$$F_{net} = F_T$$

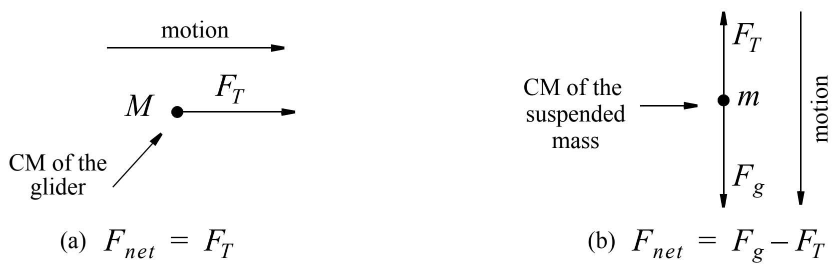


Fig (1 Force Diagrams for (a) the glider (b) the Suspended Mass

Applying Newton's second law: ($\boldsymbol{F}_{net} = m\boldsymbol{a}$), we get:

$$F_T = Ma \qquad \text{..........(4)}$$

From the force diagram of the suspended mass we get:

$$F_{net} = F_g - F_T = mg - F_T$$

Applying the second law we get:

$$mg - F_T = ma$$

$$F_T = mg - ma \qquad \text{..........(5)}$$

From Eqns (3) & (4), eliminating T we get:

$$Ma = mg - ma$$

Rearranging and solving for a, we get:

$$a = \left(\frac{m}{m+M}\right)g \qquad \text{..........(6)}$$

This is the theoretical or the calculated value of acceleration. Please note that as $m/(m+M)$ is a dimensionless quantity, masses may be left in grams. Converting to kg will be waste of effort.

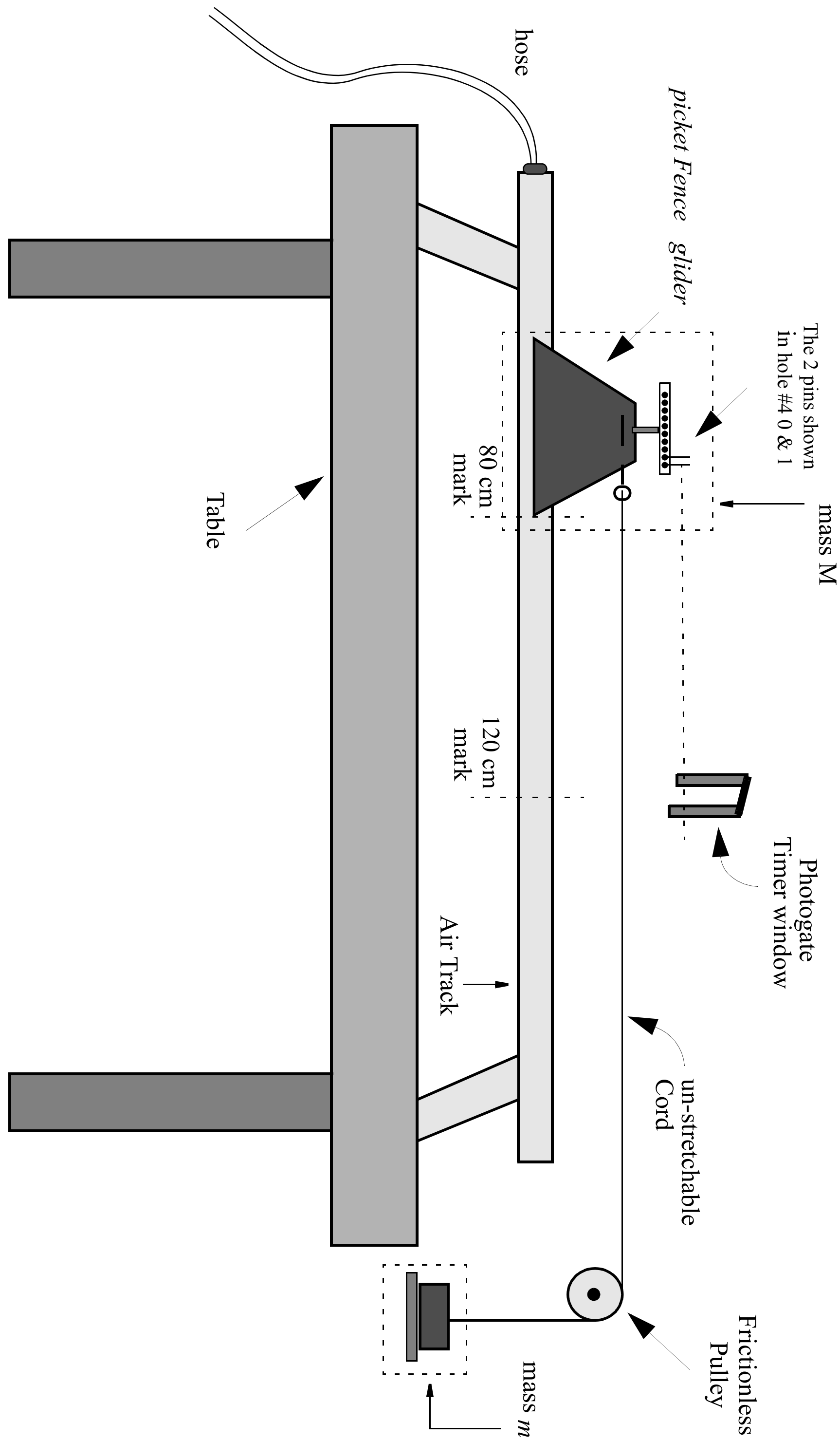


Fig (2) Horizontally Accelerated System

Apparatus Required

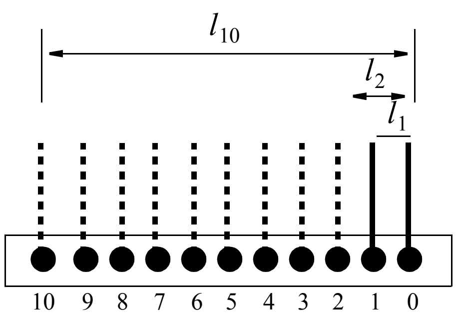
(1) Linear air track with air source
(2) The Picket Fence glider, with pins
(3) Electronic timer with photogate
(4) Accessories: masses, mass holder, un-stretchable cord, vernier calipers, etc.

Procedure

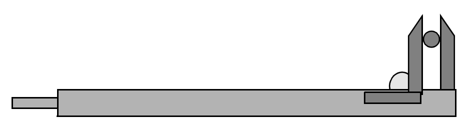
(1) Fig (3a) shows an enlarged view of the picket fence accessory of the picket fence glider. It shows the 11 holes that are numbered from right to left. The right-most hole is numbered zero. As the glider passes through the photogate, the leading edge of the first pin (in hole zero) switches on the timing process. The leading edge of the second pin stops this process. The second pin moves from hole #1 to hole #10 (one hole at a time), thereby setting the 10 flag lengths, for the ten trials. The timer automatically records the ten corresponding values of time.

To measure the lengths of flags, proceed as follows. Select one pin for the zeroth hole and leave it alone. Measure the diameter of the second pin using a digital vernier calipers, as shown in Fig (3b). Call it D and record in the data table.

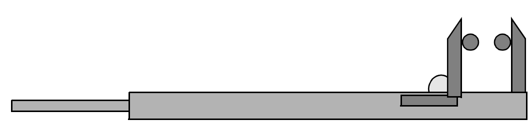
With the designated pin placed in hole zero, place the second pin in the next hole (hole #1) and measure the outside length of the two pins, using the vernier caliper, as shown in Fig (3c). This will be the outside length for the first trial. Call it L_1. Next move this pin to hole #2 and measure the outside length. This will be L_2, the outside length for the second trial. Continue till you have measured all the outside lengths from L_1 to L_{10}. From each "outside length" L_i subtract D to get l_i. Record these measurements in the data sheet as $l_1, l_2, ...l_{10}$.



(3a) Top view of the glider



(3b) measuring the diameter D of the 2nd (undesignated) pin



(3c) measuring the outside length L_1 of the 2 pins for the 1st trial

Fig (3) Measuring $l_1, l_2, ...l_{10}$

(2) Find the mass of the picket fence glider (with the two pins seated upon it) using a digital balance and call it M.

(3) A suggested value of the suspended mass is 25-g. The instructor may prefer to assign a

different value. Find the total mass of the suspended mass and the mass of the mass holder, using a digital balance and call it m.

(4) Set up the linear air track. Switch on the air flow and, with the glider floating on a cushion of air, level the track. Let the glider be at the 120-cm (or 80-cm) mark and adjust the levelling screws such that the glider stays in a state of rest. Switch off the air flow.

(5) Attach the cord (with the suspended mass at the other end) to the glider and make sure that the cord is parallel to the air track.

(6) Insert the pin designated for hole # 0, in the zeroth hole. Position the glider such that its right-most (or the left-most) edge is exactly at the 120-cm (or the 80-cm) mark. Now move the photogate (on its stand) to the left (or the right) slowly and carefully. Find the position where the timer *just about* begins to count. Leave the photogate here and do not disturb its position for the rest of the experiment.

(7) Set the electronic timer in the ***Pulse*** mode. Select 4 decimal places for the measurement of time and one memory.

(8) Now that the photogate is in place and the timer is set, place the second pin in hole # 1 of the picket fence glider. The starting position for the glider will be the 80-cm (or the 120-cm) mark. The right-most (or the left-most) edge of the glider must coincide with the 80-cm (or the 120-cm) mark. This sets d to be 40-cm exactly.

(9) Switch on the air flow and let it stay on all the time.

(10) Hold the suggested edge of the glider gently at the suggested starting position. Clear the timer and release the glider. Stop it manually soon after it has cleared the photogate. As the glider moves past the photogate window, the timer records the time $(\Delta t)_1$.

Repeat two more times for a total of three runs for the first trial. Read and record the timings as $(\Delta t)_1$, $(\Delta t)_2$, and $(\Delta t)_3$.

(11) Move the second pin to the next hole and repeat step (10).

(12) Repeat step (11) for the remaining 8 holes.

(13) The instructor may require you to repeat the entire experiment for a second value of the suspended mass.

(13) The experiment ends; switch off the timer and unplug it. Switch off the air flow and arrange everything neatly on the table.

Note: Air track positions given in parentheses apply if your table position is such that the air inlet hose is on your left hand side

Calculations & Graphs

(1) Calculate the value of a using Eqn (6). This is the “expected” value of a.

(2) For each of the 10 l values, calculate (i) $(d + l)$, and (ii) $[\sqrt{(d+l)} - \sqrt{d}]^2$.These calculations can easily be done on the computer, using the Cricketgraph program.

(3) For each of the 10 trials (i) find the average of the three recorded values of Δt. Call it $(\Delta t)_{av}$ and (ii) calculate $(\Delta t)^2_{av}$.

(4) Plot the values of $[\sqrt{(d+l)} - \sqrt{d}]^2$ on the x-axis and the values of $(\Delta t)^2_{av}$ on the y-axis. Instruct the computer to fit a straight line and print its equation together with the value or r^2, (the coefficient of determination). The straight line equation should have 5 decimal digits.

(5) The slope equals $(2/a)$. Solve it for a. This is the “experimental” value of a.

(6) Compare the two values of a and find percent error.

(7) If you repeated the experiment for a second suspended mass, repeat steps (1) to (6).

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given.

Data & Data Tables

Name.................................... **Date**..............................

Instructor.............................. **Lab Section**...................

Partner.......................... **Table #**..........................

Total mass of the picket fence glider, M: g

Total mass of the suspended mass and the mass holder, m: g

Diameter of pin # 2, the undesignated pin:, D: m

The initial distance of the glider from the photogate beam, d: 0.40 m

Table 1: Recording the 10 l values

second pin placed in hole #	outside lengths L_i, (m)	$l_i = L_i - D$ (m)	second pin placed in hole #	outside lengths L_i, (m)	$l_i = L_i - D$ (m)
1			6		
2			7		
3			8		
4			9		
5			10		

Table 2 Recording the Δt values.

Serial #	$(\Delta t)_1$ (sec)	$(\Delta t)_2$ (sec)	$(\Delta t)_3$ (sec)	$(\Delta t)_{av}$ (sec)	Serial #	$(\Delta t)_1$ (sec)	$(\Delta t)_2$ (sec)	$(\Delta t)_3$ (sec)	$(\Delta t)_{av}$ (sec)
1					6				
2					7				
3					8				
4					9				
5					10				

For repeating the experiment for a second given suspended mass (if required)

Total mass of the suspended mass and the mass holder: kg

Table 2a Recording the Δt values.

Serial #	$(\Delta t)_1$ (sec)	$(\Delta t)_2$ (sec)	$(\Delta t)_3$ (sec)	$(\Delta t)_{av}$ (sec)	Serial #	$(\Delta t)_1$ (sec)	$(\Delta t)_2$ (sec)	$(\Delta t)_3$ (sec)	$(\Delta t)_{av}$ (sec)
1					6				
2					7				
3					8				
4					9				
5					10				

Additional data or remarks if any: