Experiment # 3

Kinematic Equations in x-Mode

Principles

Defining x-mode Motion & its Parameters

When an object moves on a flat surface and is being supported by that surface, it is said to be moving in x-mode. It may move with uniform velocity (or speed), or with non-uniform (accelerated) velocity. The parameters that describe the motion fully and completely are: (i) displacement x or $\boldsymbol{d}$, (ii) velocity $\boldsymbol{v}$, (iii) acceleration $\boldsymbol{a}$, and (iv) time t.

In what follows, acceleration will be deemed to be uniform, at all times.

Inter-relating the Four Parameters

Consider a change of velocity of the object from some initial velocity v_o to some final velocity v. The agency that caused the object to change its velocity is, of course, acceleration $\boldsymbol{a}$. We may represent the process graphically, as shown below. The graph will successfully incorporate all four parameters if we let the y-axis represent the velocities v_o and v; and let the x-axis represent the time t that the object took to change its velocity. The slope of the straight line will represent acceleration $\boldsymbol{a}$ and the area under the straight line will be the displacement $\boldsymbol{d}$.

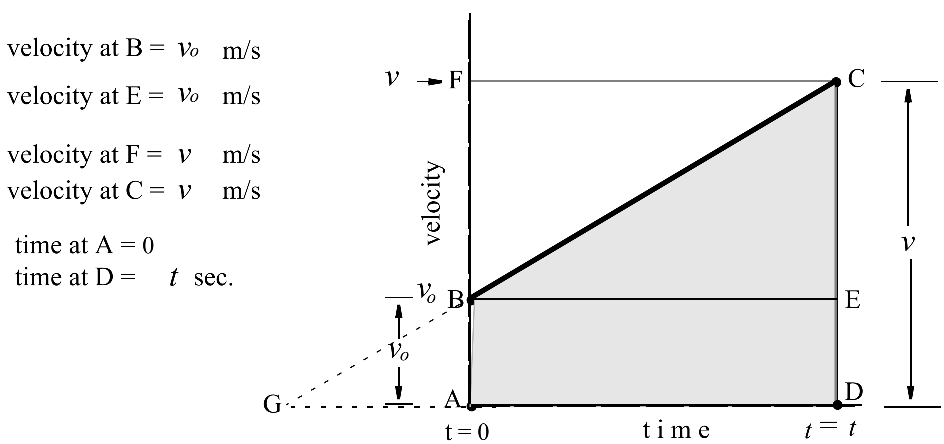

Fig (1) The Graph

From the slope we find:

$$v = v_o + at \qquad \text{..........(1)}$$

The area under the straight line *BC* i.e. *ABCDA*, can be calculated in three different ways.

(a) Letting the area *ABCDA* be a trapezoid, we find:

$$d = (1/2)(v + v_o)\,t \qquad \text{..........(2)}$$

(b) Letting the area *ABCDA* be the sum of (i) a triangle *BCE*, and (ii) a rectangle *ABED*, we find

$$d = v_o\, t + (1/2)at^2 \qquad \text{..........(3)}$$

Eqn. (3) is known as the *time dependent* equation.

(c) Letting the area *ABCDA* be the difference of two triangles (i) *GDC*, and (ii) *GAB*, we find:

$$d = \frac{(v^2 - v_o^2)}{2a} \qquad \text{..........(4)}$$

Eqn. (4) is known as the *time independent* equation.

Eqns: 1, 2, 3, and 4 are known as the set of four kinematic equations.

If the distance of travel d, is measured with respect to some arbitrary reference position, x_o, we may express d as $(x - x_o)$ and write;

$$d = (x - x_o) \qquad \text{..........(5)}$$

If the object starts from rest or comes to a complete stop from motion, we may use a set of four *short form* equations given below. These formulae are based on the premise that if a certain acceleration takes an object from rest to velocity v in a certain period of time and moving through a certain distance, then it will also bring the object to a full stop from velocity v, in exactly the same period of time and moving through exactly the same distance. In other words, in the absence of dissipative elements, the motion is completely *reversible*.

$$v = at \qquad \text{..........(6)}$$

$$d = (1/2)at^2 \qquad \text{..........(7)}$$

$$d = (1/2)vt \qquad \text{..........(8)}$$

$$d = v^2/(2a) \qquad \text{..........(9)}$$

It will always be possible to use $(x - x_o)$ for d in the above short form equations.

The velocity appearing in Eqn (6) through (9) is the *surviving* velocity of the object (the one that is not zero). The necessary algebraic signs have been taken into consideration and one need use the *magnitudes* of variables only. *All algebraic signs must be dropped* when using an equation from the above set of *short form* equations.

Objectives of the Experiment

(1) To determine the acceleration of a glider on an air track directly, using a time dependent method

(2) To determine the acceleration of the glider indirectly, using the time independent method

Setting Up

The time dependent method would use the time dependent equation, Eqn. (3), which is a quadratic equation (or a second order polynomial). Recall Eqn. # 3:

$$d = v_o\, t + (1/2)at^2 \qquad \text{Eqn. \# 3}$$

Replacing d by $(x - x_o)$, and rearranging we get:

$$x = x_o + v_o t + (1/2)at^2 \qquad \text{.........(10)}$$

It is possible to study this equation directly, without any further deliberations, mathematical or otherwise. This is because we have, at our disposal, an advanced electronic timer that is programmed to fit a second order polynomial curve to a set of data points in position x and time t. Position values are pre-programmed in the timer. As for the time values, the timer collects them (with the help of a photogate), stores them, and processes them. The timer's pre-programmed positions are "translated" into *windows* of a *windows plat*e. The *windows plat*e is "embedded" in the moving object. As the object moves, the windows plate "interacts" with the strategically placed photogate. The photogate measures the time discretely for each window of the windows plate. The timer does the rest. There are 16 windows in our windows plate and the timer solves a 16×16 matrix to arrive at the three coefficients.

The timer displays all three coefficients and the value of the coefficient of determination, r^2. The three coefficients are the values of x_o, v_o and $(1/2)\,a$. We expect x_o to be zero and r^2 to have the value unity (for a perfect fit).

As stated in "Objectives", we shall use a glider on an air track to eliminate dissipative elements. One would mount the windows plate on the glider and let it pass through the photogate. The timer will record 16 values of time for the 16 windows of the windows plate, all in a single run of the plate past the infrared beam of the photogate. By pressing the *calc* key, the values of v_o and $(1/2)\,a$ will be displayed instantly.

After making sure that the coefficient x_o is indeed zero and that the coefficient of determination r^2 has the value unity, the values of v_o and $(1/2)\,a$ will be recorded.

Technically, the first part of the "Objectives" will be achieved by a single run of the glider past the photogate and it will be of our least concern to bother where the glider started from on the air track. However, from the point of view of learning more about (i) the kinematic equations, and (ii) the techniques of experimentation, we *will* bother about "where the glider started from on the air track". If we repeat the experiment (running the glider past the photogate once) *several* times by letting the glider start (from rest) from a number of different position, we shall get following data;

(1) *several* values of half-acceleration of the glider, that should all be the same within reasonable experimental error, and

(2) *several* values of the initial speed of the glider, that should all be different! Each will depend on the position of the glider on the air track, where it started from.

From (1) above, we may calculate the mean value of acceleration and its standard deviation (using a calculator or a computer). From (2) above, we can find yet another value of acceleration using the time independent kinematic equation, Eqn (9). It will then be interesting to cross-match the values of acceleration and comment on the validity of kinematic equations involved.

One would choose the starting position of the glider nearest the photogate beam as x_o. If we then plot v^2 against $2(x - x_o)$, we shall get the full acceleration a of the glider (and not half-acceleration) from the slope of the straight line. There will be no y-axis intercept. If we do get a small intercept (positive or negative), it will indicate presence of systematic error. The intercept should be really very small.

But we can be creative. Recall Eqn. (9):

$$(x - x_o) = \frac{v_o^2}{2a} \qquad \text{Eqn. \# 9}$$

Re-arranging:

$$v_o^2 = -2ax_o + (a)(2x) \qquad ....(11)$$

Comparing eqn (11) with the equation of a straight line:

$$y = b + mx$$

we find that if we plot v^2 against $2x$, not only that we shall get the value of full acceleration, a but now we shall definitely get a y-axis intercept. The comparison also tells us that this intercept will be negative. We shall, however, ignore the negative sign. If we divide the intercept by the slope and further divide the quotient by 2, we should get the value of x_o ! This value will be chosen by us and thus will be known to us in advance. If the two values match one another reasonably, our work will be authenticated and we shall have faith in the correctness and accuracy of other results obtained from the *same* data.Additionally, our faith in the technique that we developed, and the equations that we used, will all be reaffirmed.

Apparatus

(1) Linear air track;

(2) Air source,

(3) Advanced electronic timer with windows plate,

(4) Photogate,

(5) Accessories: pulley, masses, mass holder, cord, sensitive balance, ruler, etc.

Diagram

The diagram is NOT made to scale. The purpose is to highlight important aspects of the system, such as the positions x and x_o, the position of the photogate, and so on.

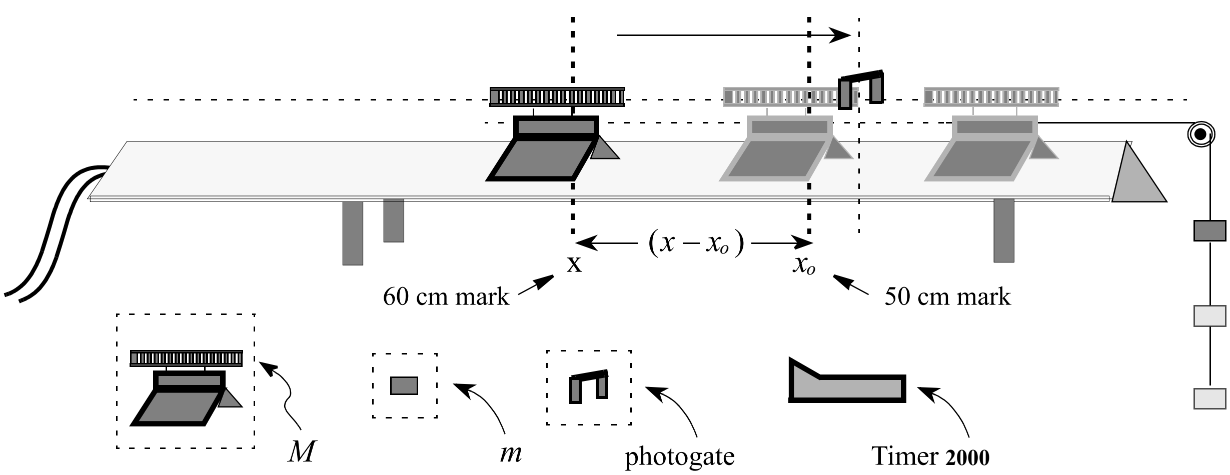

Fig (2) Experimental Set-up

Procedure

1. Set up the air track and level it around its mid-point.
2. Attach the windows plate to the glider and make sure the glider is balanced and runs smoothly on the air cushion.
3. Use a mass of 50 g as the suspended mass. The total mass whose weight force will pull the glider will be 50 g plus the mass of the mass holder.
4. Set up the photogate on its stand and adjust its height such that the windows plate cuts the beam comfortably. The beam is neither too high or too low.
5. Set up the timer and connect the photogate cord
6. Set the glider assembly such that the right-most edge (shown with a dotted line) is exactly at the 50 cm (or the 150 cm) mark. This will set the value of x_o to 50 cm.
7. Press the "start" key on the timer. The timer will start counting. Now slowly move the photogate toward the leading edge of the windows plate until a green rectangle appears in the timer's display screen, at its extreme right hand side. Leave the photogate in this position and press the "stop" key. Make sure that the photogate does not move during the experiment.
8. Now the system is ready for data collection. For the first trial, hold the glider on the air track (gently) such that its right-most edge is at 60 cm (or 140 cm) position.
9. Release the glider gently. As it glides on the air track, the windows plate will pass through the photogate. Immediately after the windows plate has cleared the photogate, stop the glider manually, gently but firmly. Do not let it run off the track. Again, it will be healthy if the suspended mass does not hit the floor.
10. Press the *calc* key on the timer. Check the display of coefficients. If the value of r^2 is 1.000 and that of x_o is zero, then enter the values of all 4 coefficients in the respective columns in the data table. Sometime $x_o = 0.0001$ may be tolerated. If r^2 is not 1.000, and/or x_o is greater than 0.0001, repeat the trial.
11. For subsequent trials increase x in steps of 4 cm. The subsequent positions will be 64 cm, 68 cm,......... 120 cm (or 136 cm, 132 cm,......120 cm). Hold the right most edge of the glider at these positions and repeat steps (9) and (10). There will be 16 trials altogether.
12. The experiment ends. Switch off everything and place all apparatus neatly on the table.

Note: Air track positions given in parentheses apply if your table position is such that the air inlet hose is on your left hand side

Calculations& Graph

Pregraph Calculations (Steps (ii) and (iii) should be carried out on the computer)

(i) Find the average of all 16 values of half-accelerations and then multiply by 2 to get the value for the acceleration of the windows plate. Call it a_1.

(ii) Multiply all x values by 2 to get $2x$

(iii) Calculate the square of all 16 initial velocities v_o to get v_o^2

Graph

Plot a graph of v_o^2 (on y-axis), against $2x$, using the "cricketgraph" program, on a computer. Instruct the computer to fit a straight line and calculate the value of r^2. The straight-line equation and r^2 should contain five significant decimal places. Record the values of slope and the y-axis intercept from the computer print out into your lab journal.

Postgraph Calculations (cannot be done on the computer).

(i) The slope of the straight line is in fact the acceleration of the system. Call it a_2.

(ii) Compare the two values of acceleration (a_1 and a_2) and find percent deviation, using the equation:

$$\frac{|a_1 - a_2|}{(a_1 + a_2)} \times 200 \qquad \text{..........(13)}$$

(iii) Divide the intercept by the slope and then divide again by 2. This is x_o. It should be 50 cm. Find percent difference.

Results

(i) Enter the values of acceleration found (i) using time dependent method, and (ii) time independent method and enter their percent deviation.

(ii) Enter the values of actual offset distance x_o and its experimental value. Also enter their percent difference.

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Data & Data Tables

Name................................... **Date**..............................

Instructor............................. **Lab Section**...................

Partner......................... **Table #**..........................

Position on the air track where the photogate gets activated x_o : 50 cm

Table (1) Accelerations & Initial Velocities

#	Position of the leading edge of the glider	Timer 2000			
		half-acceleration	initial velocity	offset distance	r^2 value
1	60				
2	64				
3	68				
4	72				
5	76				
6	80				
7	84				
8	88				
9	92				
10	96				
11	100				
12	104				
13	108				
14	112				
15	116				
16	120				
average half-acceleration					

Additional Table.

If the instructor suggests different settings, you may use the following table.

Position on the air track where the photogate gets activated x_o : cm

Table (1) Accelerations & Initial Velocities

#	Position of the leading edge of the glider	Timer 2000			
		half-acceleration	initial velocity	offset distance	r^2 value
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
average half-acceleration					