Experiment #2

Simple Pendulum (B)

Principles

There are no new principles. For enquiries or clarification, refer to: "Simple Pendulum (A)". We, however, reproduce the general equation for the time period of a simple pendulum:

$$T = 2\pi\sqrt{\frac{l}{g}}\left[1 + \frac{1}{4}\sin^2(\theta/2) + \frac{9}{64}\sin^4(\theta/2) + \dots\dots\dots\right] \qquad \dots\dots\dots\dots(1)$$

Objectives of the experiment

To study the dependence of the time period T of the given simple pendulum on the angle θ, through which it is oscillated.

Setting up

Study of the Dependence of T on θ.

For a given angle of oscillation θ, the length of the cord l can be treated as the hypotenuse of a right angled triangle, with the mean-position line as the vertical side P, and the amplitude of oscillation A, as the horizontal side. Fig (1) shows such a triangle. However, for experimental convenience, we consider a smaller triangle; one with half the hypotenuse, (length $l' = l/2$). Such a triangle is shown in Fig (2). The two sides are marked as p and a. The dependence of the time period of the pendulum on the angle of oscillation θ, will be translated into its dependence on the vertical side p, of the said triangle.

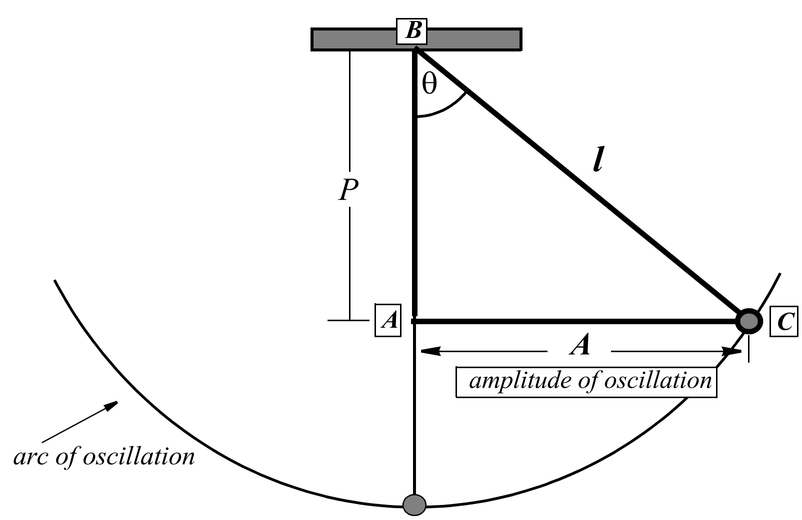

Fig (1) The Geometry showing A and P for a Given Length l

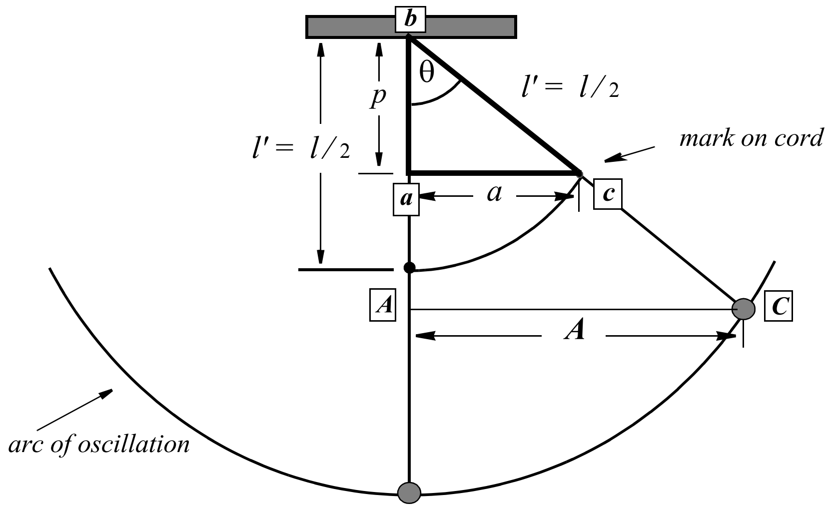

Fig (2) The Geometry Showing a and p for Half the Given length $l' = l/2$

From Fig (2), we find that

$$\cos\theta = \frac{p}{l'} = \frac{p}{0.5l} = \frac{2p}{l}$$

Eqn (1) is found to contain even powers of $\sin(\theta/2)$. We bring to our attention an identity from the *Archives of Trigonometry*:

$$\sin(\theta/2) = \sqrt{\frac{1-\cos\theta}{2}} \qquad \text{..........(2)}$$

Squaring:

$$\sin^2(\theta/2) = \frac{1-\cos\theta}{2} = \frac{1-\frac{2p}{l}}{2} = \left(0.5-\frac{p}{l}\right) \qquad \text{..........(3)}$$

where we inserted $\frac{2p}{l}$ for $\cos\theta$. Squaring again;

$$\sin^4(\theta/2) = \left[0.5-\frac{p}{l}\right]^2 = \left(\frac{p^2}{l^2}-\frac{p}{l}+0.25\right) \qquad \text{..........(4)}$$

We plug-in these values in Eqn (1) and replace $(2\pi)/(\sqrt{g})$ by its value 2.00709:

$$T = (2.00709)\left\{1+\left(\frac{1}{4}\right)\left(0.5-\frac{p}{l}\right)+\left(\frac{9}{64}\right)\left(\frac{p^2}{l^2}-\frac{p}{l}+0.25\right)\right\}(\sqrt{l})$$

$$T = (2.00709)\left\{1+(0.25)\left(0.5-\frac{p}{l}\right)+(0.140625)\left(\frac{p^2}{l^2}-\frac{p}{l}+0.25\right)\right\}(\sqrt{l})$$

$$T = (2.00709)\left\{1+\left(0.125-\frac{0.25}{l}p\right)+\left(\frac{0.140625}{l^2}p^2-\frac{0.140625}{l}p+0.03515625\right)\right\}(\sqrt{l})$$

$$T = (2.00709)\left\{(1 + 0.125 + 0.03515625) - \frac{(0.25 + 0.140625)}{l}\,p + \frac{0.140625}{l^2}\,p^2\right\}(\sqrt{l})$$

$$T = (2.00709)\left\{1.16015625 - \frac{0.390625}{l}\,p + \frac{0.140625}{l^2}\,p^2\right\}(\sqrt{l})$$

$$T = \left\{(2.328538) - \frac{(0.78401953)}{l}\,p + \frac{(0.28224703)}{l^2}\,p^2\right\}(\sqrt{l})$$

$$T = (2.328538)\,\sqrt{l} - \left(\frac{0.78401953}{\sqrt{l}}\right)p + \left(\frac{0.28224703}{l\sqrt{l}}\right)p^2$$

Or, rounding off to 5 decimal places, we get our final equation:

$$\boldsymbol{T = (2.32854)\,\sqrt{l} - \left(\frac{0.78402}{\sqrt{l}}\right)p + \left(\frac{0.28225}{l\sqrt{l}}\right)p^2} \qquad \text{..........(5)}$$

For $l = 1.00$ m, we get:

$$T = \boldsymbol{(2.32854) - (0.78402)\,p + (0.28225)\,p^2} \qquad \text{..........(6)}$$

This is a second order polynomial in p! For experimental verification, one will choose different values of p and for each, will determine the time period T. A second order polynomial will then be fitted to the pairs of values of p and T. If we choose $l = 1.00$ m, the fit should yield the numerical coefficients of Eqn (6). For other values of l, the coefficients will have to be calculated from Eqn (5). Setting up of the pendulum and the arrangement to select values of p experimentally, are given in "Procedure".

Apparatus Required

(1) A pendulum bob,

(2) Vertical support and a piece of an unstretchable cord, approximately 2 meters long,

(3) Electronic timer with photogate,

(4) A Laboratory Jack, with a rod of medium length mounted on it

(5) Two half-meter long meter-sticks and one 6" ruler

(6) A third half-meter long meter stick, cello-taped to a small wooden block

Procedure

Dependence of T on θ

Following instructions are written for a pendulum length l of 1.00 m. For other lengths these instructions should be modified.

(1) Place the pendulum bob and the cord on the table. The cord should be in stretched state. With the help of the half-meter long meter stick, measure a length of half meter, on the cord (from the mid-line of the ball). Make a mark here on the cord using a ball pen. Place a piece of paper under the cord. so as not to make a mark on the surface of the table. The mark on the cord should be all around the cord, and not just in front.

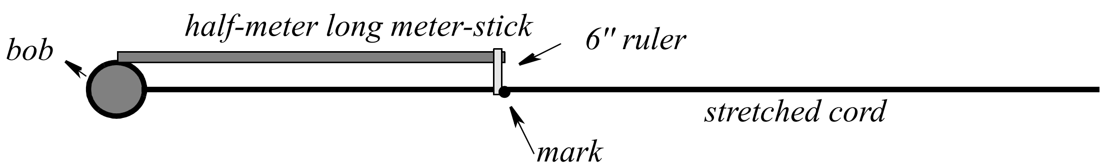

Fig (3) Marking the position at 0.5 m position for $L = 1$ m

(2) Suspend the pendulum from its clamp. Hold the half-meter long meter stick against the clamp and pull the cord up (or down) until the mark you made on the cord coincides with the bottom edge of the half-meter long meter stick. This procedure will ensure a length of one meter for the pendulum cord. See Fig (4a).

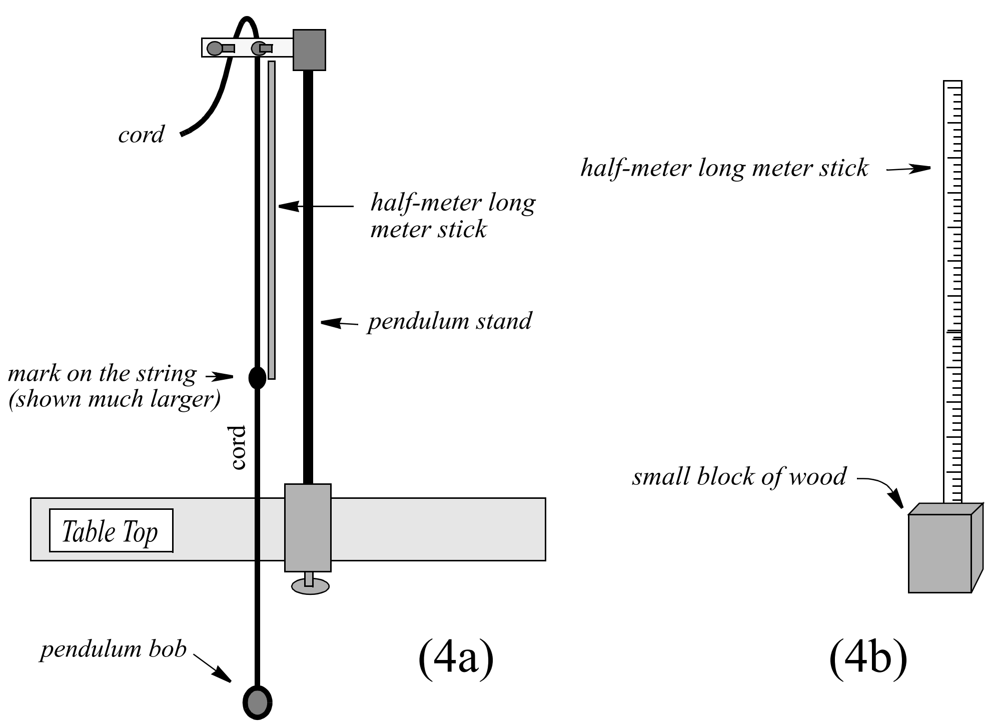

Fig (4) A Pendulum of Length 1.0 m and the "Scale"

(3) Attach a half-meter long meter stick to a block of wood, as shown in Fig (4b). This arrangement will henceforth be called *scale* and will be used for measuring the height by which lab jack is raised (or lowered).

(4) Clamp a half-meter long meter stick (horizontally) on the rod mounted on the lab jack, as shown in Fig (5). Test the horizontally clamped half-meter stick for being horizontal. This can be done by using another half-meter long meter stick to make sure that the height of the horizontally clamped half-meter stick, above the table, is the same throughout its length.

Place the *scale* at the side of the lab jack.

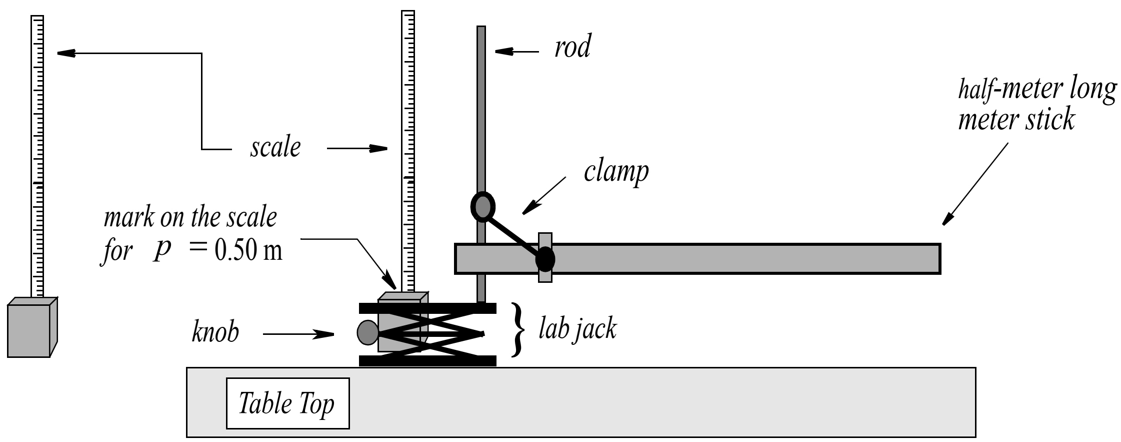

Fig (5) Details of the Experimental arrangement

(5) Position the jack with all its paraphernalia, such that nearly all of the horizontally clamped half-meter long meter stick is to the right of the pendulum cord. Adjust the height of the jack such that the mark on the pendulum cord lies at the center of the width of the meter stick. The pendulum, at this time, is at rest. Record the position of the top of the jack on the scale. This represents $p = 0.5$. Call this position, the “reference point”. See Fig (6a) on the next page.

When the pendulum is in its mean position and is not oscillating then $\theta = 0$, $l = p = 0.5$ m and $a = 0$. The position of the top surface of the lab jack, as read on the scale, should be treated as the initial or the zero position for measuring p. The lab jack (and hence the meter stick) is now ready to be raised. If raised by one centimeter with respect to the reference line on the scale, p will decrease by one centimeter. If the jack is raised by 5 cm, p will decrease by 5 cm and so on. The arrangement allows us to choose different values of p directly and accurately. When the jack has been raised by 25 cm, p will be $0.50 - 0.25 = 0.25$ m. The angle of oscillation, at this time, will be $\cos^{-1}(0.25/0.5)$ or 60°.

(6) Place the photogate such that the bob, when at its mean position, rests in the window of the photogate, with bob’s mid-line coinciding with the photogate window.

(7) Set the electronic timer in the ***Pendulum*** mode. Select 4 decimal places for measuring time and select just *one* memory. The timer will measure the time period of only *one* oscillation. It will be measured to an accuracy of one tenth of one millisecond.

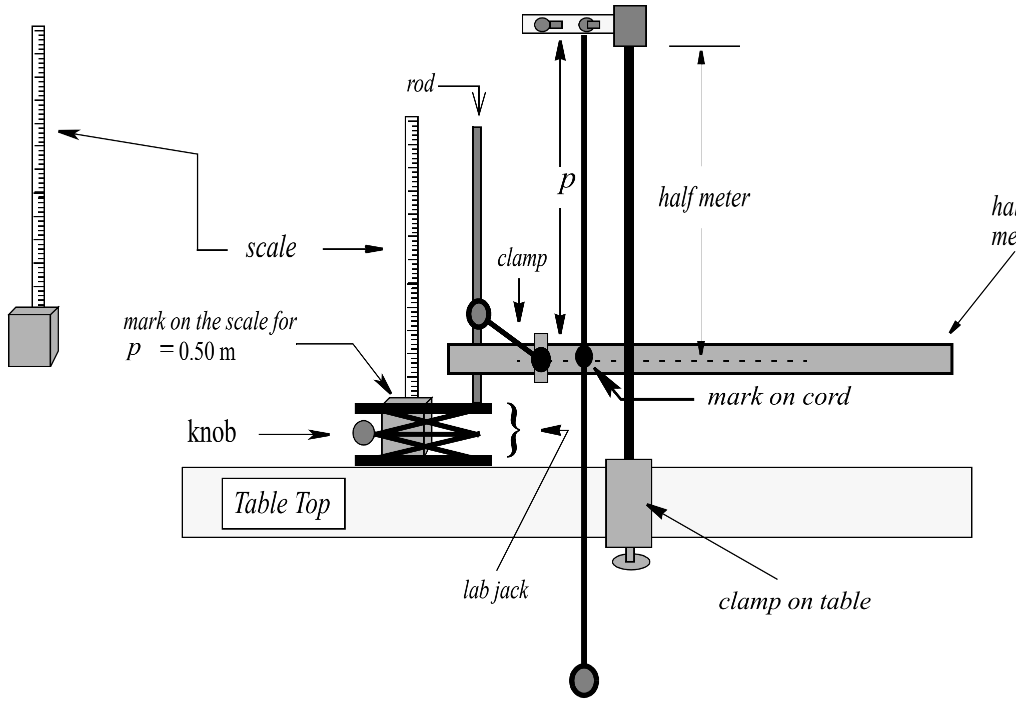

Fig (6a)

Fig (6a) Details of the Experimental arrangement

(8) Measure the time period at this time for a *very* small oscillation. It should be in the close vicinity of 2.0071 sec. This is your first trial. Enter 0.50 m for p. Enter the corresponding value of T that you found, in the appropriate column.

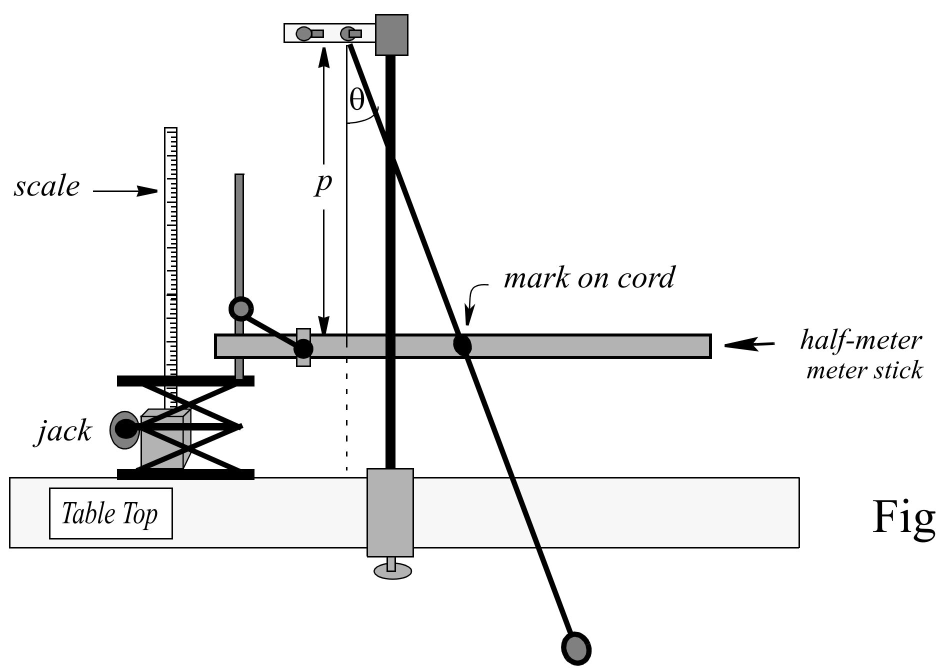

Fig (6b)

Fig (6b)Setting the Pendulum Bob into Oscillations from Different p Positions

(9) Raise the jack (and hence the half-meter long meter stick) slowly and carefully until the reading on the scale increases by one centimeter. Meter stick must be raised exactly by one cm! It is critical. You should look in the same line of vision as the top surface of the jack and the scale. In particular, you should not look at the scale standing up. The error can be quite substantial.

The value of p is now 0.49 m. Press *clear* or *start* on the timer. Raise the pendulum bob such that the mark on the pendulum cord lies at the center of the width of the meter stick. Your partner should help. Release the bob from this position and when the timer has measured the time period, record the value of T, as shown by the timer.

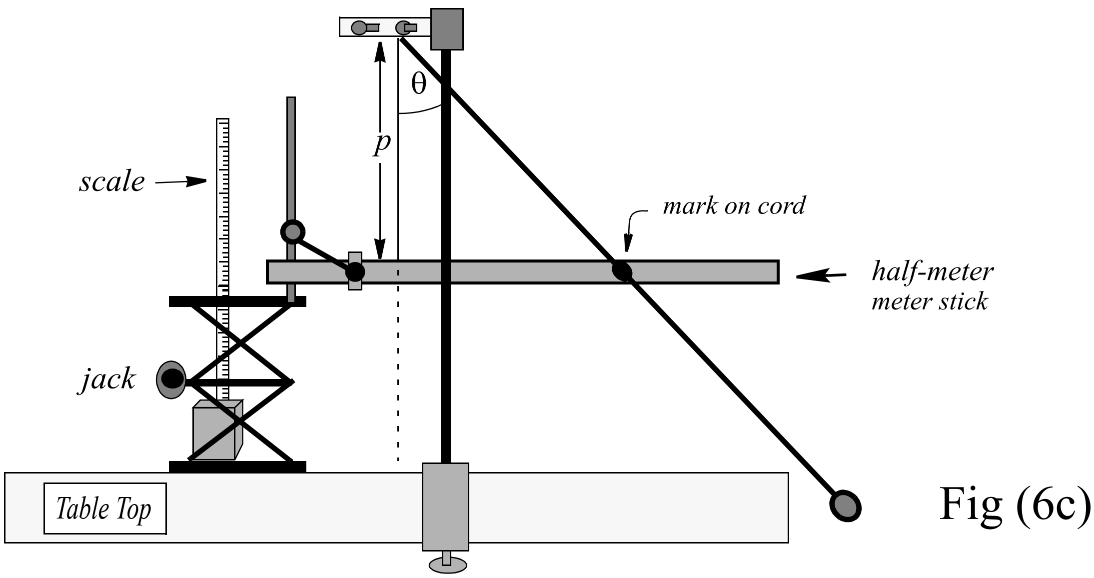

Fig (6c) Smaller and Smaller Values of p Resulting in Larger and Larger Oscillations

Note that the only purpose of the meter stick is to set the height from which the pendulum bob is released. By raising/lowering the bob, one can position the mark (on the cord) at the center of the width of the meter stick. Meter stick itself is not being used to collect any data.

(10) Raise the jack again by one centimeter, as carefully and as precisely as you can, with the help of the scale. This will raise the meter stick also by one centimeter. The value of p will now be 0.48 m. As the meter stick is now higher, the pendulum bob will have to be raised to a higher level, for the mark on the cord to lie at the center of the width of the meter stick. Press *clear* or *start* on the timer, raise the bob to the correct height (with the assistance of your partner) and release it gently. Record the values of p and T in the data sheet. Figs (6b) & (6c) show two other positions for p, obtained by raising the jack.

(11) Repeat step 10 by raising the jack in steps of 1 cm, every time, until a total of 20 trials have been completed. You would have recorded 20 pairs of values of p and T.

Calculations and Graphs

Dependence of T on θ

(1) Pre Graph Calculations:

Plug-in the value of l that you used in the experiment in Eqn (5) and solve to bring it in the form of Eqn (6). Call it Eqn (6a).

(2) Graph: Plot p values on the x-axis and the corresponding T values on the y-axis. Instruct the computer to fit a second order polynomial and print its equation together with the value of r^2. As always, the computer must print the equation with 5 significant decimal digits. Copy the values of the three coefficients and the value of the coefficient of determination r^2, from the graph in your lab journal.

(3) Post Graph Calculation:

Calculate percent differences for the three coefficients from their expected values.

Interpreting the Coefficients

It should be noted that the three numerical coefficients, (namely 2.32854, 0.78402 and 0.28225) are not unit-less or dimensionless numbers. Each carries the unit $second/(\sqrt{meter})$, necessary to make the equation homogeneous. Each of the three terms, then individually have the unit *second*.

(1) As $\angle\theta$ increases, the perpendicular side p of our right angled triangle decreases. When $\theta = 90°$, then $p = 0$ and Eqn (5) reduces to:

$$T = 2.32854\sqrt{l} \qquad \text{..........(7)}$$

The coefficient of p^0, is therefore, the time period for $\theta = 90°$, for (of course) the chosen length of thee pendulum. *This is the* ***maximum*** *time period.*

(2) As p grows from zero, the time period T decreases. The second coefficient indicates the amount of decrease in the time period, for a given angle, translated in terms of p. For the maximum value of p, (the half-length of the pendulum cord), the time period for an infinitesimally small amplitude of oscillation, will be given by:

$$T = 2.32854\sqrt{l} - \frac{0.78402}{\sqrt{l}}(0.5\,l) = (2.32854 - 0.78402x0.5)\sqrt{l} = 1.93653\sqrt{l} \qquad \text{..........(8)}$$

Taking only the first two terms, we have, inadvertently, used a linear equation. The dependence of the time period on the angle is, however, far from being linear. Experimental observations tell us that we have taken off too much. The observed time is somewhat greater than the one given by Eqn (8).

(3) The correction for non-linearity is provided by the third term. The correction is of the second order and one needs to add a small amount of time. Consider again the maximum value of p, the time period for an infinitesimally small oscillations should be increased by an amount:

$$\left(\frac{0.28225}{l\sqrt{l}}\right)(0.5l)^2 = 0.070563\sqrt{l} \qquad \text{..........(9)}$$

The corrected time period will then be:

$$1.93653\sqrt{l} + 0.070563\sqrt{l} = 2.007093\sqrt{l} \qquad \text{..........(10)}$$

It is amazingly interesting to see that the time period of a pendulum of length $l = 1$ m, for infinitesimally small oscillations, is given by

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1}{9.80}} = 2.0070899 \text{ seconds} \qquad \text{..........(11)}$$

The interpretation of coefficients can now be written down as:

Angular Dependence of the Time Period of a Simple Pendulum

	coefficient of	Interpretation	Expected value	Experimental value	% error
1	p^0	Time period of the pendulum for $\theta = 90°$			
2	p^1	Time period of the pendulum for $\theta < 90°$, assuming a linear relationship between T and θ			
3	p^2	Additional correction for the time period of the pendulum for $\theta < 90°$, for the non-linearity of the said relationship			

Results

Dependence of T on θ.

Compare experimental values of the three coefficients, with their expected values, as outlined in the above table and find percent errors.

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?

Write a hearty and thoughtful account of what you learned in this experiment by way of the setting up and the techniques of experimentation. Elements of the principle need not be given

Data & Data Tables

Name...................................   **Date**.............................

Instructor.............................   **Lab Section**...................

Partner..........................   **Table #**..........................

Table (3A) Dependence of T on θ; $l = 1.00$ m

#	Height by which the jack is raised cm)	p (m)	Time Period $(\sec)$	#	Height by which the jack is raised cm)	p (m)	Time Period $(\sec)$
1	1	0.49		11	11	0.39	
2	2	0.48		12	12	0.38	
3	3	0.47		13	13	0.37	
4	4	0.46		14	14	0.36	
5	5	0.45		15	15	0.35	
6	6	0.44		16	16	0.34	
7	7	0.43		17	17	0.33	
8	8	0.42		18	18	0.32	
9	9	0.41		19	19	0.31	
10	10	0.40		20	20	0.30	

Table (3B) Dependence of T on θ ; $l \neq 1.00$ m

#	Height by which the jack is raised cm)	p (m)	Time Period (sec)	#	Height by which the jack is raised cm)	p (m)	Time Period (sec)
1	1			11	11		
2	2			12	12		
3	3			13	13		
4	4			14	14		
5	5			15	15		
6	6			16	16		
7	7			17	17		
8	8			18	18		
9	9			19	19		
10	10			20	20		