Experiment #1

Simple Pendulum (A)

Principles

Definition

A “simple pendulum” consists of a “bob” suspended by a cord, as shown in Fig (1a). The bob, in general, is spherical and has a finite mass. The cord, though flexible, is un-stretchable; it does not get stretched by the weight of the bob or by its swinging motion.

Function

The function of the pendulum is to “oscillate” i.e. to move to-and-fro about its mean position of suspension, as shown in Fig (1b). To set the pendulum into oscillations, the ball is pulled to one side through some angle θ (keeping the cord stretched) and then released gently. The time, the bob takes to make one complete to-and-fro motion, is called its “time period”, T and is measured in seconds.

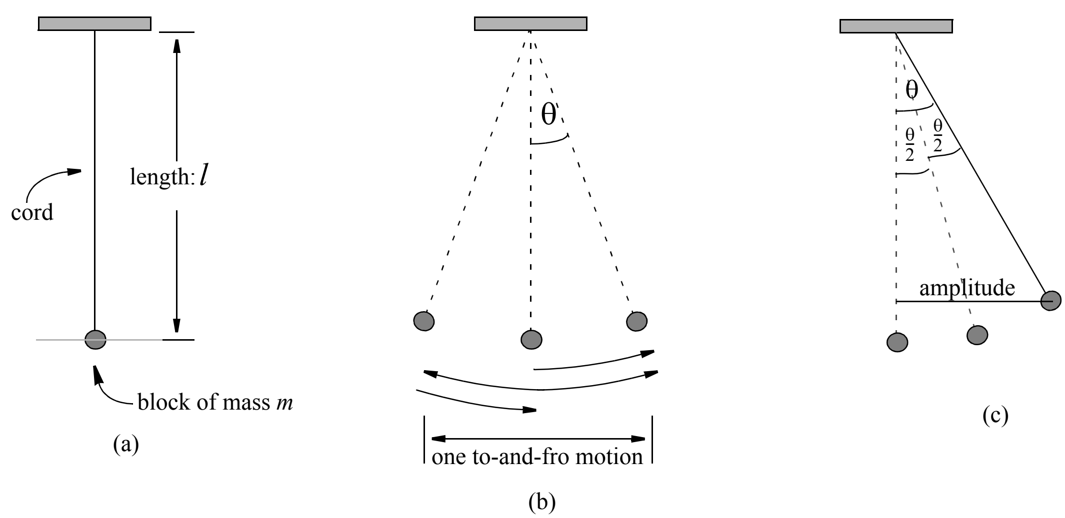

Fig (1) The pendulum and its function

Mathematical Analysis

The system has three variables: the mass of the pendulum m, the length of the cord l, and the angle θ through which the bob is initially displaced. These are identified as the *input parameters* of the pendulum. The time period T, which depends on the magnitudes of the input parameters, is identified as the *output parameter* of the pendulum.

The purpose of the mathematical analysis is to relate or correlate the output parameter to the three input parameters. The analysis is not as simple as one would like it to be. By introducing the constraints: (1) the cord is massless (2) the cord is unstretchable (3) the mass of the system is concentrated at the center of the bob (or more precisely, at its center of mass), the following formula is developed:

$$T = 2\pi \sqrt{\frac{l}{g}} \left[1 + \tfrac{1}{4}\sin^2(\theta/2) + \frac{9}{64}\sin^4(\theta/2) + \ldots\ldots\ldots \right] \qquad \ldots\ldots\ldots\ldots(1)$$

The above result shows that the time period, T, does not depend on the mass of the bob! (Surprised?) It, however, does depend on the remaining two input parameters: the length l of the cord, and the angle θ through which the bob has been displaced.

Objectives of the experiment

To study

(a) the dependence of the time period T of a simple pendulum on its length l,
(b) its independence of the mass m of the bob.

Setting up

Experimentally, one can study the dependence of T only on one input parameter at one time. This requires that we perform two separate experiments.

(A) Dependence of the Time Period of the Pendulum on its Length.

To eliminate the simultaneous dependence on θ, we shall swing the pendulum to an angle $\theta < 5°$. To ensure this, we shall displace the bob by about 3 cm from its mean position. For such small angles, the terms involving powers of "$\sin(\theta/2)$" will be negligibly small. Eqn. (1) then reduces to:

$$T = 2\pi \sqrt{\frac{l}{g}} \qquad \ldots\ldots\ldots\ldots(2)$$

A suitable technique to convert Eqn (2) into the equation of a straight line, $y = mx + b$, will be to take *logs*. First step, however is to separate out the constant terms on the right hand side. We get:

$$T = \left(\frac{2\pi}{\sqrt{g}}\right)(\sqrt{l})$$

Taking *logs* on both sides, Eqn (2), we get:

$$\log T = \log\left(\frac{2\pi}{\sqrt{g}}\right) + \tfrac{1}{2}\log l \qquad \ldots\ldots\ldots\ldots(3)$$

Eqn (3) is comparable to the equation of a straight line: $y = mx + b$ and it is now possible to obtain a straight line graph from experimental data. The straight line will have a slope of 0.500 and a y-axis intercept of:

$$\log\left(\frac{2\pi}{\sqrt{g}}\right) \qquad \ldots\ldots\ldots\ldots(4)$$

To interpret the above intercept, we set $l = 1$ in Eqn (3). As $\log 1 = 0$, we get:

$$\log T = \log\left(\frac{2\pi}{\sqrt{g}}\right)$$

Taking antilog on both sides, we get:

$$T = \frac{2\pi}{\sqrt{g}} \qquad \text{..........(5)}$$

Eqn (5) tells us that $(2\pi)/(\sqrt{g})$ is the time period, of a pendulum of length *one* meter, when oscillated at very small angles. Inserting values of π and g, we get a magnitude for this time of 2.00709 sec.

(B) Independence of the Time Period of the Pendulum of its Mass.

This study is simple and requires no additional mathematical treatment. In fact it is based on common sense. We select a length l of the pendulum, say approximately 1 m (no need to measure it), and find the time period. Then we place a 20 g mass upon it and find the time period again. Then we repeat for 3 additional 20 g masses. We expect the five time periods to be the same up to 2 decimal places. Minor differences will appear in subsequent decimal places because of the slight increase in the length of the cord due to increased tension.

Apparatus Required

(1) A pendulum bob of mass about 80 g,

(2) Vertical support and a piece of an unstretchable cord, approximately 2 meters long,

(3) Advanced electronic timer (multiple memories) with photogate,

(4) One two-meter long and one one-meter, long meter-stick.

Procedure

(A) Dependence of the Time Period of the Pendulum on its Length.

(1) Select 12 values of length l, between the general limits of 1.6 m and 0.3 m. These values of length may not be equally spaced; but should cover the given range. Each length is to be measured from the center of bob to the point of suspension of the cord, as shown in Fig (1a).

(2) Place the photogate in such a way that the bob, when at its mean position, rests in the window of the photogate with the window of the photogate coinciding with the *equator* of the bob. This will enable the infra-red beam to lie along the mid-line of the ball.

(3) Set the electronic timer in the ***Period*** mode and choose (i) 4 decimal places for the measurement of time, (ii) 5 memories. The timer will automatically record five consecutive time periods of a trial to an accuracy of one tenth of one millisecond.

(4) Displace the bob by about three centimeters to one side and gently release it. As the pendulum oscillates, press the *start* key. The timer will measures the designated number of time periods and stop. You may stop the pendulum bob manually at this point. If you

now press the *Enter* key, the timer will automatically calculate and display the average (mean) of the 5 values of time that it recorded. Record this value in the data sheet. By pressing the *Enter* key again, the timer will go back to the normal mode and be ready for the second (and subsequent) sets of data. When a new length of the pendulum has been selected, press the *clear* key. The timer will reset itself and record a fresh set of 5 time periods.

(5) Repeat step 4 for the remaining 11 lengths of the pendulum.

(B) Independence of the Time Period of the Pendulum of its Mass.

(6) Set the length of the pendulum to approximately one meter. Do not measure it.

(7) Reset the timer to record time to 2 decimal places only.

(8) Follow step 4 above and record the average time period.

(8) Place a 20 g mass on the pendulum bob, follow step 4 and record the average time period.

(9) Place a second, a third and finally a fourth 20 g mass upon the pendulum bob (one at a time), follow step 4 (each time) and record the average time period.

(10) The experimental work is completed. Switch off the timer, unplug the adapter and re-arrange apparatus neatly on the table.

Calculations and Graphs

(A) Dependence of the Time Period of the Pendulum on its Length.

(1) Pre Graph Calculations:

(i) take logs of all twelve values of T,

(ii) take logs of all twelve values of l (in meters).

(2) Graph: Plot $\log l$ on x-axis, and $\log T$ on y-axis, using a computer. Instruct the computer to fit a straight line and print (i) its equation, and (ii) the r^2 value. As always, the computer must print the equation with 5 significant decimal digits. Copy the values of (a) slope, and (b) y-axis intercept from the graph in your lab journal.

(3) Post Graph Calculations:

(i) leave the slope alone

(ii) take antilog of the y-axis intercept.

Interpreting "Slopes" and "Intercepts"

The slope represents the nature of dependence of T on l. Eqn. (3) tells us that T is proportional to the square root of l. So we expect the slope of the $\log T$, $\log l$ graph to be one-half or 0.500. The intercept, as shown above, gives us (after taking antilog) the time period of a pendulum of length $l = 1$ m, the magnitude of this time being 2.00709 sec.

The interpretation is now complete. For *slope* we shall write the *power of l* and for *intercept* we shall write *the time period of a pendulum of length one meter.*

Results

(A) Dependence of the Time Period of the Pendulum on its Length.

Compare experimental values of *power of l* and *time period of the pendulum of length one meter*, with their respective expected values. Find percent differences between expected and experimental values.

(B) Independence of the Time Period of the Pendulum of its Mass.

Make a table with two columns. One showing the masses (or sizes) of the 4 pendulum bobs, and the other for their average time periods.

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Data & Data Tables

Name.................................... **Date**..............................

Instructor.............................. **Lab Section**....................

Partner.......................... **Table #**..........................

Table (1) Dependence of T on l

#	length (cm)	length (m)	Average Time Period (sec)
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

Table (2) Independence of T of m.

#	mass	Average Time Period (sec)
1	just the bob	
2	bob + 20 g	
3	bob + 40 g	
4	bob + 60 g	
5	bob + 80 g	

An additional table, just in case........

Table (1) Dependence of T on l

#	length (cm)	length (m)	Average Time Period ($\sec$)
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			