CHAPTER 9

Straight Line Motion On Inclined Planes

AKA

Motion on Ramps or on Slanting or Tilted Surfaces

Henceforth Called the

$\angle\theta$ - Mode Motion

9-00 Prelude

(a) Motion along inclined planes, ramps, tilted or slanting surfaces, represent motion of objects on a surface which is inclined to the horizontal, at some angle $\angle\theta$.

Henceforth we shall call all of the above as $\angle\theta$-mode motion

(b) Euclidean space is spanned by Cartesian coordinates comprising of x-, y- and z-axes. It is customary to assume that the x-axis lies east-west, y-axis lies north-south, and z-axis lies up-down. The three axes make three planes. These are the (x,y) plane, the (y,z) plane and the (x,z) planes.

There are *two* vertical planes.

When only one plane is needed, one generally uses the (x,z) plane. Thus $\angle\theta$-mode will essentially lie in the (x,z) plane. This means that $\angle\theta$ will be measured with respect to the x-axis. If, however, $\angle\theta$ were given to be with respect to y-axis, we shall quickly rotate our reference frame so that our x-axis coincides with the y-direction. Thus, in our reference frame, the inclined plane will always be in the (x,z) plane.

(c) The two trigonometric functions: $sin\theta$ and $cos\theta$ are scaling factors for us. Many physical entities and systems are orientation-dependent. If a physical entity assumes minimum value at $\angle\theta = 0°$, and maximum value at $\angle\theta = 90°$, the scaling factor $sin\theta$ will be used. If, however, its orientation dependence is opposite of the above, the scaling factor $cos\theta$ will be used.

(d) Acceleration due to gravity $\boldsymbol{g}$, acts in a vertically downward direction (i.e. it is directed toward the center of the earth).

(e) An object is affected by $\boldsymbol{g}$ (i.e it accelerates downward) only if it is *not* supported. If the object is supported by another solid object, then it does not accelerate and the effect of $\boldsymbol{g}$ is nullified.

(f) The supporting surface (the inclined plane) is a next-door neighbor of the object because the two are in constant contact with one another. In this chapter we shall treat this neighbor as an un-neighbor. In a future chapter, this *ban* on interaction will be lifted.

(g) All objects and systems of objects are represented by their centers of mass. It is the center of mass that follows the path of motion.

(h) The general form of a quadratic equation is:

$$ax^2 + bx + c = 0$$

The two solutions for x are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quantity $\sqrt{b^2 - 4ac}$ is known as the *discriminant* of the quadratic equation. It has some interesting applications.

(i) Mothers have recipes, chemists have formulae and physicists have equations

9-1 ***In this chapter we shall study the following***

Motion of objects that slide up or down an inclined plane.

When an object is projected upward on an inclined plane, it keeps losing speed until the speed becomes zero, bringing the object to a state of rest. The object stays in the state of rest for an infinitesimally small interval of time and then begins to slide down on its own; it keeps gaining speed as it travels down the incline. Objects that slide down the inclined plane, when released from rest or pushed downward (using an unglued force), move freely and accelerate down the inclined plane. They keep gaining speed until they hit the floor (or something) with a thud. Their motion, therefore, is not uniform and objects are in non-equilibrium states (activity regions).

9-2 ***In this chapter, we shall not study the following:***

(A) Uniform motion on the inclined plane

Escalators are a good example.They move at some convenient angle with respect to the horizontal and travel at uniform speeds. Moving loads on ramps is another example. They are usually pulled up (or down) with uniform speeds. In all such cases the effect of $\mathbf{g}_{eff}$ is neutralized by acceleration obtained from other sources. We shall talk of them when we have learned more about forces.

(B) Motion with uniform accelerations that are either greater or smaller than $\mathbf{g}_{eff}$ i.e. $\mathbf{a}_z < \mathbf{g}_{eff}$ or $\mathbf{a}_z > \mathbf{g}_{eff}$.

(C) Motion on an inclined plane when the supporting surface is a neighbor.

Not that motions of types (B) and (C) do not exist; they do. We shall not deal with them here. This is like being a store owner who tells you that the store does not carry *that* brand of merchandise.

9-3 ***Calculating Effective Acceleration***

An object left on an inclined plane tends to slide downward on its own. As it slides, it picks up speed. The motion of the object, therefore, is an accelerated motion. This apparent *self-induced* motion is due to the earth's pull on the object i.e. $\boldsymbol{g}$. The magnitude of this sliding motion depends on how steep the incline is. If it is sufficiently steep, the slide will be brisk but if the surface is barely inclined, the sliding motion will be minimal.

This observation leads us to conclude that, on an inclined plane, the effectiveness of $\boldsymbol{g}$ is not constant. The smaller the angle of inclination, the smaller is the effectiveness of $\boldsymbol{g}$, and the larger the angle of inclination, the greater is the effectiveness of $\boldsymbol{g}$. We conclude that effectiveness of $\boldsymbol{g}$ is orientation-dependent.

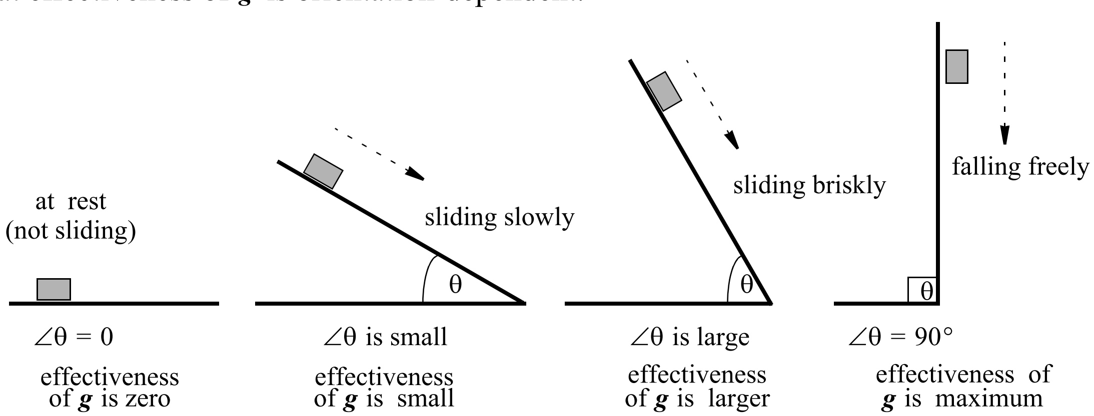

Fig (1) Dependence of the Effectiveness of $\mathbf{g}$ *on the Angle of Inclination of the Inclined Plane.*

It is obvious that for $\angle\theta = 0°$, the inclined plane becomes a horizontal plane where the object is in x-mode and hence the effectiveness of g is zero. We also note, without hesitation, that for $\angle\theta = 90°$, the inclined plane will become a vertical plane. The object will lose contact with the plane and find itself in z-mode. Consequently, it will fall freely; the effect of g will be maximum.

This calls for the use of the scaling factor $sin\theta$. The effective value of g is promptly given by

$$g_{eff} = g sin\theta \qquad ....(1)$$

9-4 Placing the Reference Frame

As observers, we are free to place our reference frames in any orientation we find convenient for us. As all the activity is taking place along the inclined plane, we shall rotate our reference frame in such a way that the x-axis coincides with the inclined plane. We shall call it x'-axis. This will enable us to avoid confusion with the x-axis on a flat surface. The z'-axis will be perpendicular to the x'-axis. This is shown in Fig (2).

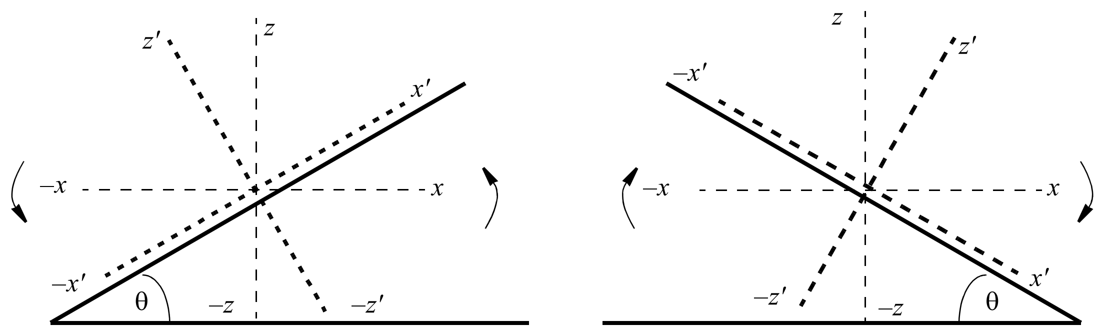

Fig (2) Placement of the Reference Frame For The Two Orientations of the Inclined Plane

9-5 Kinematics of Motion in $\angle\theta$-mode: The Variables

The variables of motion in $\angle\theta$-mode motion are:

d : displacement, along the inclined plane, in a straight line; hence forth written as d

a : uniform acceleration, on the inclined plane, in a straight line, $g sin\theta$

v_o : uniform initial velocity of an object, on the inclined plane, in a straight line, $v_{ox'}$

v : uniform final velocity of an object, on a flat surface in a straight line, $v_{x'}$

t : time, t

9-6 Kinematics of Motion in $\angle\theta$-mode: Algebraic Sign of g_{eff}

A table for the sign of acceleration, relative to that of velocity, is given here for ready reference:

Table 1: Determining the Algebraic Sign of g_{eff}

Direction of motion of the object	Algebraic sign of the velocity vector	Is the speed of the object increasing or decreasing?	Algebraic sign of g_{eff}
Up the inclined plane	+	Decreasing	–
Down the inclined plane	–	Increasing	–

9-7 Importing the "Kinematic" Equations.

We shall now import the kinematic equations from Chapter (6). Both, the regular and the short-form sets will be imported. To adapt those equations to the $\angle\theta$-mode environment, we should paste the subscript x' to velocities. Acceleration $\boldsymbol{a}$ will be replaced by $\boldsymbol{g}_{eff}$. Variable $\boldsymbol{d}$ does not need any alteration.

9-8 Importing the "Regular" Equations

Here are the equations:

Table 2: The Four Kinematic Equations: The "Regular" Equations

Eqn. #	Equation	Identifying the "absentee" parameter in the equation				Name of equation	Is the name useful?
		d	$v_{x'}$	g_{eff}	t		
1	$t = \frac{(v_{x'} - v_{ox'})}{g_{eff}}$ or $v_{x'} = v_{ox'} + g_{eff}\,t$	no	yes	yes	yes	distance independent	not really
2	$d = v_{ox'}\,t + (1/2)\,g_{eff}\,t^2$	yes	no	yes	yes	time dependent	yes
3	$d = (1/2)(v_{x'} + v_{ox'})\,t$	yes	yes	no	yes	acceleration independent	not really
4	$d = \frac{(v_{x'}^2 - v_{ox'}^2)}{2g_{eff}}$	yes	yes	yes	no	time independent	yes

As the left hand sides of these kinematic equations must remain positive (for reasons of being real physical quantities), it is imperative that negative $(v_{x'} - v_{ox'})$ or $(v_{x'}^2 - v_{ox'}^2)$ be accompanied by negative acceleration. Again, should you need to replace $v_{ox'}$ to $v_{x'}$ or vice versa, the algebraic sign of g_{eff} must also be reversed.

9-9 Importing the "Short Form" Equations.

When either $v_{ox'} = 0$ or $v_{x'} = 0$, we should use the *short form* equations. While using these equations, we insert the magnitudes of parameters only and leave out their algebraic signs. The set of four equations is given in table (3), below:

Table 3: The Four Kinematic Equations: The "Short Form" Equations

Eqn. #	Equation	Identifying the "absentee" parameter in the equation				Name of equation	Is the name useful?
		d	$v_{x'}$	g_{eff}	t		
1	$t = v_{x'}/g_{eff}$ or $v_{x'} = g_{eff}\,t$	no	yes	yes	yes	distance independent	not really
2	$d = (1/2)\,g_{eff}t^2$	yes	no	yes	yes	time dependent	yes
3	$d = (1/2)\,v_{x'}\,t$	yes	yes	no	yes	acceleration independent	not really
4	$d = v_{x'}^2/2g_{eff}$	yes	yes	yes	no	time independent	yes

9-10 The Principle of "Matching the Absentee Parameter". The "Deviate" Format

Following diagram is designed to help you with matching the absentee parameter. Please note that when final velocity is not specified, the initial velocity may still be present. Presence of initial velocity will not interfere with our scheme.

∠θ -mode kinematics

Table 4: The Four Kinematic Equations: The "All in One" Table

Eqn. #	Regular Equation	Short Form Equations	Identifying the "absentee" parameter in the equation				Name of equation	Is the name useful?
			d	$v_{x'}$	g_{eff}	t		
1	$t = \frac{(v_{x'} - v_{ox'})}{g_{eff}}$ or $v_{x'} = v_{ox'} + g_{eff}\,t$	$t = v_{x'}/g_{eff}$ $v_{x'} = g_{eff}\,t$	no	yes	yes	yes	distance independent	not really
2	$d = v_{ox'}\,t + (1/2)g_{eff}t^2$	$d = (1/2)g_{eff}t^2$	yes	no	yes	yes	time dependent	yes
3	$d = (1/2)(v_{x'} + v_{ox'})t$	$d = (1/2)v_{x'}\,t$	yes	yes	no	yes	acceleration independent	not really
4	$d = \frac{(v_{x'}^2 - v_{ox'}^2)}{2g_{eff}}$	$d = v_{x'}^2/2g_{eff}$	yes	yes	yes	no	time independent	yes

The "Deviate" Format

Table 5: The Four Kinematic Equations: The "Deviate" Table

Eqn. #	Regular Equation	Short Form Equations	The Deviate Format	Enter values from problem
1	$t = \frac{(v_{x'} - v_{ox'})}{g_{eff}}$ or $v_{x'} = v_{ox'} + g_{eff}\,t$	$t = v_{x'}/g_{eff}$ $v_{x'} = g_{eff}\,t$	h	
2	$d = v_{ox'}\,t + (1/2)g_{eff}t^2$	$d = (1/2)g_{eff}t^2$	$v_{x'}$	
3	$d = (1/2)(v_{x'} + v_{ox'})t$	$d = (1/2)v_{x'}\,t$	g_{eff}	
4	$d = \frac{(v_{x'}^2 - v_{ox'}^2)}{2g_{eff}}$	$d = v_{x'}^2/2g_{eff}$	t	
			$v_{ox'}$	