Straight Up & Straight Down Motion

AKA

Vertical Motion or Motion along z-axis

Henceforth Called the

Z-Mode Motion

(The Story of "Free Fall")

8-0 Prelude

(a) *Up & down*, *into-the-page & out-of-the-page*, *vertical*, *free fall, along z-axis* and *z-mode* are all synonyms and convey the sense of motion being vertically (straight) up or down or both. The word "vertically" excludes up and down motion on inclined surfaces or ramps. An object undergoing such a motion is neither in contact with, nor is supported by another solid surface. For this reason, the object is said to be "falling freely"; even if it is going up. Textbooks discuss *vertical motion* under the title *Free Fall*.

Henceforth we shall call all of the above as "z-mode" motion

(b) Euclidean space is spanned by Cartesian coordinates comprising of x-, y- and z-axes. The z-axis always implies the *straight* up and the *straight* down direction. In this textbook, we shall never use y-axis for describing z-mode motion.

There is *one* vertical axis.

(c) The axes are all independent of one another and this independence is very important indeed. Because of this, goings-on in one axis are completely unknown to other axes.

(d) The three axes make three planes. These are the (x,y) plane, the (y,z) plane and the (x,z) planes. Of these, two planes: the (x,z) plane and the (y-z) plane, are vertical planes.

There are *two* (prominent) vertical planes.

When only one plane is needed, one generally uses the (x,z) plane.

(e) Objects moving in z-mode freely, cannot be in a state of equilibrium. Thus, for the entire duration of their free-fall motion, objects remain in *activity regions*.

(f) All objects and systems of objects are represented by their centers of mass. It is the center of mass that follows the path (trajectory) of motion

(g) While executing the z-mode motion, an object is in direct contact (all the time) with air. Thus air is a neighbor of our object. However, we shall treat air as an un-neighbor.

(h) The general form of a quadratic equation is:

$$ax^2 + bx + c = 0$$

The two solutions for x are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quantity $\sqrt{b^2 - 4ac}$ is known as the *discriminant* of the quadratic equation. It has some interesting applications.

(i) Mothers have recipes, chemists have formulae and physicists have equations

8-1 ***Why is this chapter so long?***

Because z-mode is different from all other modes

8-2 ***Why is z-mode different?***

Unlike other modes, z-mode has a built-in acceleration. This acceleration arises from the basic property *attraction* of material things (chapter 1). Earth, being a very large mass, pulls every material thing toward irs center of mass (its geometrical center). The force of pull produces an acceleration of magnitude 9.80 m/s^2, directed toward its center of mass. This acceleration is known as *acceleration due to gravity*, and we write $\boldsymbol{g}$ for it. We know $\boldsymbol{g}$ from our school days. The words *directed toward the center of earth* translates into a vertically downward direction for this acceleration. The value 9.80 m/s^2 holds good for all material things that lie on or near the surface of earth. Because the center of earth is 6.38 million meters below earth's surface, a departure of a couple of hundred meters (from 6.38 million meters) does not change the value of $\boldsymbol{g}$ to any measurable extent. We can confidently say that the value of $\boldsymbol{g}$ remains 9.80 m/s^2 not only on the surface of earth but couple of hundred meters above or below it.

For the moon, which is roughly 384 million meters away from the center of mass of the earth, the magnitude of the acceleration is only 2.73 millimeters/s^2, instead of 9.8 meter/s^2. Had the moon been at rest (i.e. not circling earth), it would have ended up as a shooting star, long, *long* time ago.

8-3 ***In this chapter we shall study the following:***

(A) Motion of objects that are *dropped*.

Objects that are dropped, fall freely and accelerate toward the center of earth. The motion, therefore, is not uniform and objects are in *activity regions*.

(B) Motion of objects that are *thrown* straight up or straight down.

Such objects do not travel with uniform velocities either. If an object is thrown vertically downward, it will keep gaining speed until it hits the floor (or something) with a thud. Similarly when an object is thrown vertically upward, it keeps losing speed until the speed becomes zero, bringing the object to a state of rest. The object stays in the state of rest for an infinitesimally small interval of time and then begins to fall down on its own; it keeps gaining speed as it falls. It should be noted that in all these situations, the acceleration remains strictly at 9.80 m/s^2.

8-4 ***In this chapter, we shall not study the following:***

(A) Uniform motion in z-mode

Elevators are a good example.They move strictly in z-mode and travel at uniform speeds. Cranes operate in z-mode with uniform speeds. Parachute enthusiasts attain *terminal* velocities that are uniform too. In all such cases the effect of $\mathbf{g}$ is neutralized by acceleration obtained from other sources. We shall talk of them when we have learned more about forces.

(B) Motion with uniform accelerations that are either greater or smaller than $\mathbf{g}$ i.e. $\mathbf{a}_z < \mathbf{g}$ or $\mathbf{a}_z > \mathbf{g}$.

(C) Motion with non-uniform accelerations along the z-axis.

Not that motions of types (B) and (C) do not exist; they do. We shall not study them here. This is like store owners who tell us that they do not carry *that* brand of merchandise.

8-5 Kinematics of Motion in z-mode: The Variables

The variables of motion in *z-mode* are:

d : displacement in z-mode; henceforth called *displacement height* and written as h , m

a : acceleration due to gravity g , m/s^2

v_o : uniform initial velocity of an object in z-mode, v_{oz} , m/s

v : uniform final velocity of an object in z-mode, v_z , m/s

t : time, t , sec

8-6 Kinematics of Motion in z-mode: Determining Displacements

In order to measure heights, we need a reference frame or a reference level. All heights above the reference level are deemed positive and those below it are deemed negative. Displacement h , as per definition, is the shortest straight line distance between two given points, along z-axis.

Figs (1), (2) and (3) illustrate the measurement of heights. These diagrams show a stone which is thrown (vertically) upward from point A , with some initial velocity v_{oz} . The stone stops momentarily at the top of its flight (point B) and then falls down. It hits the ground at point C with some final velocity v_z ; called the *impact* velocity. Our concern with the stone ends here and we do not seek to know what happens to the stone or to its impact velocity [*].

The position of the reference line is arbitrary and is chosen by the observer. The final outcome of observations is independent of this choice. The results of investigations will be the same no matter where the reference line is placed. Figs. (1), (2) and (3) show three (of the many) choices for the placement of the reference line. The distance travelled by the stone is exactly the same in each of the three cases (as expected). The displacement of the stone, however, has the same magnitude in each case but the algebraic signs are different.

The motion of the stone takes place in the (x,z) plane. The reference line itself becomes the x-axis of the (x,z) plane and carries the origin of the 2-D coordinate system.

It is recommended that the reference line be placed at the starting position of the object (stone, in the present discussion) i.e. at "A". This will minimize algebra and maximizes efficiency.

When we study the *energy approach*, this recommendation will be withdrawn.

total distance travelled by the stone

$= 2h_1 + h_2$

displacement of the stone

$= -h_2$

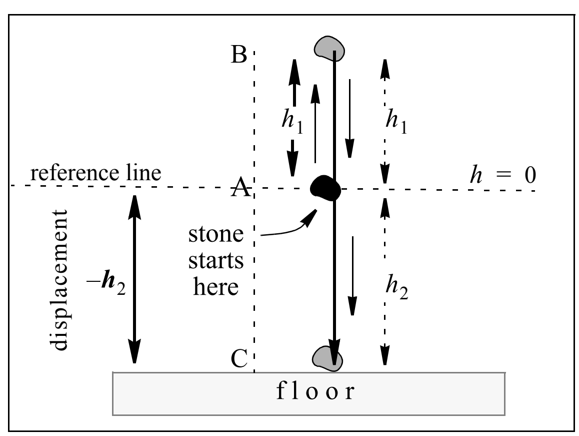

Fig (1) Reference Line placed at A

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[1] To quench the curiosity, however, it will be in order to mention that the impact velocity is quickly absorbed by the earth and the stone comes to rest in few brief moments.

distance travelled by the stone

$= 2h_1 + h_2$

displacement of the stone

$= -\boldsymbol{h}_2$

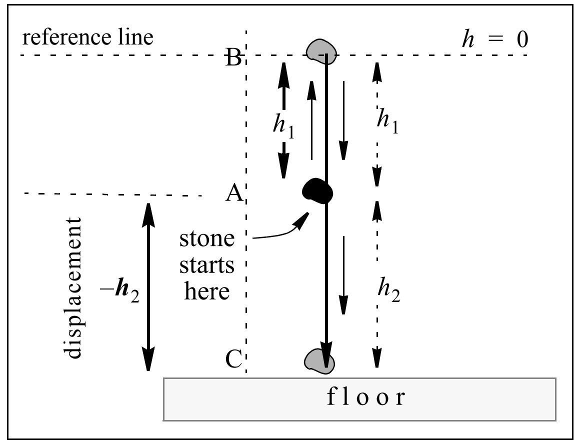


Fig (2) Reference Line placed at B

distance travelled by the stone

$= 2h_1 + h_2$

displacement of the stone

$= +\boldsymbol{h}_2$

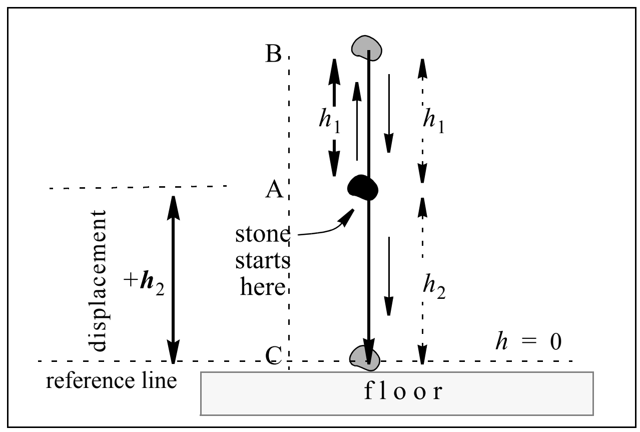


Fig (3) Reference Line placed at C

8-7 *Kinematics of Motion in z-mode: Algebraic Sign of g, "Acceleration due to Gravity"*

Acceleration due to gravity, like any other acceleration, is *acceleration* if it causes the velocity of the object to *increase*. In this case $\boldsymbol{v}_z$ and $\boldsymbol{g}$ will have the **same** algebraic sign. They will either both be positive or both be negative. Again, $\boldsymbol{g}$ is deemed to be *deceleration* if it causes the velocity of the object to decrease. In this case $\boldsymbol{v}_z$ and $\boldsymbol{g}$ will have **opposite** algebraic signs. Thus if $\boldsymbol{v}_z$ is positive, $\boldsymbol{g}$ will be negative and vice versa. As mentioned earlier, this rule is true for *all* accelerations, irrespective of their cast or creed, and has no exceptions.

Whenever an object moves upward, its initial velocity $\boldsymbol{v}_{oz}$ is positive. The acceleration $\boldsymbol{g}$, however, causes the object to slow down. Hence $\boldsymbol{v}_{oz}$ and $\boldsymbol{g}$ need be assigned opposite algebraic signs. Since $\boldsymbol{v}_{oz}$ is positive, $\boldsymbol{g}$ must be negative. When the object moves downward, its velocity $\boldsymbol{v}_z$ is negative, simply because it is moving downward in its own reference frame (which is glued to it). The acceleration $\boldsymbol{g}$, however, now causes the speed of the object to increase. Hence $\boldsymbol{v}_z$ and $\boldsymbol{g}$ need be assigned the same algebraic signs. As $\boldsymbol{v}_z$ is negative, $\boldsymbol{g}$ must also be negative. Fig (4) illustrates the role of $\boldsymbol{g}$ as *acceleration* and as *deceleration*.

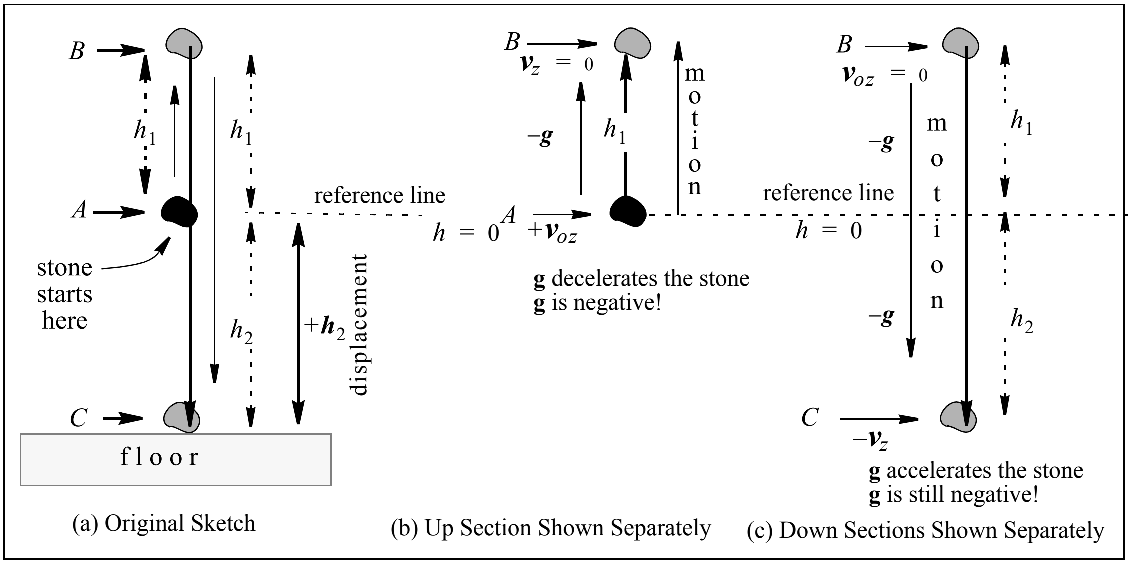


*Fig (4) We have added "**g**" to Fig (1)*

Here is the interesting outcome of this discussion. For the z-mode motion of objects, velocities and heights may be positive and negative but g will always be negative. The only exception will be the case when velocity and height are also negative. In this case all three parameters will be negative. We shall ignore the negative signs (multiply throughout by "-1"). The apparent positive sign of g, in such cases, should not confuse you.

Some people err here. They tell us that if acceleration is along the positive y-axis (positive z-axis according to us), it should be positive. This may lead to serious errors. Please be reminded that in this textbook we shall not use a reference frame for determining the algebraic sign of acceleration.

8-8 Preparing to Solve Problems in z-mode: Translating English into Mathematics

We shall begin by translating some English into Mathematics, for z-mode motion.

Table 1: Translation

	English	Algebraic Signs	Comments
1	object is *dropped*	Here v_{oz} of the object is zero but it will grow in the negative z-direction and be negative; h is negative as it is below the reference line. As explained earlier, g is also negative.	h, v_z and g are all negative. **Ignore signs altogether** v_z is *apparently* positive
2	object is *thrown downward*	Here v_{oz} of the object is negative as the object is going down; and h is negative as it is below the reference line. As explained earlier, g is also negative.	h, v_{oz}, v_z and g are all negative. **Ignore signs altogether** v_z is apparently positive
3	object is *thrown upward*	Here v_{oz} of the object is positive as the object is going up; and h is positive as it is above the reference line. As explained earlier, g is negative.	v_{oz} is positive v_z is negative **Use signs**

8-9 Kinematic Equations in z-Mode

We now import all the kinematic equations from Chapter (6). To adapt these equations to the z-mode environment, we shall paste the subscript z to the velocities. Acceleration $\boldsymbol{a}$ will be replaced by the z-mode acceleration $\boldsymbol{g}$ and the displacement $\boldsymbol{d}$ will be replaced by z-mode displacement $\boldsymbol{h}$.

Table 2: The Four Kinematic Equations:

Eqn. #	Regular Equations	Short Form Equations	Identifying the "absentee" parameter in the equation				Name of equation	Is the name useful?
			$\boldsymbol{h}$	$\boldsymbol{v}_z$	$\boldsymbol{g}$	t		
1	$t = (\boldsymbol{v}_z - \boldsymbol{v}_{oz})/\boldsymbol{g}$ $\boldsymbol{v}_z = \boldsymbol{v}_{oz} + \boldsymbol{g}t$	$t = \boldsymbol{v}_z/\boldsymbol{g}$ $\boldsymbol{v}_z = \boldsymbol{g}t$	no	yes	yes	yes	displacement independent	not really
2	$\boldsymbol{h} = \boldsymbol{v}_{oz}t + (1/2)\boldsymbol{g}t^2$	$\boldsymbol{h} = (1/2)\boldsymbol{g}t^2$	yes	no	yes	yes	time dependent	yes
3	$\boldsymbol{h} = (1/2)(\boldsymbol{v}_z + \boldsymbol{v}_{oz})t$	$\boldsymbol{h} = (1/2)\boldsymbol{v}_z t$	yes	yes	no	yes	acceleration independent	not really
4	$\boldsymbol{h} = \dfrac{(\boldsymbol{v}_z^2 - \boldsymbol{v}_{oz}^2)}{2\boldsymbol{g}}$	$\boldsymbol{h} = (\boldsymbol{v}_z^2)/(2\boldsymbol{g})$	yes	yes	yes	no	time independent	yes

Remember when an object hits the floor, its velocity is not zero!

8-10 Preparing to Solve Problems in z-mode: "Matching the Absentee Parameter" The "Deviate" Format.

Table 3: The Four Kinematic Equations: The *Deviate* Format

Eqn. #	Full Length Equation	Short Form Equations	The Deviate Principle	Enter values from problem
1	$t = (\boldsymbol{v}_z - \boldsymbol{v}_{oz})/\boldsymbol{g}$ $\boldsymbol{v}_z = \boldsymbol{v}_{oz} + \boldsymbol{g}t$	$t = \boldsymbol{v}_z/\boldsymbol{g}$ $\boldsymbol{v}_z = \boldsymbol{g}t$	$\boldsymbol{h}$	
2	$\boldsymbol{h} = \boldsymbol{v}_{oz}t + (1/2)\boldsymbol{g}t^2$	$\boldsymbol{h} = (1/2)\boldsymbol{g}t^2$ $t = \sqrt{h/(4.9)}$	$\boldsymbol{v}_z$	
3	$\boldsymbol{h} = (1/2)(\boldsymbol{v}_z + \boldsymbol{v}_{oz})t$	$\boldsymbol{h} = (1/2)\boldsymbol{v}_z t$	$\boldsymbol{g}$	
4	$\boldsymbol{h} = \dfrac{(\boldsymbol{v}_z^2 - \boldsymbol{v}_{oz}^2)}{2\boldsymbol{g}}$	$\boldsymbol{h} = \dfrac{(\boldsymbol{v}_z^2)}{2\boldsymbol{g}}$ $\boldsymbol{v}_z = \sqrt{2\boldsymbol{g}\boldsymbol{h}}$	t	
			$\boldsymbol{v}_{oz}$	

8-11 Q: What Determines the Duration of Z-Mode Motion (The Activity Time)?

A: The Earth!

We want to tell you now that the duration of time, *for free fall* (in z-mode), is determined by the earth. Strange as it may seem but it is true. The earth pulls everything and this is why things fall. The time of fall (up or down) is therefore, found from the acceleration $\boldsymbol{g}$ that the earth's force of pull imparts to falling objects. The time dependent equation:

$$\boldsymbol{h} = \boldsymbol{v}_{oz}t + (1/2)\boldsymbol{g}t^2$$

assumes special importance and is used generously. One often ends up with quadratic equations but it is a blessing in disguise, as we shall soon find out.

8-12 Neglecting Air Resistance

In what follows, we shall neglect the effect of air resistance on our systems, thereby making air an un-neighbor. It should be pointed out that the effect of air resistance is really not negligible. In general, air resistance will reduce the magnitude of the final velocity significantly. The same will be true of the vertical distance to which an object would rise, for a given initial velocity. With these perspectives, advising one to solve problems to three or five decimal places, appears to be logically insane. With some grin on our chin, we shall not change our policy. The approximation process may be a bit more liberal.

8-13 Types of z-mode Motion

As the z-mode motion, in general, has *straight-up* and *straight-down* sections, we shall come across following four types of motion:

(1) *straight-down* with no *straight-up* segment and vice versa

(2) *straight-up* with equal *straight-down* segment

(3) *straight-up* with larger *straight-down* segment

(4) *straight-up* with shorter *straight-down* segment

These are shown in Fig (6) below.

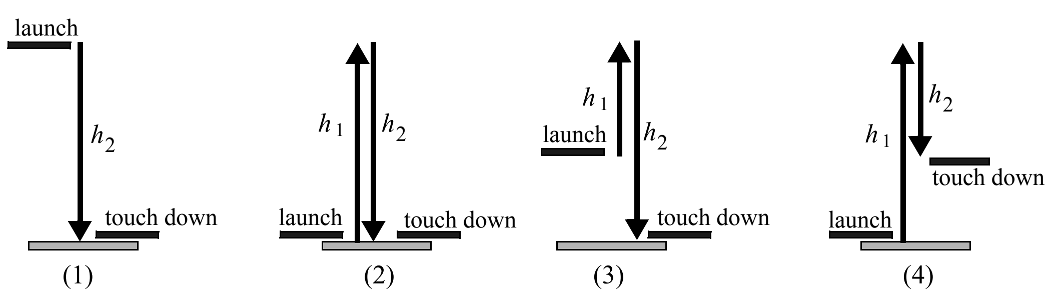


Fig (6) The Four Types of z-mode Motion

8-14 "Straight Down", with no "Straight Up" Section and Vice Versa

Such a motion is shown as type #1 in Fig (6) above. Such a motion is easiest to deal with. Here one of the two velocities ($\boldsymbol{v}_{oz}$ or $\boldsymbol{v}_z$) is usually (but not always) zero. Necessary equations are given Table #s (2), (3). No special considerations are needed.

8-15 "Straight Up" with "Straight Down" Motion

What Goes Up Must Come Down (1)

We now consider z-mode motion of type #s (2), (3) and (4) as shown in Fig (6). "*What goes up, must come down",* is a familiar and well known fact of life. Hence we expect that whenever an object is thrown upward, it must return to the thrower or fall on the ground. As we are considering the *straight* up and *straight* down motions, we expect the object to *retraces* its path during the return journey. The arrows in the diagrams above, should really be lying one on top of the other. For the sake of clarity however, the two have been slightly displaced sideways.

8-16 How Many Seconds in Air? Duration of Motion

The time of a complete (up and down) motion is called the *time of flight* or t_{fl}. It can be determined by separating the *straight-up* and *straight-down* parts of the flight. Because at the top of the flight, the object is momentarily at rest, we find that for the *straight-up* part $\boldsymbol{v}_z = 0$ while for the *straight-down* part $\boldsymbol{v}_{oz} = 0$. We can, therefore, pick a suitable equation from the list of *short form* equations. We choose:

$$\boldsymbol{h} = (1/2)\boldsymbol{g}t^2 \qquad \text{..........(1)}$$

Using the numerical value of $\boldsymbol{g}$ and rearranging, we get:

$$t = \sqrt{\boldsymbol{h}/(4.9)} \qquad \text{..........(2)}$$

We shall use Eqn (2) to find the time of travel for *each* of the two heights. Adding them we get the total time t_{fl}, for which the object stays in air.

$$t_{fl} = t_{up} + t_{down} = \sqrt{\frac{\boldsymbol{h}_1}{4.9}} + \sqrt{\frac{\boldsymbol{h}_2}{4.9}} \qquad \text{..........(3)}$$

Eqn (3) can be further simplified to yield

$$t_{fl} = t_{up} + t_{down} = \left(\sqrt{\frac{1}{4.9}}\right)(\sqrt{\boldsymbol{h}_1} + \sqrt{\boldsymbol{h}_2}) = 0.45175(\sqrt{\boldsymbol{h}_1} + \sqrt{\boldsymbol{h}_2}) \qquad \text{..........(4)}$$

If the two heights are equal, as shown in Fig (6-2), then

$$(\boldsymbol{h}_1 = \boldsymbol{h}_2 = \boldsymbol{h}) \qquad t_{up} = t_{down}$$

and

$$t_{fl} = 2t_{up} = 2t_{down}$$

We get:

$$t_{fl} = 2t_{up} = 2t_{down} = 2 \times \sqrt{\frac{\boldsymbol{h}}{4.9}} = \left(\frac{2}{\sqrt{4.9}}\right)(\sqrt{\boldsymbol{h}}) = 0.9035\sqrt{\boldsymbol{h}} \qquad \text{..........(5)}$$

Squaring both sides:

$$t_{fl}^2 = (0.9035)^2 \times \boldsymbol{h} \qquad \text{strictly for } \boldsymbol{h}_1 = \boldsymbol{h}_2 = \boldsymbol{h}$$

and solving for h we get:

$$\boldsymbol{h} = 1.225t_{fl}^2 \qquad \text{strictly for } \boldsymbol{h}_1 = \boldsymbol{h}_2 = \boldsymbol{h}$$

The distance (height) from the starting position to the *top of the flight* is indeed the maximum height to which the object rises. We call it h_{max} and rewrite the last equation as:

$$\boldsymbol{h}_{max} = 1.225t_{fl}^2 \qquad \text{..........(6)}$$

This is indeed a really interesting and immensely useful equation.The number 1.225 is an *exact* number and has not been rounded off to three decimal places.

It should however, be kept in mind that Eqn (6) is applicable only for those situations where $h_{up} = h_{down}$ i.e. $h_1 = h_2 = h$ ***and*** $g = 9.800$ m/s².

8-17 How Many Meters in Air? Distances of Travel

The distance of a complete (up and down) journey can also be calculated by separating the *up* and *down* parts of travel. As at the top of the flight, the particle is momentarily at rest, we find that for the *up* part $v_z = 0$ while for the *down* part $v_{oz} = 0$. We can, therefore, pick a suitable equation from the list of *short form* equations. We choose:

$$h = (v_z^2)/(2g)$$

and use it to find the distance of travel for *each* of the two heights. Adding them we get: the total distance, traversed by the object.

$$h_{total} = h_{up} + h_{down} = \frac{v_{oz}^2}{2g} + \frac{v_z^2}{2g} = \frac{(v_{oz}^2 + v_z^2)}{2g} = \frac{(v_{oz}^2 + v_z^2)}{19.60} \qquad ....(7)$$

If the two heights are equal, as shown in Fig (6-2), then

$(h_1 = h_2 = h)$ and $v_{oz} = v_z$.

In this case $h_{up} = h_{down}$

and

$$h_{total} = 2h_{up} = 2h_{down} = \frac{2v_{oz}^2}{2g} = \frac{2v_z^2}{2g} = \frac{v_{oz}^2}{g} = \frac{v_z^2}{g} \qquad ....(8)$$

8-18 Velocities of the Object

The initial or the final velocity of an object for a vertical motion is usually found by using either of the two equations. These are:

$$t = (v_z/g) \qquad or \qquad v_z = gt$$

$$h = v_z^2/(2g) \qquad \text{or} \qquad v_z = \sqrt{2gh}$$

In some cases, however, we may find v_z from the discriminant of the quadratic equation. This is explained in *Killing two Bugs with One Stone*, next.

8-19 Killing Two Bugs With One Stone

We now consider a general case where an object starts **upward** with some initial velocity v_{oz} at a certain height and ends up with final (impact) velocity v_z at a different height. Placing the x-axis at the starting point of the object's motion, we set the initial height as zero. The *displacement* of the object may then be positive (basket ball game i.e. h_{up} is greater than h_{down}) or negative (diving in a pool i.e. h_{down} is greater than h_{up}). Let us now choose the time dependent equation:

$$h = v_{oz}t + (1/2)gt^2$$

where h is the z-mode displacement of the trajectory of the object. In each case g is negative as we know very well by now. Rearranging, we will get one of the following two quadratic equations:

$$+h = v_{oz}t - 4.9t^2 \qquad 4.9t^2 - v_{oz}t + h = 0 \qquad h_{up} > h_{down}$$

$$-h = v_{oz}t - 4.9t^2 \qquad 4.9t^2 - v_{oz}t - h = 0 \qquad h_{up} < h_{down}$$

or

$$4.9t^2 - v_{oz}t \pm h = 0$$

The solutions for t will be either:

$$t = \frac{\boldsymbol{v}_{oz} \pm \sqrt{(\boldsymbol{v}_{oz})^2 - 19.6 \times \boldsymbol{h}}}{9.8} \qquad (\boldsymbol{h} \text{ is positive}) \qquad \text{..........(9)}$$

or

$$t = \frac{\boldsymbol{v}_{oz} \pm \sqrt{(\boldsymbol{v}_{oz})^2 + 19.6 \times \boldsymbol{h}}}{9.8} \qquad (\boldsymbol{h} \text{ is negative}) \qquad \text{..........(10)}$$

All positive values of t will be acceptable in Physics and *will* form valid *answers*.

The *fun* part of this treatment is that the discriminants of the above equations, turn out to be $\boldsymbol{v}_{impact}$:

$$\boldsymbol{v}_{impact} = \boldsymbol{v}_z = \sqrt{\boldsymbol{v}_{oz}^2 \pm 19.6 \times \boldsymbol{h}} \qquad \text{..........(11)}$$

The discriminant gives us the final or the impact velocity of the object in the z-mode! It is easy to see that the discriminant is nothing but a rearrangement of the time independent equation:

$$h = \frac{(\boldsymbol{v}_z^2 - \boldsymbol{v}_{oz}^2)}{2\boldsymbol{g}}$$

where g is replaced by 9.80. This cleverness on our part enables us to find two answers in one treatment ("*to kill two bugs with one stone*": as they say).

A summary of the formulae for each of the three types of up-down motion is given in Table (4). The table also includes formulae for the *down-only* mode of vertical motion. We must remind you that $\boldsymbol{h}$ occurring in the formulae is vertical displacement and not the actual (total) distance travelled by the object.

8-20 A Complete Arsenal of Weapons.

Following is an ***"all you need"*** table for dealing with z-mode motion:

Table 4: "Straight Up"-"Straight Down" Motion & Fun Formulae For Connoisseurs

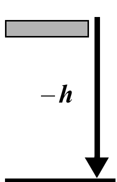
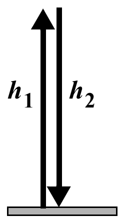
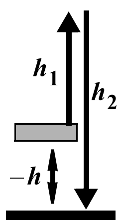
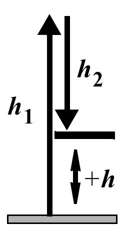
Modes of up-down motion	Description & Signs of $\boldsymbol{h}$, $\boldsymbol{v}_{oz}$, & $\boldsymbol{g}$	Formulae		
		$(\boldsymbol{v}, t)$	$(t, \boldsymbol{h})$	$(\boldsymbol{h}, \boldsymbol{v})$
	(1/2) of a Symmetrical Trajectory $h = -\boldsymbol{h}$ $-\boldsymbol{h}$ $-\boldsymbol{v}_{oz}$ $-\boldsymbol{g}$ ignore the negative sign throughout	if dropped: $\boldsymbol{v}_{oz} = 0$ $\boldsymbol{v}_z = (9.8)t_{down}$ $t_{down} = (0.1020408)\boldsymbol{v}_z$ if thrown: $\lvert\boldsymbol{v}_{oz}\rvert > 0$ $\boldsymbol{v}_z = \boldsymbol{v}_{oz} + (9.8)\,t_{down}$	if $\boldsymbol{v}_{oz} = 0$ $\quad \boldsymbol{h} = (4.9)t^2$ $t_{down} = \sqrt{(0.20408)\boldsymbol{h}}$ $= (0.45175)\sqrt{\boldsymbol{h}}$ if $\lvert\boldsymbol{v}_{oz}\rvert > 0$ $t_{down} = \dfrac{-\boldsymbol{v}_{oz} \pm \sqrt{\boldsymbol{v}_{oz}^2 + (19.6)\,h}}{(9.8)}$	$\boldsymbol{h} = (0.05102041)\,\boldsymbol{v}^2$ $\boldsymbol{v}_z = (4.42719)\sqrt{\boldsymbol{h}}$ $\boldsymbol{h} = \dfrac{\boldsymbol{v}_z^2}{2\boldsymbol{g}}$ $\boldsymbol{v}_z = \boldsymbol{v}_{oz} + \boldsymbol{g}t$ $\boldsymbol{h} = \dfrac{\boldsymbol{v}_z^2}{2\boldsymbol{g}} - \dfrac{\boldsymbol{v}_{oz}^2}{2\boldsymbol{g}}$

Table 4: "Straight Up"-"Straight Down" Motion & Fun Formulae For Connoisseurs

Modes of up-down motion	Description & Signs of h, v_{oz}, & g	Formulae		
		(v, t)	(t, h)	(h, v)
	Symmetrical Trajectory. $h = 0$ $h = 0$ $+v_{oz}$ $-g$	$t_{up} = t_{down} = (1/2)t_{fl}$ $v_{oz} = v_z = (9.8)t_{up}$ $t_{up} = (0.1020408)v_{oz}$	$h = (4.9)t^2$ $t_{up} = t_{down} = (1/2)t_{fl} = \sqrt{(0.20408)h} = (0.45175)\sqrt{h}$ $t_{fl} = t_{up} + t_{down} = (0.9035)\sqrt{h}$ $h = (1.225)t_{fl}^2$	$v_{oz} = v_z = v$ $h_1 = h_2 = h$ $h = (0.05102041)\, v^2$ $v_z = (4.42719)\sqrt{h}$ $h_1 = h_2 = \frac{v_{oz}^2}{2g} = \frac{v_z^2}{2g}$
	"Diving in a Pool" variety $h < 0$ $-h$ $+v_{oz}$ $-g$	$v_{oz} = (9.8)t_{up}$ $t_{up} = (0.1020408)v_{oz}$ $v_z = (9.8)t_{down}$ $t_{down} = (0.1020408)v_z$	$t_{fl} = \frac{v_{oz} \pm \sqrt{v_{oz}^2 + (19.6)h}}{(9.8)}$	From Discriminant $v_z = \sqrt{v_{oz}^2 + (19.6)\,h}$ $h_{max} = \frac{v_z^2}{2g}$ $h_{total} = \frac{v_{oz}^2}{2g} + \frac{v_z^2}{2g}$
	"Basket Ball" variety $h > 0$ $+h$ $+v_{oz}$ $-g$	$v_{oz} = (9.8)t_{up}$ $t_{up} = (0.1020408)v_{oz}$ $v_z = (9.8)t_{down}$ $t_{down} = (0.1020408)v_z$	$t_{fl} = \frac{v_{oz} \pm \sqrt{v_{oz}^2 - (19.6)h}}{(9.8)}$	From Discriminant $v_z = \sqrt{v_{oz}^2 - (19.6)\,h}$ $h_{max} = \frac{v_{oz}^2}{2g}$ $h_{total} = \frac{v_{oz}^2}{2g} + \frac{v_z^2}{2g}$

8-21 "straight up" with "straight down".

What Goes Up, Must Come Down (2)

Let's face it, in almost all real life cases, an object *does not* retrace its path on return journey. Objects are usually displaced sideways and they do not return to a position vertically above or vertically below their starting positions. Let us suppose that the objects moves laterally and ends up landing in a different place; as shown in Fig (7). I would now tell you what to do to solve for the flight time in such cases.

If you want the flight time or the z-mode velocities (initial or final or both) or z-mode displacements, you need do nothing! Yes, absolutely nothing! The answers we would get by using the above mentioned techniques will be truly correct. Guaranteed!

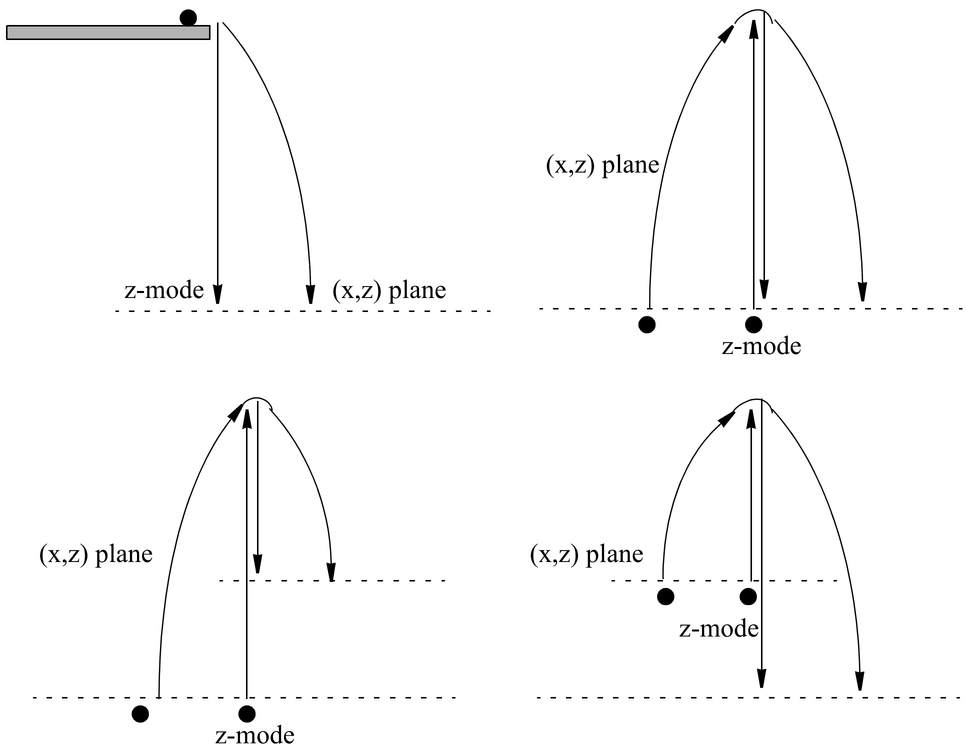


Fig (7) "Objects Do Not Retrace Their Paths

As long as we confine ourselves to z-mode, we need not worry about the lateral (sideways) displacement or sideways movement of the objects. This is a direct consequence of the independence of coordinate axes! The vertical motion of an object is independent of the object's horizontal motion. Neither is affected by the other. As long as we are considering the vertical heights (as opposed to slanting heights) we are in z-mode and as such we are entitled to the use of the formulae that belong to this mode. The fact that the object travelled sideways as well, is of no concern to us at all!

Please be informed that whether the sideways displacements of the objects is a millimeter or a mile, its z-mode parameters will not change.

In a future chapter, we shall study flights of objects of the types shown in Fig (7). Such flights will be quickly split up into x-mode and z-mode. For the z-mode part, we shall simply import Table (2) and (3) and use all the techniques that we have learned in this chapter.

8-22 *Gravity & weight*

Acceleration due to gravity is built into our system (genetic?). It can never be switched off! It may, however, be rendered ineffective by placing the object on something hard. When we are sitting on a chair or standing up on the floor or running around on the ground, we do not sink in the ground to accelerate towards the center of earth. The question is where did earth's pull go? The answer is that it reappeared as *weight*. We must understand that the earth does not stop pulling us downward. If an object does not get accelerated, it acquires weight.

This is really bizarre! Objects either accelerate at 9.8 m/s^2 toward the center of earth and have no weight, or, they do not accelerate and have weight! We state:

If we are falling down freely, we have no weight.

If we are not falling down, we have weight!

Just for fun, imagine a scale which is glued to my feet. As I look down on it, I see my weight displayed upon it. If I jump from a tall building and keep looking down on the scale, I shall find that the scale reads zero! I shall be accelerating continuously and my speed of fall will increase roughly by 22 mph, *every second* of fall but the scale will stubbornly read zero! Like astronauts in outer space, *I shall have no weight*!

Why do you think people go Bunjee jumping? (to have no weight)

Some of you may find the above to be illogical and hard to believe. On a windy day, try facing the wind by standing still. You will experience gusts of air hitting your body. Now run *with* the wind, with the same speed as the wind. You will notice that the effect of wind has disappeared, as if it stopped blowing. If you now stop running, the gusts of air will immediately begin hitting you.

8-23 Please do not read this section

A confusing way of saying the same thing is to state that: you either have weight and no $\boldsymbol{g}$ or you have $\boldsymbol{g}$ but no weight. To add to the confusion, we tell you that weight is due to $\boldsymbol{g}$; i.e. if there is weight, there must be $\boldsymbol{g}$. (Note: Having no $\boldsymbol{g}$ means that you are not accelerating toward the center of earth at all.)

It was precisely for this reason (having weight and no $\boldsymbol{g}$) that we did not talk of this ever-present acceleration in the previous chapter, while discussing the x-mode motion. For objects moving on a flat surface where they are being supported by a solid surface) $\boldsymbol{g}$ remains in limbo, or is inactive or dormant or sleeping or..... Thus for x-mode, we ignored $\boldsymbol{g}$ because it was in limbo. Since $\boldsymbol{g}$ was in limbo therefore objects had weight. Next thing you know, (still in x-mode) we ignored the weight because it is a z-mode thing and as such, it is not recognized in x-mode.

Such is life!

8-24 Alternative ways of making earth's pull ineffective

Earth's pull can be made ineffective in several ways. One of course is placing the object on a hard (solid) surface, thereby letting the *normal* force neutralize the earth's force of pull. A mountain resting on hard ground or a pencil lying on a table are some examples. Another way is to suspend the object by a cord, rope or a cable. The *tension* force developed in the cord (or the rope or cable) neutralizes earth's force of pull. Consider a chandelier as a familiar example. Still another way is to place the object in a liquid of equal or higher density. The object floats on the surface of the liquid. The force that neutralizes earth's force of pull here, is the force of *buoyancy* or the *buoyant* force" Canoes and Aircraft Carriers are good examples. Finally, the neutralization of the weight force by *centripetal* force, causes the astronauts to be weightless in space ships.

8-25 A fraction of "g" and a fraction of "weight"

In the next chapter we shall talk of *inclined planes* or *ramps*. An inclined plane is neither x-mode nor z-mode. It is an *in-between* mode where the object is in a state in between one of having weight and no $\boldsymbol{g}$ (x-mode) and another having $\boldsymbol{g}$ and no weight (z-mode). The *normal* force in such cases is not sufficient to counterbalance or neutralize the weight force. A fraction of the weight survives and so does a fraction of acceleration $\boldsymbol{g}$. The fraction of weight that survives is called *partial* weight. Its magnitude depends on the angle of inclination $\angle\theta$ and is given by $(\text{weight} \times \sin\theta)$