CHAPTER 6

Kinematics General

6-1 Why Kinematic Equations?

Kinematic equations have been developed to enable us to solve for one equilibrium state provided that the other is given. The word *kinematics* means *study of motion*, or the *study of the traits of motion*, to be more specific.

6-2 The Four Variables of Kinematics

The equilibrium states (i) of rest and (ii) of uniform motion, led us to develop the concepts of (i) velocity $\boldsymbol{v}$ and (ii) acceleration $\boldsymbol{a}$. Together with displacement $\boldsymbol{d}$ and time t, we have collected four entities which will, from now on, be called the four parameters of kinematics. Expressed in the interval form, we shall have:

$$\Delta \boldsymbol{x} = \boldsymbol{d} = \boldsymbol{x} - \boldsymbol{x}_o \qquad \Delta \boldsymbol{v} = \boldsymbol{v} - \boldsymbol{v}_o \qquad \Delta t = t - t_o$$

Please note that $\boldsymbol{x}_o$ and $\boldsymbol{x}$ represent positions of the object in the two equilibrium states of rest, associated with an appropriate activity region. Similarly $\boldsymbol{v}_o$ and $\boldsymbol{v}$ represent velocities of the object in the two equilibrium states of uniform motion, associated with a similar appropriate activity region.

As for the acceleration $\boldsymbol{a}$, in a no-jerk physics course, $\boldsymbol{a}$ will remain $\boldsymbol{a}$ and will not be expressed in the interval form.

It is important to note that the velocities $\boldsymbol{v}_o$ and $\boldsymbol{v}$ are uniform velocities. The moment $\boldsymbol{v}_o$ enters the activity region, it becomes *instantaneous* velocity. All through the activity region, it remains instantaneous as its magnitude must keep changing continuously. By the time it has reached the end of the activity region, it has gotten converted to $\boldsymbol{v}$, which is still an instantaneous velocity. At this point, we hand it over to the second equilibrium state and our concern with it ends. The equilibrium state, however, treats it as uniform velocity.

Physics is concerned with the activity region only. Like a transporter, we pick up the velocity outside of one equilibrium state and unload it outside the other equilibrium state. For us, the velocity is always instantaneous. Whatever the status of these velocities, is or was outside of the activity region, is of no concern to us.

6-3 Developing Kinematic Equations

We shall develop the kinematic equations from a consideration of the activity region diagram of Fig (1) of Chapter 5. This diagram shows an object (such as a car) initially travelling at uniform velocity $\boldsymbol{v}_o$. It enters the activity region and emerges from it with the uniform velocity $\boldsymbol{v}$. It spends an interval of time $\Delta t = t - t_o = t$ (where we set $t_o = 0$), in the activity region. The said diagram (reproduced below for ready reference) is converted into a graph by plotting time on the x-axis and velocity on the y-axis.

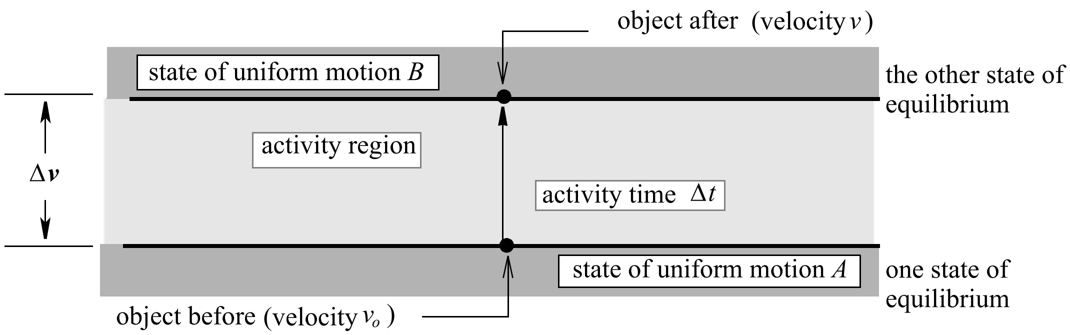

Fig (1) An Activity Region Diagram

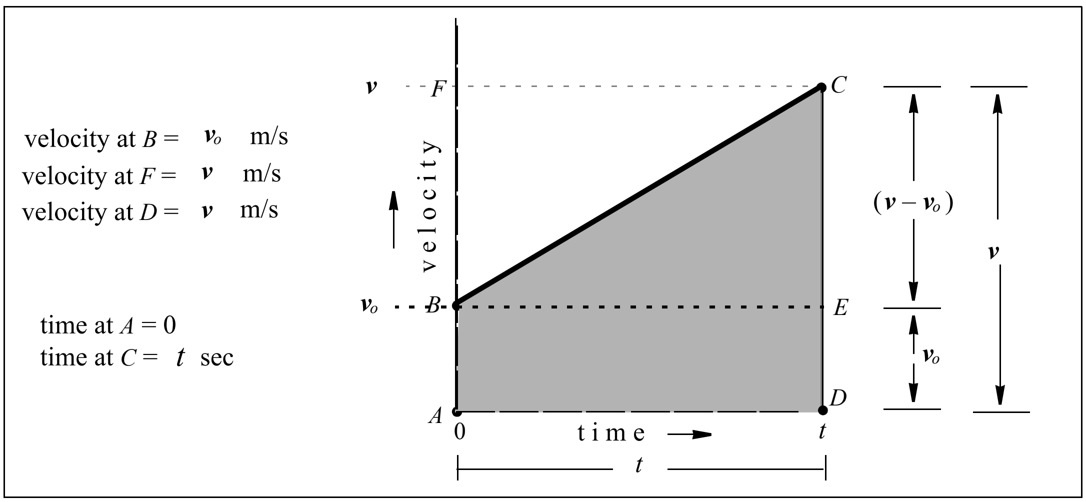

Fig (2) Converting the "Activity Region" into a Graph

It is easy to see that the slope of a velocity *versus* time graph is acceleration:

$$\frac{\Delta y}{\Delta x} = \frac{\Delta v}{\Delta t} = \frac{m/s}{s} = m/s^2 = \boldsymbol{a} \qquad \text{..........(1)}$$

while the area under the straight line equals the distance travelled by the object (its *displacement*) in the activity region:

$$(\Delta y) \times (\Delta x) = \left(\frac{m}{s} \times s\right) = m = \boldsymbol{d} \qquad \text{..........(2)}$$

In the graph, shown in Fig (2), the first equilibrium state of Fig (1) is represented by the vertical line AB. Here the object is cruising at velocity $\boldsymbol{v}_o$. The start of the activity region is marked by setting $t = 0$. The second equilibrium state of Fig (1) is represented by the vertical line DC. Here the object is cruising at velocity $\boldsymbol{v}$. The end of the activity region is marked by setting $t = t$. The activity itself is represented by the thick line BC. As shown above, the slope of line BC represents the acceleration of the object $\boldsymbol{a}$. The shaded area represents the displacement $\boldsymbol{d}$, of the object during the activity.

6-4 Step (a): The slope of the Straight Line

To find $\boldsymbol{a}$ we take the slope of the straight line BC:

$$a = \frac{CE}{BE} = \frac{CE}{AD} = \frac{(\boldsymbol{v} - \boldsymbol{v}_o)}{t} \qquad \text{..........(3)}$$

$$a = (v - v_o)/t \qquad \text{..........(4)}$$

Rearranging:

$$(v - v_o) = a\,t \qquad \text{..........(5)}$$

$$t = \frac{(v - v_o)}{a} \qquad \text{..........(6)}$$

6-5 Step (b); The Area Under the Straight Line

As stated before, the area of the region $ABCD$ in Fig (2) is the displacement d of the object in the activity region. Basically $ABCD$ is a trapezoid, but it can be configured in three different ways. Correspondingly, the displacement d can be worked out in three different forms. The configurations are:

(i) a trapezoid

(ii) the sum of a rectangle and a triangle, and

(iii) the difference of two triangles.

6-5a Trapezoid as a Trapezoid

The trapezoid is shown in Fig (3).

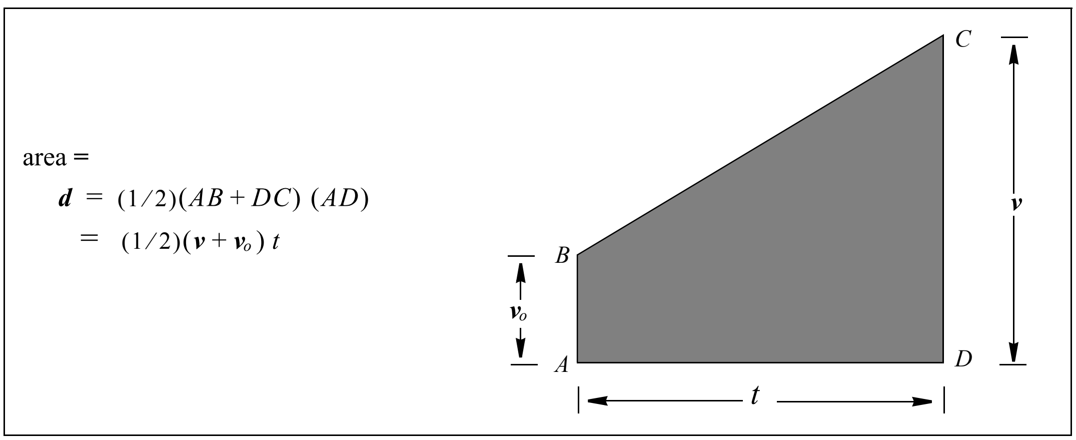

Fig (3) the Trapezoid

The area of this trapezoid is found by using the standard formula:

area of a trapezoid = (average height) x (base)

$$= \{(1/2)(AB + DC)\} \times (AD)$$

$$d = (1/2)(v + v_o)\,t \qquad \text{..........(7)}$$

6-5b Trapezoid as the Sum of a Rectangle and a Triangle

The rectangle and the triangle are shown in Fig (4).

$$d = d_1 + d_2 = \text{(area of rectangle } ABED\text{)} + \text{(area of triangle } BEC\text{)}$$

$$d = v_o t + (1/2)(t)(v - v_o)$$

Plug-in the value of $(v - v_o)$ from Eqn (5), to get:

$$d = v_o t + (1/2)a t^2 \qquad \text{..........(8)}$$

$$d = d_1 + d_2$$

$$d_1 = (AB) \times (AD) = (v_o)(t)$$

$$d_2 = (1/2)(BE)(EC)$$

$$= (1/2)(t)(v - v_o)$$

$$= (1/2)(t)(at)$$

$$= (1/2)(at^2)$$

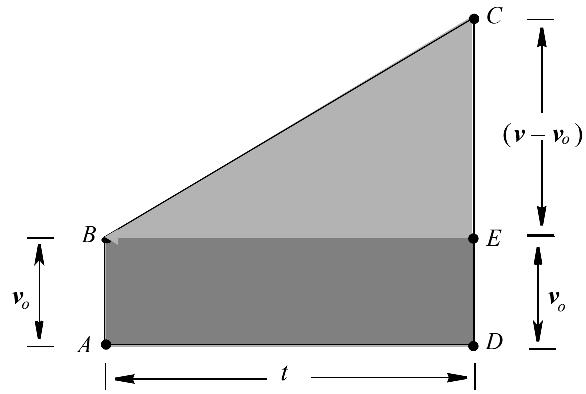

Fig (4) Trapezoid as the sum of a rectangle and a triangle

6-5c Trapezoid as the Difference of Two Triangles

The two triangles are shown in Fig (5).

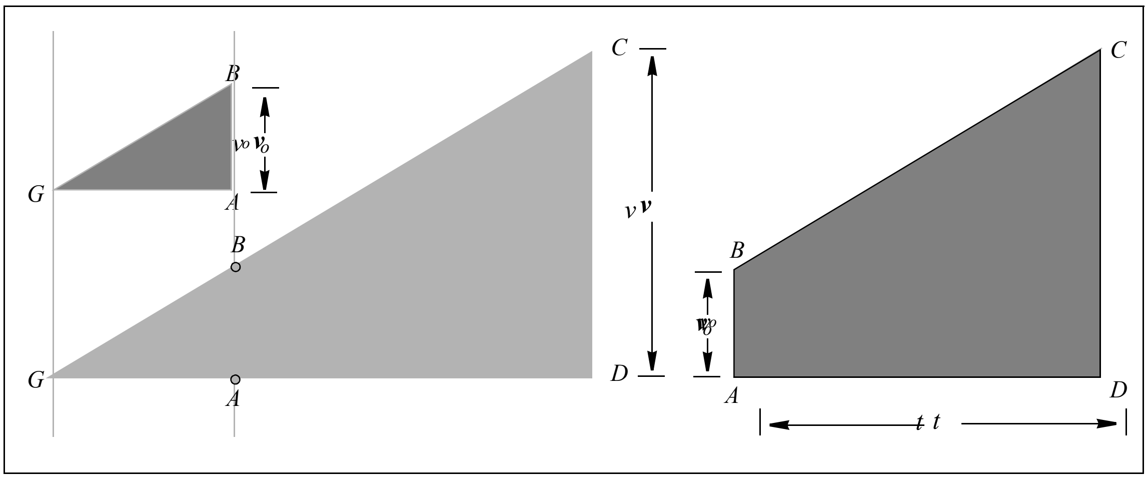

Fig (5) Trapezoid as the difference of two triangles

(area of trapezoid $ABCD$) = (area of triangle GDC) – (area of triangle GAB)

Let the area of triangle GDC be d_1 and that of the triangle GAB be d_2. Then:

$$d = d_1 - d_2 = (1/2)(GD \times DC) - (1/2)(GA \times AB)$$

$$= (1/2)(v)(GC) - (1/2)(v_o)(GA) \qquad ....(9)$$

To determine the length GC, we consider the slope of line GC which, being an extension of line BC, is acceleration:

$$a = \frac{CD}{GC} = \frac{v}{GC}$$

or

$$GC = \frac{v}{a}$$

Similarly it can be shown that:

$$GA = \frac{v_o}{a}$$

Plug-in these values of GC and GA in Eqn (9) to get:

$$d = \frac{v^2}{2a} - \frac{v_o^2}{2a}$$

Or

$$d = \left(\frac{v^2 - v_o^2}{2a}\right) \qquad \text{..........(10)}$$

This completes our investigation of kinematic equations.

6-6 *The Set of Four Kinematic Equations*

The four kinematic equations are presented in a Table (1) below. It is interesting to note that of the four parameters: *displacement* d, *final velocity* v, *acceleration* a, and *time* t, the first kinematic equation doesn't have the first parameter, the second doesn't have the second, the third doesn't have the third and the fourth doesn't have the fourth.

Table 1: The Four Kinematic Equations; The General Equations

Eqn. #	Equation	Identifying the *absentee* parameter in each equation				Name of equation	Is the name useful?
		d	v	a	t		
1	$t = \frac{(v - v_o)}{a}$ or $v = v_o + at$	*absent*	present	present	present	displacement independent	not really
2	$d = v_o t + (1/2)at^2$	present	*absent*	present	present	final velocity independent ---------------- *time dependent*	not really --------- *yes*
3	$d = (1/2)(v + v_o)\,t$	present	present	*absent*	present	acceleration independent	not really
4	$d = \left(\frac{v^2 - v_o^2}{2a}\right)$	present	present	present	*absent*	*time independent*	*yes*

As the left hand sides of these kinematic equations must remain positive (for reasons of being real physical quantities), it is imperative that negative $(v - v_o)$ or $(v^2 - v_o^2)$ be accompanied by negative a.

6-7 *The Principle of "Reversibility of Motion"*

Under the influence of uniform (or constant) acceleration, and in the absence of dissipative elements, motion is reversible. If an object accelerates from v_o to v in time t, travelling a distance d, then it will slow down from v to v_o in the same time t and will traverse the same distance d provided, of course, that the *magnitude* of acceleration is the same. In the former case the acceleration will cause the speed to increase while in the latter case, it will cause the speed to decrease. This principle acquires significant importance when dealing with situations where one of the two equilibrium states is a state of rest. The principle also tells us that if we are given v instead of v_0 (eqn 2, table 1) then all we need to do is to replace v_0 by v and $+a$ by $-a$. Please see Example (1) in section 6-12.

6-8 Kinematic Equations When One Equilibrium State is State of Rest

In a fair number of situations, objects under study, either begin to move from rest or slow down to a complete stop. In the first case, v_o is zero and acceleration is positive; in the second, v is zero and acceleration is negative. If we set $v_o = 0$ with $+a$ for objects starting from rest, or $v = 0$ with $-a$ for objects coming to a complete stop, we get one simpler set of four kinematic equations that will hold for both situations. The velocity term v will represent the non-zero velocity; (initial or final). Likewise, a will represent acceleration or deceleration, irrespective of its algebraic sign. These equations are included in table (2) below. We should regard them as *short form* equations.

To use short form equations, one needs the magnitudes of parameters only and one should not even think of using their algebraic signs.

6-9 The Principle of "Matching the Absentee Parameter" & the "Deviate" Format

Soon enough you will be solving problems. You will observe that problems (no matter how hard or how easy) only mention 3 parameters (altogether), at one rime. One is always *left out*. To solve the problem, all one has to do is to pick up the equation, the *absentee* parameter of which, is the same as the *left-out* parameter in the problem. For example, if the problem doesn't mention the final velocity, then v is the *left-out* parameter. To solve this problem, we shall pick-up the equation in the second row, whose *absentee* parameter is also v. The *Principle of Matching the Absentee Parameter* can now be stated as:

Choose a kinematic equation whose *absentee* parameter is the same as the *left-out* parameter of the problem.

This principle has been molded into a format, called the *Deviate* format. It is shown in Table (2), together with the *full length* and the *short form* equations.We enter the values of parameters in the last column from the problem to be solved. One of the 5 places in the last column, will always be found to be left blank. So we use the equation from that row (*full length* or *short form*) and solve the problem.

Table 2: The Four Kinematic Equations; The General Equations

Eqn #	Full length equation	Identifying the *absentee* parameter in each equation				Short form equations	The *Deviate* principle	Enter values from problem
		d	v	a	t			
1	$t = \frac{(v - v_o)}{a}$ or $v = v_o + at$	*absent*	present	present	present	$t = v/a$	d	
2	$d = v_o t + (1/2)at^2$	present	*absent*	present	present	$d = (1/2)at^2$	v	
3	$d = (1/2)(v + v_o)t$	present	present	*absent*	present	$d = (1/2)vt$	a	
4	$d = \left(\frac{v^2 - v_o^2}{2a}\right)$	present	present	present	*absent*	$d = \frac{(v^2)}{2a}$	t	
							v_o	

6-10 *Why "Deviate"?*

If we read out the letters d, v, a and t rather quickly, the overall sound effect will resemble the word *deviate.* So, being in a light mood, we coined the name *deviate* for our format.

6-11 *The "Deviate" Format*

Table 3: The Four Kinematic Equations; The "Deviate" Format

Eqn #	Full length equation	Short form equations	The *Deviate* principle	Enter values from problem
1	$t = \frac{(v - v_o)}{a}$ or $v = v_0 + at$	$t = v/a$	d	
2	$d = v_o t + (1/2)at^2$	$d = (1/2)at^2$	v	
3	$d = (1/2)(v + v_o)\,t$	$d = (1/2)vt$	a	
4	$d = \left(\frac{v^2 - v_o^2}{2a}\right)$	$d = \frac{(v^2)}{2a}$	t	
			v_o	

6-12 *An Example*

Sheila, while bicycling, wishes to catch up with her friend Tequila, who is moving at 8 m/s, ahead of her, on a straight road. To do so, Sheila accelerates uniformly at 0.1 m/s^2 for 12 seconds and catches up with her friend. For what distance did Sheila accelerate?

Try the *deviate* table: As both, the displacement d and initial velocity v_0 are not given, our wonderful format will not work, unless we solve the problem in 2 steps. To solve it in one step only (the *miss* principle) we look for a formula that uses only one velocity. Such a formula is formula #2 but it uses v_0 something that we do not know. So we take help from the theorem of the reversibility of motion and change v_0 to v and replace $+a$ by $-a$;

$$d = (8 \times 12) - (1/2)(0.1)(12^2) = 96 - 7.2 = 88.80 \text{ m}$$

So Sheila travelled 88.80 m during the acceleration period.

6-13 *The Role of Mass*

You may find it suspiciously intriguing that in all the preceding discussions, we never talked of mass or brought it out in our equations. This shows that traits of motion are independent of mass.

Even though *the traits of motion* are exactly the same for all objects and systems of objects, irrespective of their shapes, sizes **and even masses**, *masses* do not disappear from the scene completely. They remain in the background. You must understand that it is only the *mass* that can have *acceleration*. Hence only objects that have masses can have acceleration. Objects without masses (such as photons) can have velocities but they will never have accelerations!