CHAPTER 5

The Second State of Equilibrium

The Concept of *Acceleration*

5-1 The second State of Equilibrium

The second state of equilibrium is: *state of uniform motion in a straight line*, or simply *state of uniform motion* The words *in a straight line*, though not mentioned explicitly every time, will be implied always.

5-2 Two Equilibrium States of Uniform Motion

Let A and B be two equilibrium states of uniform motion. Objects in these states will be travelling at uniform velocities. Let these velocities be $\boldsymbol{v}_o$ and $\boldsymbol{v}$ and let them be $\Delta\boldsymbol{v}$ m/s different from one another. We construct the following activity diagram:

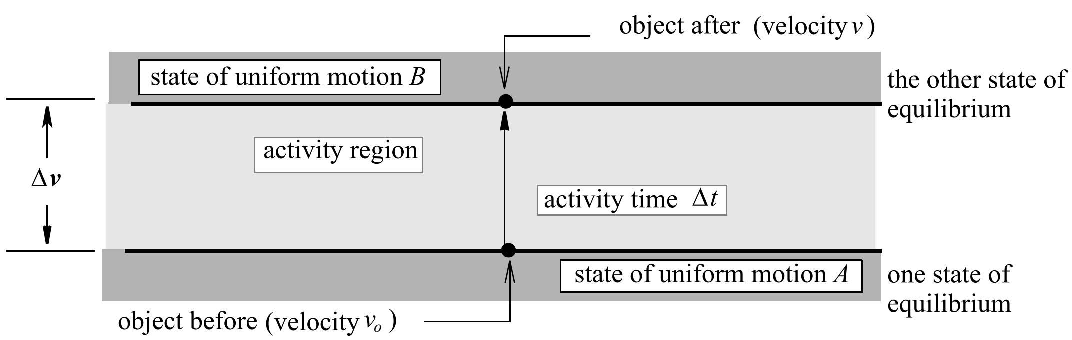

Fig (1) Defining Acceleration

Consider an object (such as a car) to be initially in the first *equilibrium state of uniform motion* (*state* A), travelling with a uniform velocity $\boldsymbol{v}_o$ m/s. After a time Δt, the object is found to be in the second *equilibrium state of uniform motion* (state B), travelling with a uniform velocity $\boldsymbol{v}$ m/s. Recalling the analogy with the river and its two banks, we ask,

Q: How did the object get to the other side (velocity)?

A: It accelerated

5-3 Defining Acceleration

Leaving the *fine print* aside, w estate:

Define "acceleration $\boldsymbol{a}$*" as a velocity-changing agency (of uniform magnitude) that, acting upon an object for an interval of time* Δt*, causes the object to change its velocity from one equilibrium state of uniform velocity to another equilibrium state of uniform velocity, thereby causing its velocity to change by an amount* $\Delta\boldsymbol{v}$*, (the shortest interval of velocity between the two equilibrium states), travelling all the time in a straight line and in one direction.*

Now the *fine print*. Why: *of uniform magnitude*?

Magnitudes of accelerations can be uniform (or constant) and non-uniform. We shall be concerned mainly with uniform accelerations. For one thing, we tend to deal with simpler systems in the beginning and then move on to more complex ones. Non uniform acceleration is called jerk (no idea why, but what can you do?). For another, it turns out that for a wide variety of applications, we use non-jerk (uniform) accelerations. Look at it this way: suppose you had the habit of accelerating and decelerating your car continuously, on a regular basis, your friends will soon find other friends and you will be left alone to be a jerk. However, we do find some jerks around us and we do find ways to deal with them but for now, let's concentrate on uniform accelerations.

As the agency $\boldsymbol{a}$ acted on the object for Δt seconds, we identify $\boldsymbol{a}\Delta t$ as the *cause* of the activity. As the velocity of the object got altered by $\Delta \boldsymbol{v}$ m/s, we identify $\Delta \boldsymbol{v}$ as the *effect* of the activity. Equating *cause* and *effect* we get:

$$\boldsymbol{a}\Delta t = \Delta \boldsymbol{v}$$

Rearranging, we get:

$$\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$

We may write down the entire string of possible values of $\boldsymbol{a}$ as:

$$\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{(\boldsymbol{v} - \boldsymbol{v}_o)}{(t - t_o)} = \frac{(\boldsymbol{v} - \boldsymbol{v}_o)}{t} = \frac{\boldsymbol{v}}{t} \qquad \text{..........(1)}$$

Mathematically speaking, these expressions represent the acceleration $\boldsymbol{a}$ as the *rate of change of velocity with respect to time*; or simply, the *time rate of change of velocity.* It should be clearly understood that *time rate of change of velocity* is not an acceptable definition of acceleration in physics. It is a mathematical *representation* of the definition of acceleration (as given above) and serves merely as a tool for solving problems.

The unit of acceleration is m/s^2 and its dimensions are L/T^2.

5-4 Velocity (a) When it is Uniform, and (b) When it is not Uniform

We hit *acceleration* only in the activity regions, where velocity is *not* uniform. It is, therefore, immensely important to know when velocity is uniform (constant) and when it is not. If the nature of velocity, is not identified correctly, disastrous consequences *will* happen. The student alone will be there to be blamed.

(a) Velocity is Uniform (Objects will be in States of Equilibrium)

One way of finding out if velocity is uniform is to keep looking at the speedometer (real or imaginary) glued to the object (whatever the object is). If the needle stays at rest in some position and doesn't stagger, the velocity *is* uniform (or constant). See Fig (2) below.

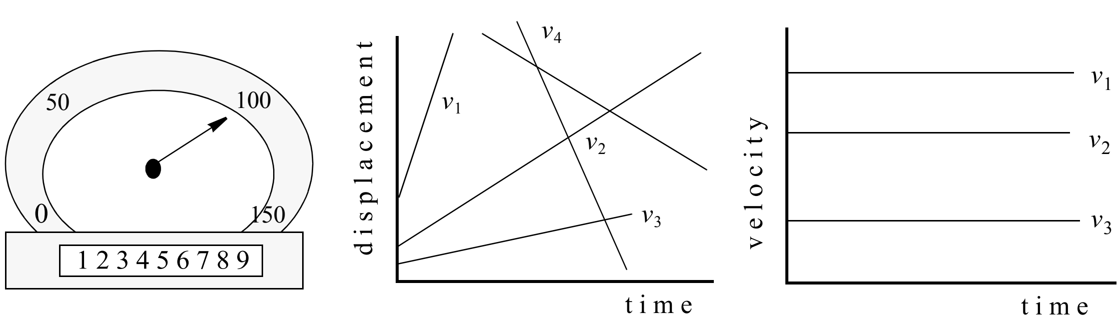

Fig (2) Uniform velocities

Another way of determining the uniformity is to plot a graph of displacement against time. If we get a straight line, the velocity is uniform. Still another way is to plot velocity against time, we shall get a straight line *parallel* to the x-axis. (Also shown in Fig 2)

(b) Velocity is Not Uniform (Objects will be in Activity Regions)

If, however, velocities are not uniform, the respective speedometer needles will be *rest-less*. The lines in the displacement-versus-time graphs will not be straight lines, those in velocity-versus-time graphs will not be parallel to the x-axis. See Fig (3).

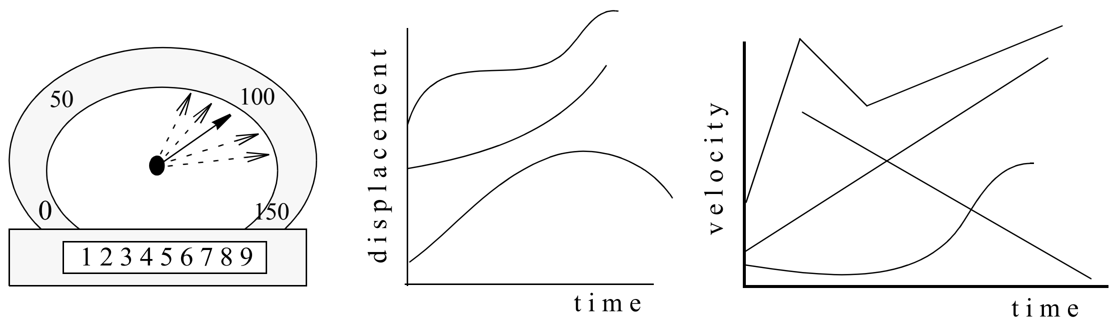

Fig (3) Non Uniform Velocities

5-5 Acceleration (a) When it is Uniform, and (b) When it is not Uniform?

Uniform or not, either way we shall be in an Activity Region

(a) Acceleration is Uniform (Objects will be in Activity Regions)

The criterion for accelerations to be uniform is the same as that for velocities. A plot of acceleration against time, will yield a straight line *parallel* to the x-axis (the time axis). Similarly, on a velocity versus time graph, we shall get *straight* lines. Activities between equilibrium states will be described by straight lines. See Fig (4).

Examples of uniform acceleration abound us. All living beings, vehicles and objects reach their optimum speeds using uniform acceleration. The same is true of them for reducing their velocities or stopping. Again, living beings and objects imparted upward or downward motion acquire uniform accelerations. The list goes on.

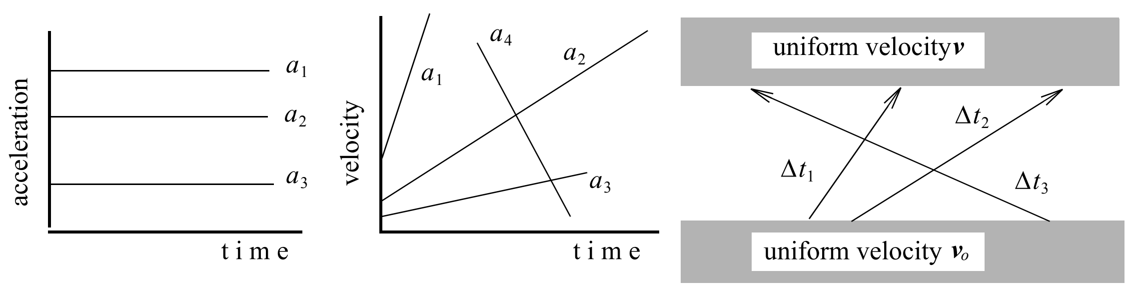

Fig (4) Uniform Accelerations

(b) Acceleration is Not Uniform (Objects will still be in Activity Regions)

If, however, accelerations are not uniform, the lines in the acceleration-time graphs will not be straight lines parallel to the x-axis, those in the velocity-time graphs will not be straight lines, and the activity lines joining the two equilibrium states will be zig-zag or curvy. See Fig (5).

Examples of non-uniform acceleration are also rampant. Rubber bands, elastic cords, springs of all types, bungee cords, trampolines, are some everyday examples.

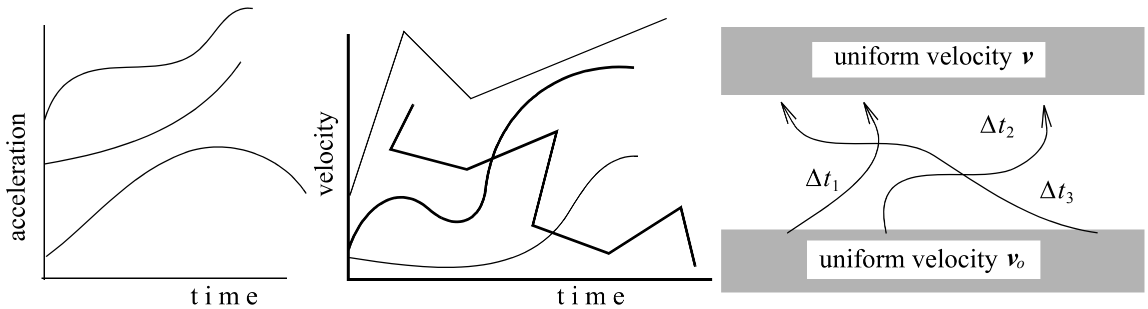

Fig (5) Non Uniform Accelerations

Currently, we shall be concerned with uniform accelerations only. Special techniques will be used when dealing with non-uniform accelerations.

5-6 *When is Acceleration "Acceleration" and When it is "Deceleration"?*

(a) Acceleration

An acceleration is deemed to be *acceleration* if it causes the velocity of the object to *increase*. In this case velocity and acceleration vectors will have the *same* direction and as such, they will be assigned the *same* algebraic sign; either both positive or both negative.

Consider a car. If it is going in the forward direction in the reference frame, glued to its center of mass, and the magnitude of its velocity is increasing, then the car is said to be *accelerating*. In this case, both, the velocity and the acceleration vectors, will be assigned *positive* algebraic sign. We get: $+\boldsymbol{v}$ and $+\boldsymbol{a}$.

If on the other hand, the car is going backward (in reverse gear) in the reference frame glued to its center of mass, and the magnitude of its velocity is increasing, the car will still be said to be *accelerating* but now both, the velocity and the acceleration, will be assigned *negative* algebraic sign. We get: $-\boldsymbol{v}$ and $-\boldsymbol{a}$. (accelerating and $-\boldsymbol{a}$? wow!)

(a) Deceleration

Acceleration is deemed to be *deceleration* if it causes the velocity of the object to *decrease*. In this case velocity and acceleration will have *opposite* directions and as such, will be assigned opposite algebraic signs; one positive, the other negative.

Consider a car. If it is going in the forward direction in the reference frame glued to its center of mass and the magnitude of its velocity is decreasing, then the car is said to be *decelerating* (or braking). As the velocity of the car is assigned a *positive* algebraic sign, the acceleration must be assigned a *negative* algebraic sign. We get: $+\boldsymbol{v}$ and $-\boldsymbol{a}$.

If on the other hand, the car is going backward (in reverse gear) in the reference frame glued to its center of mass, and the magnitude of its velocity is decreasing, the car will still be said to be *decelerating* but now the velocity and the acceleration, must be assigned opposite algebraic signs. Since the velocity is assigned a negative algebraic sign, the acceleration must be assigned a positive algebraic sign! We get: $-\boldsymbol{v}$ and $+\boldsymbol{a}$.

(decelerating and $+\boldsymbol{a}$? wow!)

This is true for *all* accelerations, irrespective of their cast or breed and has no exceptions. The above discussions are summarized in the following table

Table 1: The Algebraic Sign of the Acceleration Vector

Direction of motion of the object in a reference frame glued to the object's center of mass		Algebraic sign of the velocity vector in the said reference frame	Is the speed of the object increasing or decreasing?	Algebraic sign of *acceleration*
x- and y-modes	z-mode			
Forward	up	$+\boldsymbol{v}$	Increasing	$+\boldsymbol{a}$
Forward	up	$+\boldsymbol{v}$	Decreasing	$-\boldsymbol{a}$
Backward	down	$-\boldsymbol{v}$	Increasing	$-\boldsymbol{a}$
Backward-	down	$-\boldsymbol{v}$	Decreasing	$+\boldsymbol{a}$

We find that the direction of acceleration is tied to the direction of velocity. Thus it cannot be determined or represented *independently* in a reference frame. For example if an object is decelerating, we *cannot* categorically announce that the acceleration is negative. Similarly if the acceleration happens to be along a negative coordinate axis, we again cannot categorically announce that the acceleration is negative. Such a *careless* statement can be catastrophic and may lead to chaos. We suggest that you study the contents of Table (1) very carefully and remember them. In this textbook we shall avoid the use of a reference frame for monitoring acceleration. As shown above, the algebraic sign of acceleration depends on the algebraic sign of velocity. Thus if an object is moving along the -x axis, its acceleration is not necessarily negative.

It must also be noted, very carefully, that the *acceleration* we have developed, is parallel to the velocity. Here we used equilibrium states of uniform motion in a straight line: *equal lengths in equal intervals of time,* It is very important to realize (and memorize) that the said acceleration can *only* change the *magnitude* of the velocity and is totally incapable of changing its *direction*.

The reason for the emphasis is that in a future chapter we shall develop another species of acceleration which will be *perpendicular* to the velocity (and not parallel to it). In this case, we shall use two *lame-duck equilibrium* states of uniform motion. The object will not traverse *equal lengths in equal intervals of time.* Instead, it will traverse *equal areas in equal intervals of time.* The trajectory of such a motion is a *conic section*. This new acceleration will *only* change the *direction* of the velocity and will be totally incapable of changing its *magnitude*.

One type of acceleration changes the magnitude of velocity but not its direction, the other type of acceleration changes the direction of velocity but not its magnitude

5-7 To Remind You

The official name of *non-uniform acceleration* is ***JERK***!

5-8 A Summary

In Table (2), we have listed all parameters of motion as discussed in chapters 4 and 5. The table also gives all necessary units and dimensions.

Table 2: Parameters of Motion & their Units

Parameters of motion	Dimensions	M.K.S. units	Nature	Use of coordinate axes for assigning algebraic signs
Distance	L	m	Scaler	allowed
Displacement	L	m	Vector	allowed
Speed	L / T	m/s	Scaler	not recommended
Velocity	L / T	m/s	Vector	not recommended
Acceleration	L / T^2	m/s^2	Vector	not recommended