*CHAPTER 4*

# The First State of Equilibrium

# The Concept of *Motion*

## 1-1 The First State of Equilibrium

The first state of equilibrium is the *state of rest*.

## 1-2 Defining Distance

Distances do not really need to be defined. If we ask: how *far* is (some location) $A$ from (another location) $B$, the answer will be in terms of their *distance* apart. Thus *distance* is the length of the path between $A$ and $B$. This path length may not be in a straight line. In fact it can be traversed in many different ways. The magnitude of the path length will be different in each case.

Distances are measured in meters or kilometers, even millimeters and nanometers. We may measure a distance using our reference frame and call it $(x - x_o)$, or $\Delta x$, or simply $x$ or even $d$ (for distance).

The M.K.S. unit of distance is *meter*. We write $m$, for it and its dimension is $L$.

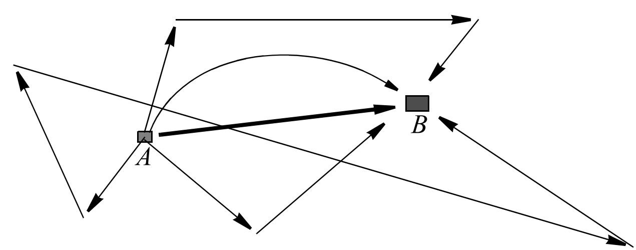


*Fig (1) Distances as Path Lengths*

## 1-3 Defining Displacement

As shown in Fig (1), we may go from $A$ to $B$ either in a straight line or in a round about manner. When going in a straight line, we go in the *same* direction and travel the *shortest* distance between the two points. When going in a round about manner, we go in many different *directions* and travel a *larger* distance. The straight line distance stands out from the rest for being the shortest and being uni-directional. We call such a distance a *displacement*. It is indicative of "how much did one get *displaced* from one's starting position and in what direction". Thus a *displacement* has a magnitude and a specific direction.

## 1-4 Vectors & Scalers

Entities in physics that have (in addition to magnitudes) specific orientations in space, glued to themselves, are called *vectors*. They are represented by *arrows*. Arrows have *tips* and *toes*. The length of the arrow (from tip to toe) is chosen to represent the magnitude of the physical entity, while the tip of the arrow points in the direction of that entity's orientation in space. To distinguish vectors from non-vectors, we shall use bold letters for them. Thus, if *distance* is $d$, then *displacement* will be $\boldsymbol{d}$. Non-vectors are called *scalers*. There are no tips or toes. No special arrangements are needed to represent them.

### 1-5 *Two Equilibrium States of Rest*

Let $A$ and $B$ be two equilibrium states of rest. Objects in these states will be in states of rest. Let the two locations be some distance $d$ apart. We measure $d$ in our reference frame and call it $\Delta x$ (meters). Then we construct the following activity diagram:

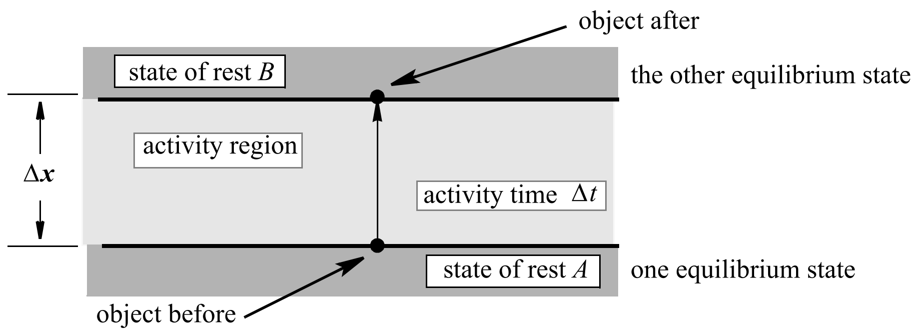


*Fig (2) Defining Velocity*

Consider an object (such as a car) to be initially in the first *equilibrium state of rest (state* $A$). After a time $\Delta t$, the object is found to be in the second *equilibrium state of rest (state* $B$). Recalling the analogy with the river and its two banks, we ask,

*Q: How did the object get to the other side?*

*A: It moved.*

### 1-6 *Defining Velocity*

Leaving the *fine print* aside, we state:

*Define "velocity* $\boldsymbol{v}$*" as a moving agency that, acting upon an object for an interval of time* $\Delta t$*, causes the object to change its position from one equilibrium state of rest to another equilibrium state of rest,* $\Delta \boldsymbol{x}$ *meters away, travelling through the shortest path length (a straight line), moving in one direction only.*

According to the above statement, the path length between $A$ and $B$ is *displacement,* which is a vector quantity, so it will be natural to expect velocity to be a vector quantity as well. It will, therefore, be written in bold letter "$\boldsymbol{v}$".

Now the *fine print*. Why displacement?

The time interval $\Delta t$ is of a fixed nature and will always have the same (given) value. But not the path-lengths. The total path-length of a particular mode will vary from mode to mode. The shortest path-length will be the straight line path from $A$ to $B$ consisting of only one segment. The longest path-length could be infinitely long with infinitely many segments. No upper limit can be assigned to it. It is also obvious that the object will be moving at different rates in the many segments of a mode, so as to keep $\Delta t$ undeterred from its given value. In view of these infinite number of possibilities, we make a bold decision. Let's choose he shortest path with one segment only. We also surmise that the object rises from rest at $A$ (to acquire the necessary motion), and falls to rest at $B$, in infinitesimally small intervals of time (in zero seconds, so to speak).

As the agency $v$ acted upon the object for $\Delta t$ seconds, we identify $\boldsymbol{v}\ \Delta t$ as the *cause* of the activity. As the position of the object got altered by $\Delta \boldsymbol{x}$ meters, we identify $\Delta \boldsymbol{x}$ as the *effect* of the activity. Equating *cause* and *effect* we get:

$$\boldsymbol{v}\ \Delta \boldsymbol{t} = \Delta \boldsymbol{x}$$

Rearranging:

$$\boldsymbol{v} = \frac{\Delta \boldsymbol{x}}{\Delta t} \qquad \text{..........}(1)$$

Writing down the entire string of possible expressions for $v$, (as permitted by the rules of mathematics), we get:

$$v = \frac{\Delta \boldsymbol{x}}{\Delta t} = \frac{(\boldsymbol{x} - \boldsymbol{x_o})}{(t - t_o)} = \frac{(\boldsymbol{x} - \boldsymbol{x_o})}{t} = \frac{\boldsymbol{x}}{t} = \frac{\boldsymbol{d}}{t} \qquad \text{..........}(2)$$

Mathematically speaking, these expressions represent velocity $\boldsymbol{v}$ as the *rate of change of displacement with respect to time*; or simply, the *time rate of change of displacement*. It should be clearly understood that *time rate of change of displacement* is not an acceptable definition of velocity in physics. It is a mathematical *representation* of the definition of velocity (given above) and serves merely as a tool for solving problems.

The unit of velocity is *meters per second* or *m/s* and its dimensions are $L/T$.

## *1-7 Defining Speed*

In case our assumptions in the above discussion were to be too stringent and the object did not really travel in one segment but it side-stepped and made few (momentary) stops here and there, then the path-length will not be eligible to be called displacement. The total path length between $A$ and $B$, in this case, could be much longer, and what's more, the constituting segments could all have been traversed in different directions. The total path-length has now become a *distance* (instead of being a displacement).

The time of activity is still $\Delta t$.

*We define "speed* $\boldsymbol{v}$ *" as a moving agency that, acting upon an object for an interval of time* $\Delta t$*, causes the object to change its position from one equilibrium state of rest to another equilibrium state of rest, moving through a total path-length of* $\Delta x$ *meters, consisting of any or many segments between the two equilibrium states, moving in any or many directions.*

Mathematically speaking we can express *speed* $v$ as the *time rate of change of distance.* Equations (1) and (2) can be used for *speeds* as well.

$$v = \frac{\Delta x}{\Delta t} = \frac{(x - x_o)}{(t - t_o)} = \frac{(x - x_o)}{t} = \frac{x}{t} = \frac{d}{t} \qquad \text{..........}(3)$$

The unit of speed is *also meters per second* or *m/s* and its dimensions are $L/T$ .

## *1-8 The Concept of Average Speed*

The speed $v$ as determined using Eqn. (3) will, in general, be treated as the *mean* speed of the object as it travelled in the activity region. The object must have started from rest at $A$ and then would have come to a complete stop at $B$. Remember $A$ and $B$ are states of rest. We have not been given enough information to know how the object moved in between. We could, therefore, allow for the possibility of the object having passed through intermediate equilibrium states, that were of no worthwhile importance. To be fair to the above and other similar possibilities, we introduce the concept of an *overall* speed, called the *average speed,* in terms of *total distance* $d_{total}$ and *total time* $t_{total}$. We define *average speed* as:

$$v_{av} = \frac{d_{total}}{t_{total}} = \frac{d_1 + d_2 + d_3 + .......}{t_1 + t_2 + t_3 + \ ......} \qquad ..........(4)$$

In no case should average speed be determined as the average of several speeds. In particular, the following equation is totally illegal in our scope of work:

$$v_{av} = \frac{v_1 + v_2 + v_3 \ .... + v_n}{n} \qquad ..........(5)$$

It is unfortunate to note that several textbooks use the following equation:

$$v_{av} = \frac{v_1 + v_2}{2}$$

as a genuine member of the set of the four standard kinematic equations. They then (indirectly) ask the students not to use it. Readers are seriously advised, to ignore the above equation completely and *never*, repeat *never* even think of using it

Fig (4) shows the anatomy of average speed.

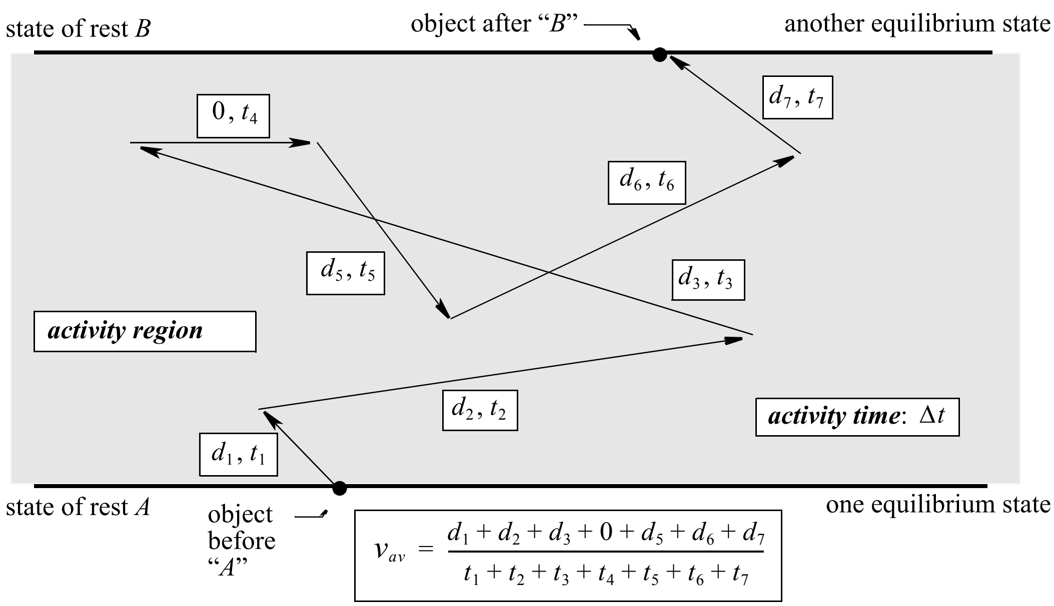


*Fig (4) Finding Average Speed*

## 1-9 What About Average Velocity?

The above could, in principle, also hold true for velocity $\boldsymbol{v}$. However, unlike speed for which there are infinite number of paths, there is only one path for velocity: the displacement $\boldsymbol{d}$. If the object has moved through many distances, in many intervals of time, we must determine the *displacement* and then divide it by the total of the *many intervals of time*. Thus $\boldsymbol{v}_{av}$ is given by the following equation:

$$\boldsymbol{v}_{av} = \frac{displacement}{t_{total}} = \frac{\boldsymbol{d}}{t_{total}} \qquad ..........(6)$$

Fig (5) shows the anatomy of average velocity $v_{av}$.

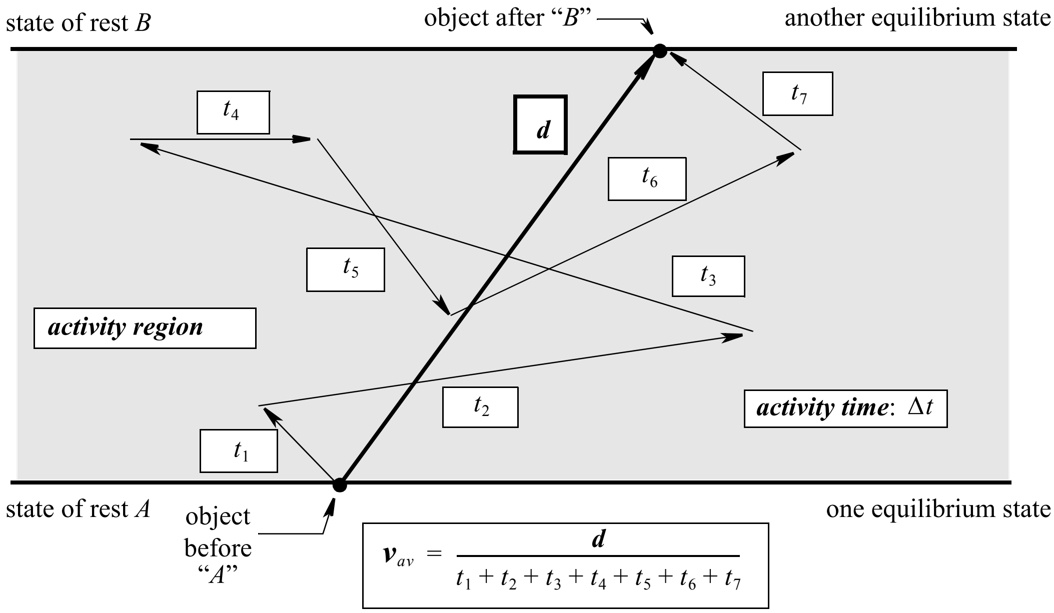


*Fig (5) Finding Average Velocity*

## 1-10 Piece-wise Trips with Uniform Motion

Single piece trips, better known as direct or non-stop journeys, are not always possible. An object (including living beings) may break a journey in any number of places for any lengths of time, for any reason. As the motion itself is uniform, the object will be said to be in piece-wise equilibrium. In all such case, the time intervals needed by the object, to rise to its uniform motion, from rest or fall back to rest from uniform motion, will be deemed to be negligible. Average speeds and velocities can be calculated using Eqns (4) and (6).

Following is a format for computing average speed or average velocity.

**Table 1: Calculating Average Speed or Velocity**

| trip # | parts of the trip | speed $v = d/t$ | distance $d = vt$ | time $t = d/v$ |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| | | Total | $d_{total} =$ | $t_{total} =$ |
| Average speed = | | | | |

First, enter the data from the problem, then fill up the blanks in the distance and time columns using the equations given in those columns. Adding the numbers in the two columns, we shall obtain total distance and total time. Average speed can now be determined easily but for finding average velocity, we will have to find the displacement which may not be quite as easy.

### *An Example*

A family travels from their home to a grocery store, to a park (for picnic lunch), to a play ground and then back home. On the way back, they drive through a longer but scenic route. At the request of the family, some stop-over times have not been recorded.

Data: for the grocery store, they travelled at 30 mph for 20 minutes
for the park, they travelled 10 miles in 15 minutes
for picnic lunch, they spend 1 hour and 21 minutes
for the play ground, they travelled for 25 minutes at 45 mph
for returning home, they travelled at 35 mph and travelled 70 miles.
Find: (i) average speed (ii) average velocity

Solution: (a) fill up the boxes of the format, using the given data.

**Table 2: Filling up Boxes Using Data**

| | parts of the trip | speed $v = d/t$ | distance $d = v\,t$ | time $t = d/v$ |
|---|---|---|---|---|
| 1 | to grocery store | 30 mph | | 20/60 hours |
| 2 | to park | | 10 miles | 15/60 hours |
| 3 | stop-over for picnic | | | 1 hour and 21 minutes |
| 4 | to play ground | 45 mph | | 25/60 hours |
| 5 | to home | 35 mph | 70 miles | |
| | | Total | $d_{total}$ = | $t_{total}$ = |

(b) complete the table by computing the missing information in the *distance* and the *time* columns only, using the formulae given in the table. Once the totals are found, Eqns 4 & 6will be used to find the answers.

**Table 3: Filling up Boxes Using Data**

| | parts of the trip | speed $v = d/t$ | distance $d = v\,t$ | time $t = d/v$ |
|---|---|---|---|---|
| 1 | to grocery store | 30 mph | 10 miles | 20/60 = 0.3333 hours |
| 2 | to park | | 10 miles | 15/60 = =0.250 hours |
| 3 | stop-over for picnic | | | 1 hour, 21 minutes<br>or 1.350 hours |
| 4 | to play ground | 45 mph | 18.750 miles | 25/60 = 0.4167 hours |
| 5 | to home | 35 mph | 70 miles | 2 hours |
| | | **Total** | $d_{total}$ **=108.750 miles** | $t_{total}$ **= 4.350 hours** |

(c) ***average speed = 108.75/4.350= 25.000 mph***

(d) obviously the displacement is zero as the family returns home.

(e) ***Hence average velocity = 0***

## *1-11 Algebraic Signs of Distances and Displacements*

Distances are scalers and as such they are just numbers. Negative distances generally do not exist but may be assigned special interpretations. For example, negative focal lengths signify diverging lenses and diverging mirrors. Likewise negative image distances signify virtual images that cannot be placed on screens but our eyes can see them.

Displacements, on account of the many virtues that the *vectors* have, cannot have a negative magnitude. Should per chance we need to find an interval of displacement $\Delta \boldsymbol{d}$,

$$\Delta \boldsymbol{d} = \boldsymbol{d}_2 - \boldsymbol{d}_1$$

we would construct the negative of the vector $\boldsymbol{d}_1$ by switching its tip and toe and then combine it with $\boldsymbol{d}_2$ vectorially. Should we get a negative value of $\Delta \boldsymbol{d}$ in the process, then it would imply that $\Delta \boldsymbol{d}$ is directed oppositely to $\boldsymbol{d}_2$. The magnitude of $\Delta \boldsymbol{d}$ would still be positive.

## *1-12 Algebraic Signs of Speeds and Velocities*

Speeds are scalers and as such they are just numbers. The concept of negative sped does not really exist.

Like displacements, velocities are also vectors and as such whatever is stated above for displacements, also applies to velocities. On occasions, an object rebounds and its direction is switched by 180°. By virtue of this switch, the velocity (upon rebound) is assigned a negative algebraic sign. The magnitude of this velocity will still be positive.

Again, if you are driving in the forward direction in your reference frame then your velocity is assigned a positive algebraic sign (magnitude remaining positive). If you see another person driving in a direction opposite to yours, then in your reference frame, the velocity of the other person will be assigned a negative algebraic sign (magnitude remaining positive). Obviously, the tip and toe of the other person's velocity vector are opposite to those of your velocity vector.

If you were to be driving your car in reverse gear, you are driving backward in your reference frame. The velocity vector in this case, will be assigned a negative algebraic sign (magnitude remaining positive).

When an object (including living beings) is moving upward (any which way), the object is moving in the forward direction in the reference frame located at its center of mass. As such the velocity must be assigned a positive algebraic sign (magnitude remaining positive). When the object moves downward (again, any which way), it has *rebounded* from the above (whatever it may be) and as such its velocity must be assigned a negative algebraic sign (magnitude remaining positive). No reference frame is used here.

These algebraic signs have nothing to do with observational reference frames. Thus if an object is descending while in the +z region, the velocity will be assigned a negative algebraic sign (magnitude remaining positive) and vice versa. Use of observational reference frames for monitoring the algebraic signs of velocities is illegal and their use is, therefore, strictly prohibited.