Chapter 17

Newton's Second Law of Motion - II
The Work - Energy Theorem

17-1 Prelude

Now we recall the *time-independent* activity diagram from Chapter 15:

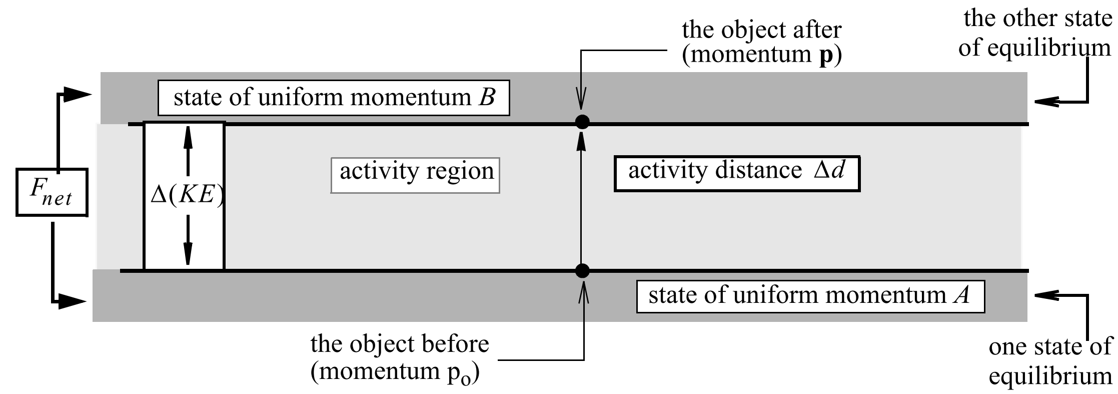

Fig (1) The Time-Independent Activity Diagram

It was also found that the change in the status of the target object was not $\Delta \boldsymbol{p}$ any more.

17-2 Getting to the Other Side

According to Fig (1) above, the agency $\boldsymbol{F}_{net}$ drives the target object through a linear distance (displacement) of magnitude Δd. The cause of change will, therefore, be:

$$(\boldsymbol{F}_{net})(\Delta d) \qquad \text{..........(1)}$$

So far the nature of this entity is not readily known. This is quite like the analysis of the time-dependent case, where the entity $(\boldsymbol{F}_{net})(\Delta t)$ was not immediately perceivable and we *defined* it to be *impulse*. So, the entity $(\boldsymbol{F}_{net})(\Delta d)$ also needs to be defined. It has been defined as ***work***. We write W for it. The unit will be (kg) (m/s^2) (m), or (kgm^2)/s^2.Using the unit N for force, we may write *work* as Nm. But that is not all. There is yet another short name for *work*. It is *Joule,* written as J.

We have established that the cause of the activity is:

$$(\boldsymbol{F}_{net})(\Delta d) = W_{net} \qquad \text{..........(2)}$$

To investigate the effect of the activity which is equally (if not more) vague, we use a neat trick. We can write $(\boldsymbol{F}_{net})(\Delta d)$ as:

$$(\boldsymbol{F}_{net})(\Delta t)\left(\frac{\Delta d}{\Delta t}\right)$$

where $(\boldsymbol{F}_{net})(\Delta t)$ is the left hand side of Eqn (3) of Chapter 16, reproduced below:

$$(\boldsymbol{F}_{net})(\Delta t) = \Delta \boldsymbol{p} \qquad \text{.......Eqn (3), Chapter 16}$$

So, if we are multiplying the left hand side by $\Delta d/(\Delta t)$, then the right hand side must also be multiplied by the same factor:

$$(\boldsymbol{F}_{net})(\Delta t)\left(\frac{\Delta d}{\Delta t}\right) = \Delta \boldsymbol{p}\left(\frac{\Delta d}{\Delta t}\right)$$

Or

$$(\boldsymbol{F}_{net})(\Delta d) = \Delta \boldsymbol{p}\left(\frac{\Delta d}{\Delta t}\right) \quad \text{..........(3)}$$

Now $\Delta d/(\Delta t)$ is velocity v. The question is what velocity? It cannot be a velocity from any of the equilibrium states, because there are two of those and we will not know which one to choose. The obvious solution is to take the average of the two velocities. As acceleration is constant, the average velocity will be given as:

$$\left(\frac{\Delta d}{\Delta t}\right) = \frac{\boldsymbol{v} + \boldsymbol{v_o}}{2} \quad \text{..........(4)}$$

As for $\Delta \boldsymbol{p}$, it is

$$:\Delta \boldsymbol{p} = \boldsymbol{p} - \boldsymbol{p}_o = m\boldsymbol{v} - m\boldsymbol{v}_o = m(\boldsymbol{v} - \boldsymbol{v_o}) \quad \text{..........(5)}$$

From (4) and (5) we get:

$$\Delta \boldsymbol{p}\left(\frac{\Delta d}{\Delta t}\right) = m(\boldsymbol{v} - \boldsymbol{v_o})\left(\frac{\boldsymbol{v} + \boldsymbol{v_o}}{2}\right)$$

$$= (1/2)(m)(\boldsymbol{v}^2 - \boldsymbol{v}_o^2)$$

$$= (1/2)(m)(\boldsymbol{v}^2) - (1/2)(m)(\boldsymbol{v}_o^2)$$

$$= \frac{m}{2}(\boldsymbol{v}^2) - \frac{m}{2}(\boldsymbol{v}_o^2)$$

We find that

$$\Delta \boldsymbol{p}\left(\frac{\Delta d}{\Delta t}\right) = \frac{m}{2}(\boldsymbol{v}^2) - \frac{m}{2}(\boldsymbol{v}_o^2) \quad \text{..........(6)}$$

Eqn (6) tells us that the effect of the activity is expressed as the difference of two terms: one with respect to the velocity of the target object in equilibrium state A and the other with respect to the velocity of the target object in equilibrium state B. There are no cross terms, no mingling and no confusion. All that remains for us to do is to give this entity a name. This name is *kinetic energy*. We write KE for it. The word *kinetic* emphasizes the fact that the target object is in motion. Writing:

$$\Delta \boldsymbol{p}\left(\frac{\Delta d}{\Delta t}\right) = \frac{m}{2}(\boldsymbol{v}^2) - \frac{m}{2}(\boldsymbol{v}_o^2) \qquad = (KE)_f - (KE)_i = \Delta(KE) \quad \text{..........(7)}$$

Inserting the value of $\Delta \boldsymbol{p}\left(\frac{\Delta d}{\Delta t}\right)$ from Eqn (7) into Eqn (3), we get our final result:

$$(\boldsymbol{F}_{net})(\Delta d) = \Delta(KE) \quad \text{..........(8)}$$

Or

$$W_{net} = \Delta(KE) \quad \text{..........(9)}$$

Eqn (8) and (9) are known as the *Work-Energy Theorem*.

Work-energy theorem tells us that when a net work is done on a target object, its kinetic energy changes by an amount $\Delta(KE)$.

We shall learn a lot more about the energy or the time independent method in later chapters. But for now we go back to the force approach.