CHAPTER 16

Newton's Second Law of Motion - I
The Impulse-Momentum Theorem

16-1 Prelude

From Newton's First Law (Chapter 15), we learned:

Momentum $\boldsymbol{p}$ and Net Force $\boldsymbol{F}_{net}$ are intimately related to one another.

We also developed an activity diagram, which is being reproduced here for ready reference:

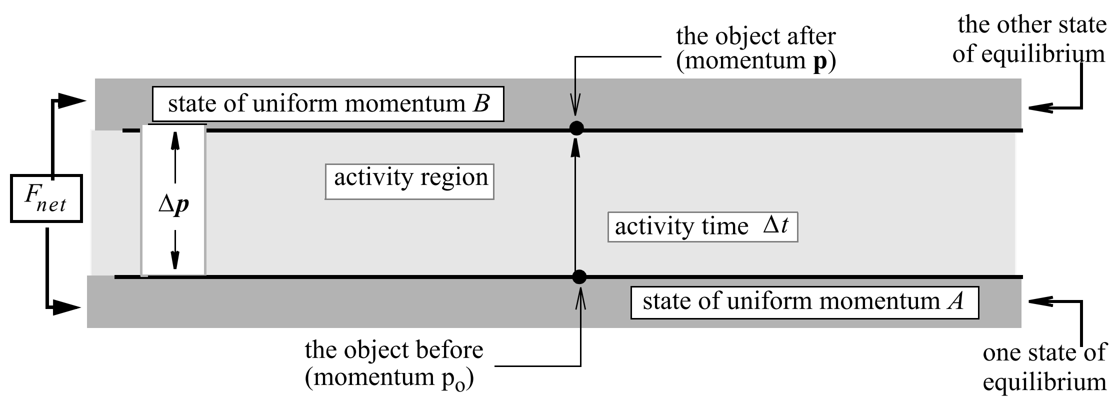

Fig (1) The Time-Dependent Activity Diagram

16-2 What Next?

The obvious next step is to *get to the other side,* by using the tried and true technique of equating *Cause and Effect* to one another. The agency is, of course, the net force $\boldsymbol{F}_{net}$, as mandated by Newton' First Law. It can act on the target object in two possible modes. (i) with respect to the *time* for which $\boldsymbol{F}_{net}$ acts on the target object. (ii) with respect to the *linear distance* through which the target object is driven by $\boldsymbol{F}_{net}$. We shall study both these modes, the *time* mode first.

16-3 Getting to the Other Side
The Impulse-Momentum Theorem

The net force $\boldsymbol{F}_{net}$, acts on the target object for the duration of time Δt. We, therefore, recognize:

$$\boldsymbol{F}_{net}\,\Delta t \qquad ....(1)$$

as the agency that caused the ***change*** of the *state-of-uniform-momentum* of the object, or simply, the *cause* of the change. The effect of the activity is the change in the momentum $\Delta\boldsymbol{p}$ of the target object. We recognize $\Delta\boldsymbol{p}$ as the *effect* of activity, where:

$$\Delta\boldsymbol{p} = p - p_o \qquad ....(2)$$

Equating *cause* and *effect* to one another, we get:

$$\boldsymbol{F}_{net}\,\Delta t = \Delta\boldsymbol{p} \qquad ....(3)$$

This is the Impulse-Momentum Theorem

We define or recognize the left hand side of Eqn (3): $\boldsymbol{F}_{net}\,\Delta t$, as *Impulse.* and the right hand side of Eqn (3): $\Delta \boldsymbol{p}$, as *Momentum*. (We already knew that $\Delta \boldsymbol{p}$ was momentum but it had to be re-stated here to complete an argument.) Writing I for impulse, we get:

$$I = \boldsymbol{F}_{net} \Delta t \qquad ....(4)$$

Impulse Momentum Theorem tells us that when a net force $\boldsymbol{F}_{net}$ acts on a system for a time interval Δt, the momentum of the target object changes by an amount $\Delta \boldsymbol{p}$.

16-4 Impulse-Momentum Theorem

(1) Newton's Second Law

Rearranging Eqn (3), we get:

$$F_{net} = \frac{\Delta p}{\Delta t} \qquad ....(5)$$

Here force is expressed as the *time rate of change of momentum*. This expression stems from Newton's first law of motion and has nothing to do with kinematics. It does not even mention *acceleration* by name! This definition, however, should not misguide you to think that force does not produce acceleration. A change of momentum is basically a change of velocity. Kinematics tells us that a change of velocity cannot occur without acceleration. We now proceed to bring acceleration to surface. Knowing that $\boldsymbol{p}_o = m\boldsymbol{v}_o$ and $\boldsymbol{p} = m\boldsymbol{v}$ we get:

$$\Delta \boldsymbol{p} = \boldsymbol{p} - \boldsymbol{p}_o = m\boldsymbol{v} - m\boldsymbol{v}_o = m(\boldsymbol{v} - \boldsymbol{v}_o) = m(\Delta \boldsymbol{v})$$

We may rewrite Eqn (5) as:

$$\boldsymbol{F}_{net} = \frac{\Delta \boldsymbol{p}}{\Delta t} = \frac{m(\Delta \boldsymbol{v})}{\Delta t} = m\left(\frac{\Delta \boldsymbol{v}}{\Delta t}\right)$$

But

$$\frac{\Delta \boldsymbol{v}}{\Delta t} = \boldsymbol{a}$$

Therefore

$$\boldsymbol{F}_{net} = m\boldsymbol{a} \qquad ....(6)$$

The unit of $\boldsymbol{F}$ or $\boldsymbol{F}_{net}$, as inferred from Eqn (6), is (kg) (m/s^2) or $\frac{kgm}{s^2}$. A shorter name for this unit is *Newton* and we write N for it. If a force produces an acceleration of one (m/s^2) on a mass of one kg, then this force is one N strong. Alternatively, a one N strong force will produce an acceleration of one (m/s^2) on a mass of one kg.

Eqn (6) can be rearranged to express acceleration in terms of $\boldsymbol{F}_{net}$:

$$\boldsymbol{a} = \frac{\boldsymbol{F}_{net}}{m} \qquad ....(7)$$

Eqns (6) and (7) are the classical forms of the prestigious *Newton's Second Law of Motion.* This is one of the fundamental laws in Physics. It is valid in all branches of physics. It can easily be shown that these results also hold good if either $\boldsymbol{v}_1$ or $\boldsymbol{v}_2$ were to be zero.

16-5 Newton's Second Law of Motion: Enunciation

The acceleration of an object is directly proportional to the net force acting upon it and is inversely proportional to its (inertial) mass. Acceleration inherits the direction of the net force.

16-6 Newton's Second Law: Alternate Derivation

As an alternate derivation, for the weak-hearted readers who may be getting chest pains with the *cause-and-effect* based derivation, we have an alternate work-out. We convert Fig (1) into a graph. This is shown in Fig (2). We plot *time* along the x-axis and *momentum* along the y-axis. In the graph, the equilibrium state of the object with momentum $\boldsymbol{p}_o$ is represented by line AB along y-axis. The other equilibrium state where the momentum of the object is $\boldsymbol{p}$, is represented by line CD, also drawn along the y-axis. Line

BD is the resulting graph. It represents the activity region that is traversed in time Δt. Because a change of state can only occur if a net force acts on the target object, (Newton's first law), we argue that the slope of line BD must represent the net force $\boldsymbol{F}_{net}$ which acted upon the object for time Δt.

The slope of line BD is (as always):

$$\frac{\Delta y}{\Delta x} = \frac{BE}{AC} = \frac{(\boldsymbol{p}-\boldsymbol{p}_o)}{(t-t_o)} = \frac{\Delta \boldsymbol{p}}{\Delta t} = \frac{m(\boldsymbol{v}-\boldsymbol{v}_o)}{\Delta t} = m\left(\frac{\Delta \boldsymbol{v}}{\Delta t}\right) = m\boldsymbol{a}$$

But, as dictated by the first law:

$$\frac{\Delta y}{\Delta x} = \text{slope} = \boldsymbol{F}_{net}$$

Equating the two right hand sides we obtain the result given in Eqn (6)

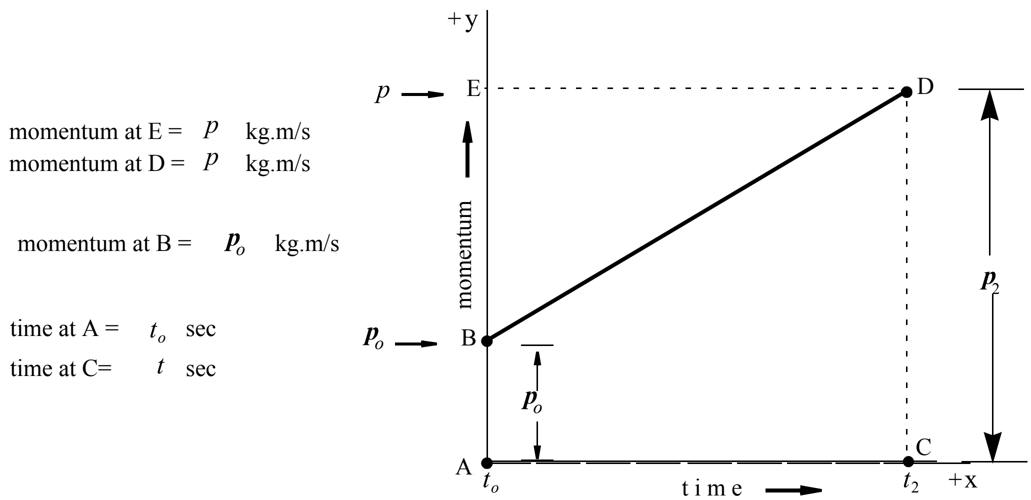

Fig (2) The Activity Diagram as a Graph.

16-7 *Net Force $\boldsymbol{F}_{net}$ and Acceleration $\boldsymbol{a}$*

From all of the above deliberations, we conclude:

Net Force $\boldsymbol{F}_{net}$ is the agency that produces acceleration $\boldsymbol{a}$.

It is pertinent at this time to explore all the duties and responsibilities of the net force in regard to producing acceleration. Here is the complete list;

(a) *net force* is the agency that produces *acceleration*.

(b) *net force* is the *only* agency that produces *acceleration*.

(c) *net force* does nothing else but produce *acceleration*.

(d) *net force* necessarily produces *acceleration*.

We find that net force has the monopoly of producing acceleration and that it has no other role, no other responsibility, no other ambition in life, and no second job. We find that force is the ***"one and only"*** source of acceleration. There is, and there will never be another agency that would produce acceleration. Again, net force *always* produces acceleration. If a force is applied, acceleration is guaranteed. It is impossible to have force and no acceleration.

When an object begins to move from rest, we know that acceleration has been present. As acceleration comes from force, we conclude that a force must also have been present. Conversely, if a force is present, the object *must accelerate* i.e. begin to move from rest, or begin to change its state of uniform motion.

Some of us may not agree with this statement. If for example, we push a wall with our hands, a force is applied to the wall but we do not see any sign of displacement of the wall. The wall simply did not budge from its position.

We would like to tell you that if the object cannot move, it gets deformed. The deformation may not be visible to our eyes but it *will* be there! Some microscopic change always occurs. We state:

A force necessarily produces deformation

An interesting demonstration of the above effect, makes use of a thick solid glass rectangular bottle such as that of *Tropicana* orange juice. It is filled to the brim with a colored liquid, such as potassium permanganate solution. The bottle is then sealed with a cork. A fine capillary tube is inserted into the bottle through the cork. The liquid rises to some convenient level in the capillary tube and stays there. If we press the *wide* sides of the bottle using thumb and fingers of one hand, so as to squeeze the bottle, we shall be tending to decrease the volume of bottle by an infinitesimal amount. As the volume decreases, the bottle expels some liquid into the capillary tube. The level of the liquid in the capillary tube rises by few millimeters and is (obviously) easily visible to our eyes. Again, if we press the *narrow* sides of the bottle, in the same manner as before, we shall be causing the bottle to tend to become circular, thereby increasing the volume of the bottle. As the volume increases, the bottle draws a tiny amount of solution from the capillary tube. The level of the liquid in the capillary tube now falls by few millimeters.

The application of force on the solid glass bottle by our hands, did produce a deformation in the bottle.

16-8 Nature of Forces

(1) Forces are vectors. This is evident from Eqn (3) where the left hand side, momentum, is known to be a vector quantity. The right hand side must, therefore, be a vector quantity also. As time is a scaler entity, force must necessarily be vector.

(2) Force is exerted by one object (called the "**source**" object) on another object (called the "**target**" object).

(3) Forces effectively act at the center of mass of the target objects. All force vectors must necessarily originate from the center of mass of target objects.

16-9 Non-Importance of Forces

You must be thinking that with all the fanfare, ostentatious display of its superiority, notoriety, royalty, publicity, pomp and show, grandeur, magnificence etc., etc. shown to ***forces,*** they must be the biggest single phenomenon not only in the realm of dynamics, but also in all domains of physics.

Unfortunately, it is not so.

We must inform you that *force* is not the *end-product* in physics. The *end-product* is ***acceleration***. We need acceleration to arrange, rearrange velocities for our needs. However, since the only source of acceleration is *force*, we owe it to ourselves to know forces inside out and learn how to exploit them to get the accelerations that we need.

Forces are somewhat like orange trees. Even though we are *only* interested in oranges (O.J. and more), we must take good care of orange trees because they are our *only* source of oranges.

Need we say more?

16-10 Impulse-Momentum Theorem
(2) Impulse & Impulsive Forces

With regard to the definition of impulse:

$$I = F_{net}\,\Delta t \qquad \text{..........Eqn (4)}$$

F_{net} is sometimes called *Impulsive Force.* Typically an impulsive force is a force of large magnitude that acts for a small interval of time. When we kick a football with our foot, we apply an impulsive force. The same is true for a tennis or ping-pong ball when hit by a racket, or a bullet fired from a gun or a pebble thrown in a pond by a kid or a marble kicked on the table by our finger, or a hockey puck set in motion by a stick or a baseball launched by a pitcher, or a soccer ball redirected in air by your head, etc., etc. An impulsive force, however, is not of constant magnitude. When plotted with respect to time, it makes a Gaussian curve. Average value of the force is then called the impulsive force. You would have, no doubt, noticed that in all these cases, the force that was applied to the target object, acted for a very short interval of time. Thus in all these cases, the target object received an impulse.

Equation (3) then tells us that if $F_{net}\,\Delta t$ is an impulse, then Δp must also be an impulse. This is quite true. In a problem, if we ask you to calculate the impulse given to a target object, you may determine it using either side of Eqn (3).

Impulsive forces are Unglued Forces

An impulse given to a target object, imparts it a uniform velocity in a straight line. The target object will keep moving with that velocity. The contact between the force and the target object, ceases and the target object moves on its own, totally independently of the force that set it in motion. We say that the impulsive force is an *unglued* force Forces that keep acting on the object for a prolonged period of time, are called *glued* or *constant* forces.

Unglued Forces and Force Diagrams

We shall, shortly, talk about forces and force diagrams to solve problems. It is good to know, at this time, that unglued forces will *never* be a part of force diagrams.

More about Impulsive Forces

We shall talk about impulses and impulsive forces later. Currently we are concentrating on uniform acceleration which demands uniform forces. So we must put impulsive forces on the back burner.