Experiment # 10

Circuit Analysis - 2b
Kirchhoff's Rules - Capacitors

Principles

Kirchhoff's Rules

Kirchhoff's Rules for capacitors, do not differ from those for resistors. These were discussed adequately in a previous experiment, where the rules were applied to a circuit containing resistors and batteries only. As such, we will not repeat them. We shall, however, discuss the necessary changes in the circuit to cater for the special needs of capacitors.

Special Needs

Capacitors do not conduct electricity. However, when fed by an AC supply, pseudo current, I_C, flows which is capable of doing everything that a real current does. (A real current is one that flows through conductors.) This calls for the use of an AC power supply. Pseudo currents activate the property *reactance* χ_C, of capacitors. We studied reactances extensively in two previous experiments. Reactances are frequency dependent and the placement of capacitors in an AC circuit produces a phase difference between the source voltage and its current. Again, multiple sources can never have identical frequencies and phases. Lack of coherence will most likely, ruin the experiment. All sources, therefore, must necessarily be of identical frequency and must be in phase with one another. These rather hard-to-comply-with conditions can (astonishingly) be met with, rather easily by using only one source, and splitting the output voltage into many *sources*.

Following are the loop and junction equations, suitable for circuits with capacitors and AC sources only (no resistors and no batteries). As you can see, there is only one change. We replaced R by χ.

$$\sum_n V_{n,\,sources} = \sum_n \chi_n I_n \qquad \text{..........(1)}$$

$$\sum_n I_{in} = \sum_n I_{out} \qquad \text{..........(2)}$$

The reactance χ_C of a capacitor is given by the equation:

$$\chi_C = \frac{1}{2\pi f C} = \frac{1}{\omega C} \qquad \text{..........(3)}$$

where, as previously stated:

$$\omega = 2\pi f \qquad \text{..........(4)}$$

The applicable Ohm's Law is:

$$V_C = \chi_C I_C = \frac{I_C}{\omega C} \qquad \text{..........(5)}$$

Objectives of the Experiment

To study Kirchhoff's Rules for the circuit given below

(i) Direct Verification: by verifying (a) all calculated values of currents and voltages, (b) all junction and loop equations

(ii) Indirect Verification: by plotting a graph of total current against ε_1.

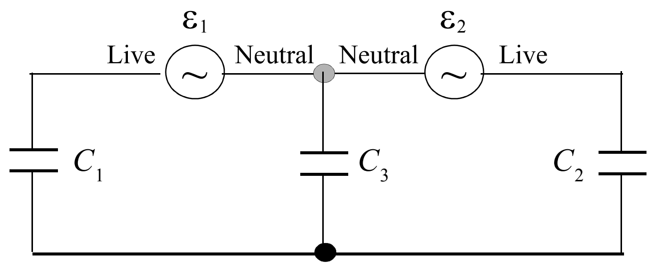

Fig (1) The Circuit

Setting Up

Applying Kirchhoff's Rules to the Assigned Circuit.

The circuit assigned for this experiment, is an exact parallel of the one we were assigned for resistors. This will allow us a direct comparison of the usefulness and applicability of Kirchhoff's rules for both types of components and both types of electricity. The circuit has three capacitors C_1, C_2 and C_3 and two supply voltages ε_1 and ε_2. Both voltages must be obtained from a single (stabilized) AC power supply (a function generator).

The circuit of Fig. (1) is reproduced in Fig. (2). We have replaced capacitors by their reactances and added some useful information.

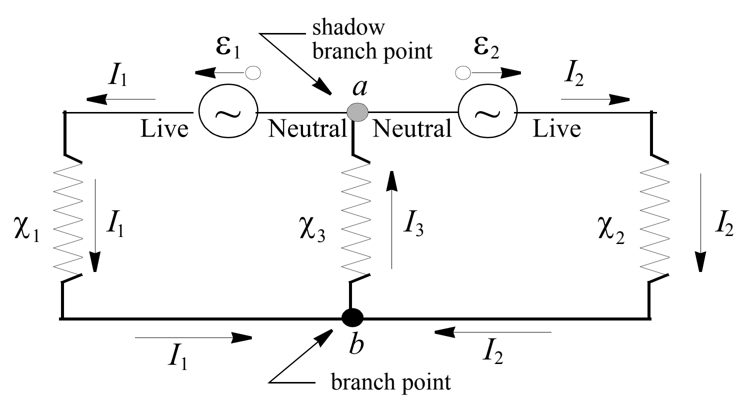

Fig (2) Circuit with Capacitors Replaced by Reactances

It should be pointed out that the *emf* directions and the directions of currents in Figs (2) show *instantaneous* voltages and currents, for some particular instant of time. This is because Kirchhoff's rules are valid only for instantaneous voltages and currents.

Analysis

The ensuing analysis is an exact parallel of that given in the experiment "Kirchhoff's Rules - Resistors", We shall, therefore, not repeat any of the arguments and only give the outlines of the analysis.

The junction (or the branch point) equation is:

$$I_1 + I_2 = I_3 \qquad \text{..........(6)}$$

The two loop equations are:

$$\varepsilon_1 = \chi_1 I_1 + \chi_3 I_3 \qquad \text{..........(7)}$$

$$\varepsilon_2 = \chi_2 I_2 + \chi_3 I_3 \qquad \text{..........(8)}$$

Rearrange Eqns (7) and (8) to express I_1 and I_2 in terms of I_3. We get:

$$I_1 = \frac{\varepsilon_1}{\chi_1} - \frac{\chi_3 I_3}{\chi_1} \qquad \text{..........(9)}$$

$$I_2 = \frac{\varepsilon_2}{\chi_2} - \frac{\chi_3 I_3}{\chi_2} \qquad \text{..........(10)}$$

Plug in these values of I_1 and I_2 in Eqn (6) to get:

$$\left(\frac{\varepsilon_1}{\chi_1} - \frac{\chi_3 I_3}{\chi_1}\right) + \left(\frac{\varepsilon_2}{\chi_2} - \frac{\chi_3 I_3}{\chi_2}\right) = I_3$$

Solving for I_3 we get:

$$I_3 = \frac{\left(\frac{\varepsilon_1}{\chi_1} + \frac{\varepsilon_2}{\chi_2}\right)}{\left(1 + \frac{\chi_3}{\chi_1} + \frac{\chi_3}{\chi_2}\right)} \qquad \text{..........(11)}$$

Unlike the experiment "Kirchhoff's Rules - Resistors", we cannot end our analysis here. We need to express the value of I_3 in terms of capacitances and not reactances. Using Eqn (3) we find:

$$\frac{\varepsilon_1}{\chi_1} = (\omega C_1)\varepsilon_1 \qquad \frac{\chi_3}{\chi_1} = \frac{\omega C_1}{\omega C_3} = \frac{C_1}{C_3} \qquad \text{..........(12)}$$

$$\frac{\varepsilon_2}{\chi_2} = (\omega C_2)\varepsilon_2 \qquad \frac{\chi_3}{\chi_2} = \frac{\omega C_2}{\omega C_3} = \frac{C_2}{C_3} \qquad \text{..........(13)}$$

We insert these values in Eqn (11) to get:

$$I_3 = \frac{(\omega C_1)\varepsilon_1 + (\omega C_2)\varepsilon_2}{1 + \frac{C_1}{C_3} + \frac{C_2}{C_3}} = \frac{\omega(C_1\varepsilon_1 + C_2\varepsilon_2)}{\frac{C_1 + C_2 + C_3}{C_3}}$$

or

$$I_3 = \frac{\omega(C_1\varepsilon_1 + C_2\varepsilon_2)(C_3)}{C_1 + C_2 + C_3} \qquad \text{..........(14)}$$

Eqn (14) gives us I_3 in terms of the known parameters. Thus I_3 stands determined. Next, we develop similar equations for I_1 and I_2:

From Eqns (9) and (12), calculating for I_1, we get:

$$I_1 = \frac{\varepsilon_1}{\chi_1} - \frac{\chi_3 I_3}{\chi_1} = (\omega C_1)\varepsilon_1 - \left(\frac{C_1}{C_3}\right) I_3 \qquad \text{..........(15)}$$

Similarly for I_2, using Eqn s (10) and (13), we get:

$$I_2 = \frac{\varepsilon_2}{\chi_2} - \frac{\chi_3 I_3}{\chi_2} = (\omega C_2)\varepsilon_2 - \left(\frac{C_2}{C_3}\right) I_3 \qquad \text{..........(16)}$$

The voltages across the three capacitors may be found by using Ohm's law, (Eqn 5), Thus:

$$V_{C1} = \frac{I_1}{\omega C_1}, \qquad V_{C2} = \frac{I_2}{\omega C_2}, \qquad V_{C3} = \frac{I_3}{\omega C_3} \qquad \text{..........(17)}$$

One can also write down the three loop equations in terms of voltages. Thus:

$$\varepsilon_1 = V_{C1} + V_{C3} \qquad \text{..........(18)}$$

$$\varepsilon_2 = V_{C2} + V_{C3} \qquad \text{..........(19)}$$

$$\varepsilon_1 - \varepsilon_2 = V_{C1} - V_{C2} \qquad \text{..........(20)}$$

This completes our analysis in full. Equations (14) through (20) can all be verified by direct measurement of the circuit currents and voltages and percent errors can be calculated.

(A) Direct Verification.

We shall set up the circuit of Fig (2) and

(i) measure the 5 voltages: ε_1, ε_2, V_{C1} , V_{C2} , and V_{C3} ; and

(ii) measure the 3 currents: I_1 , I_2, and I_3 .

(B) Indirect Verification.

We shall now verify the use of Kirchhoff's rules for our circuit, indirectly. As mandated, we shall plot a graph of total current against ε_1. Total current, in the circuit is I_3 (see Eqn 6). So, we pick up Eqn (14) and convert it into a straight-line equation.

Rewrite Eqn (14) as:

$$I_3 = \frac{\omega(C_1\varepsilon_1)(C_3)}{C_1 + C_2 + C_3} + \frac{\omega(C_2\varepsilon_2)(C_3)}{C_1 + C_2 + C_3}$$

Rearranging,

$$I_3 = \left(\frac{\omega(C_2 C_3)(\varepsilon_2)}{C_1 + C_2 + C_3}\right) + \left(\frac{\omega(C_1 C_3)}{C_1 + C_2 + C_3}\right)(\varepsilon_1) \qquad \text{..........(21)}$$

Equation (21) matches the equation of a straight line and is, therefore, the equation we were looking for. It has ε_1 as independent variable and I_3 as dependent variable (as was mandated)! The y-axis intercept:

$$\frac{\omega(C_2 C_3)(\varepsilon_2)}{(C_1 + C_2 + C_3)} \qquad \text{..........(22)}$$

represents that fraction of I_3 which is contributed by the voltage supply ε_2. This contribution stays constant for this part of the experiment. It can also be found by short-circuiting the voltage supply ε_1.

The slope:

$$\frac{\omega(C_1 C_3)}{(C_1 + C_2 + C_3)} \qquad \text{.........(23)}$$

represents the equivalent conductance G_{eq}, of C_3 (with respect to ε_1), when ε_2 is short-circuited. This is because with ε_2 shorted out, Eqn (21) will read:

$$I_3 = \left(\frac{\omega(C_1 C_3)}{C_1 + C_2 + C_3}\right)(\varepsilon_1) = G_{eq}\,\varepsilon_1 \qquad \text{.........(24)}$$

A suitable circuit is shown in *Procedur*e.

Procedure

(A) Measurement of Currents and Voltages.

(1) We shall use capacitance substitution boxes for the three capacitors. Unless otherwise instructed, set $C_1 = 0.08\ \mu\text{F}$, $C_2 = 0.03\ \mu F$ and $C_3 = 0.17\ \mu\text{F}$. Please note that the capacitor C_3 is in the middle. Also make sure you did not, mistakenly, take C_3 to be C_2 or vice versa.

(2) Check the values of the three capacitors using an LCR meter. If values are in error, consult the instructor. An LCR meter is on your table. Set it to 200 nF scale. When the value of the capacitor is being checked, the capacitor ***must not*** be in the circuit.

(3) We shall use the function generator with built-in amplifier, for both sources in our circuit. Use one multimeter as voltmeter and set it to 20 V, AC.

(4) Switch on the function generator. It should display a frequency of 1 kHz. Using the *amplitude control* knob of the generator, and the voltmeter, adjust the output voltage of the function generator to 5.00 V (as closely as possible). Disconnect the voltmeter.

(5). Set up the circuit of Fig (3).

(6) Adjust the positions of the sliding contacts in the two rheostats such that the output of the left hand side rheostat is 5.00 volts (as closely as possible) and that of the right hand side rheostat is 3.00 volts (also as closely as possible). Voltmeter in each case, should be connected between terminal 3 (live or positive) and terminal 2 (common or negative). Disconnect voltmeter after adjustment has been made. If the instructor suggests different voltages, follow the instructor.

(7) Measure the five voltages: ε_1, ε_2, V_{C1}, V_{C2}, and V_{C3}. Make sure that the leads of the voltmeter are connected properly. Fig (3) shows correct polarities of voltmeter lead wires. These polarities should be adhered to seriously.

(8) Next step is to measure currents, I_1, I_2 and I_3, at the branch point (junction) *b.* Use the other multimeter as ammeter. Select AC, A and set to 2.00 mA range. (If the instructor advises differently, follow the instructor.)

(9) Ammeter connections for the three measurements are shown in Fig (4). It should be recalled that an ammeter must be connected in series. This requires disrupting the circuit and inserting the ammeter in between. Fig (4) clearly illustrate the breaking up of the circuit and the insertion of the ammeter in between. Correct polarities have also been shown which must be followed.

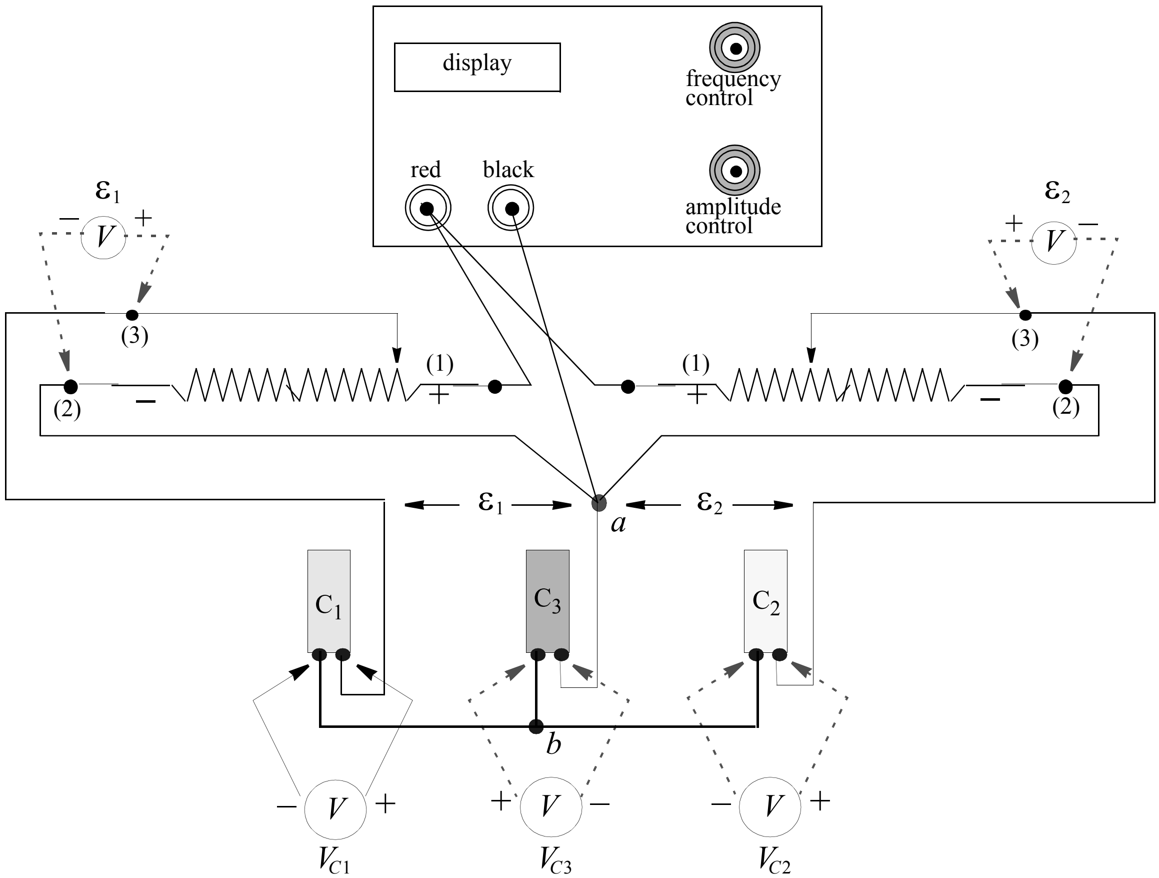

Fig (3) Assembling the Circuit and Measuring the Five Voltages

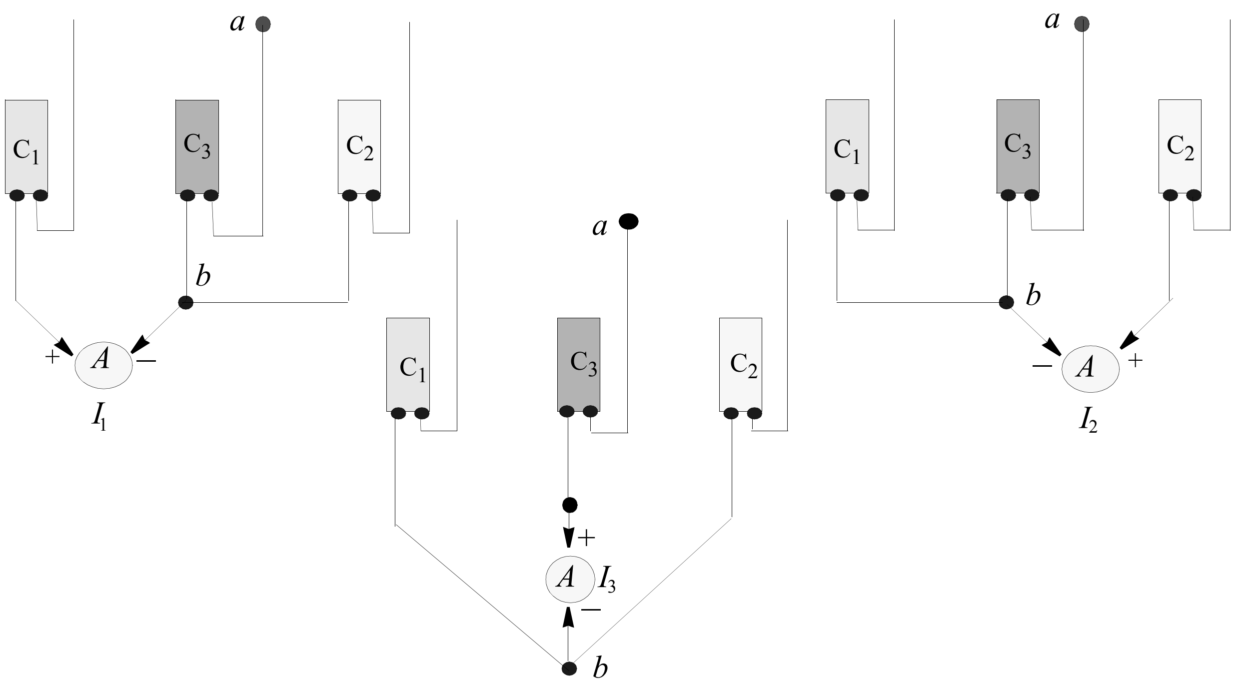

Fig (4) Measuring Currents I_1, I_2 and I_3 *at Junction b*

(10) Next step is to measure loop voltages ε_1, ε_2, and $(\varepsilon_1 - \varepsilon_2)$. The voltmeter placements for these measurements are shown in Fig (5). As always, follow the polarities very carefully.

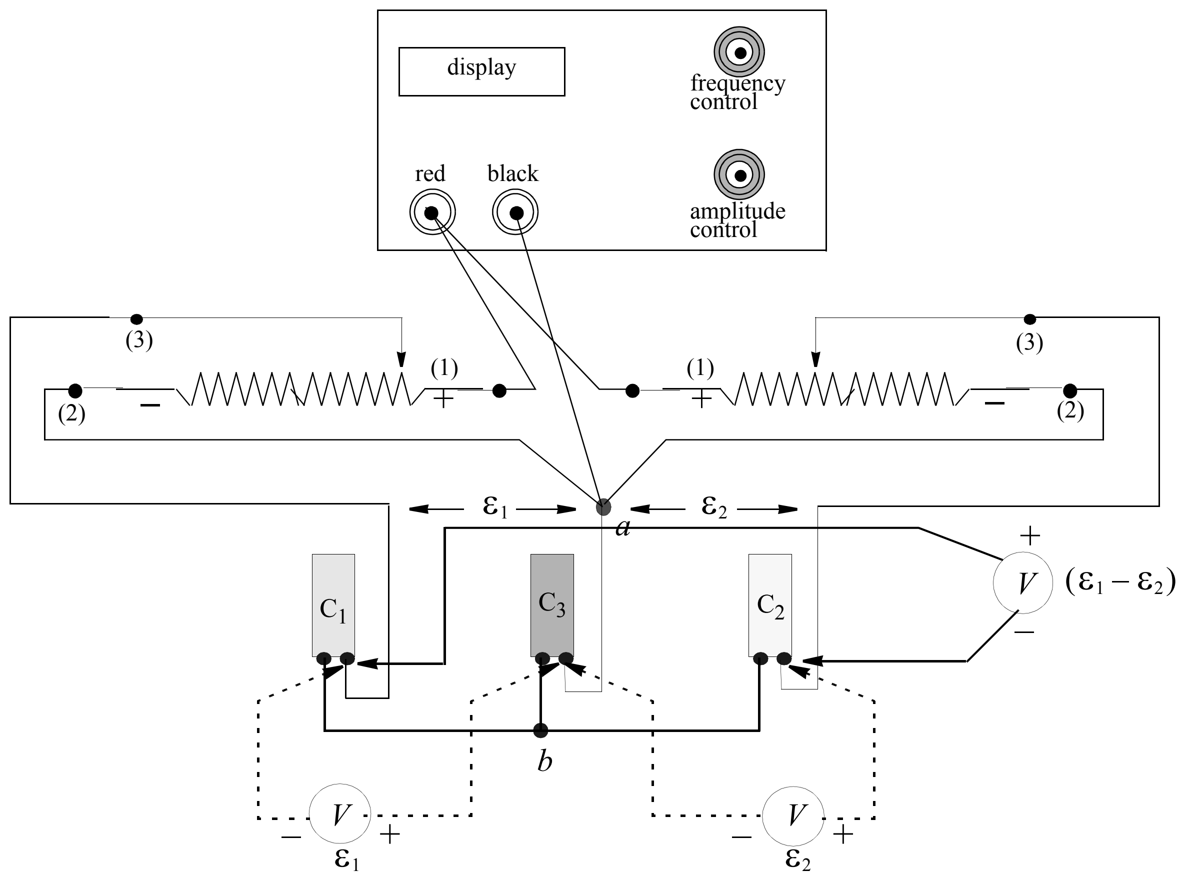

Fig (5) Measuring Loop Voltages

(11) Next (and last) step is to measure the junction current $I_1 + I_2$. Follow Fig. (6) and enter the value in the data sheet. This completes the work for the first part of the experiment.

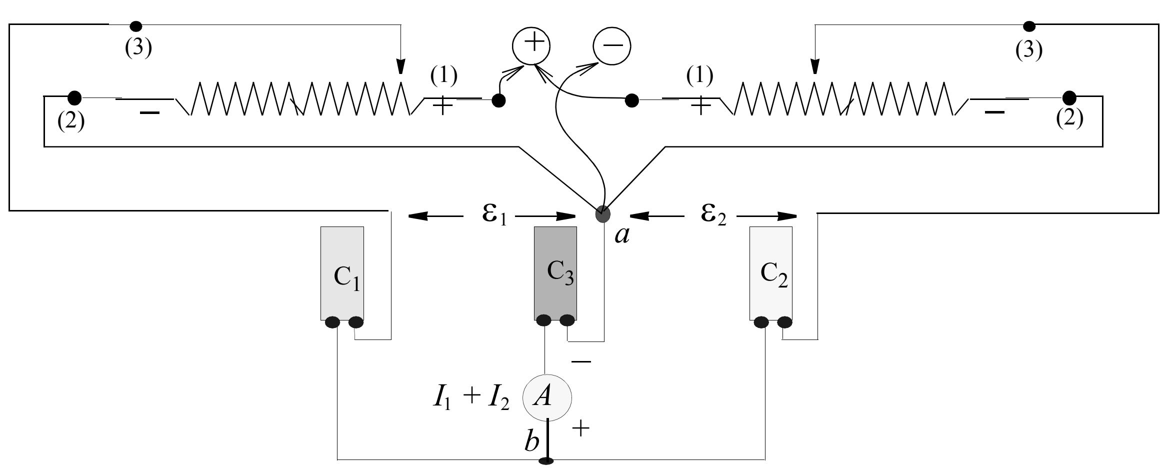

Fig (6) Measuring $I_1 + I_2$

(B) Plotting a graph of ε_1 against I_3

(1) No change in the circuit is required. We shall use the sliding contact of the left hand side rheostat to select values of ε_1. As ε_1 is already at its maximum value, we shall only be reducing its value from approximately 5.00 volt to 0.00 volt, in 16 steps. This will require moving the sliding contact to the left of where ever it is now.

(2) Use one multimeter as voltmeter and set it at 20 V, AC. Use it to measure ε_1 by connecting it to terminals (2) and (3) of the rheostat. Use the other multimeter as ammeter and set it to AC, A and select the 2.00 mA scale. Use it to measure the current I_3

(3) Switch on the function generator and for 16 different (arbitrary) positions of the sliding contact, read and record the values of ε_1 and I_3.

(4) The experiment ends. Disconnect the circuit, switch off all meters and arrange every thing neatly on the table.

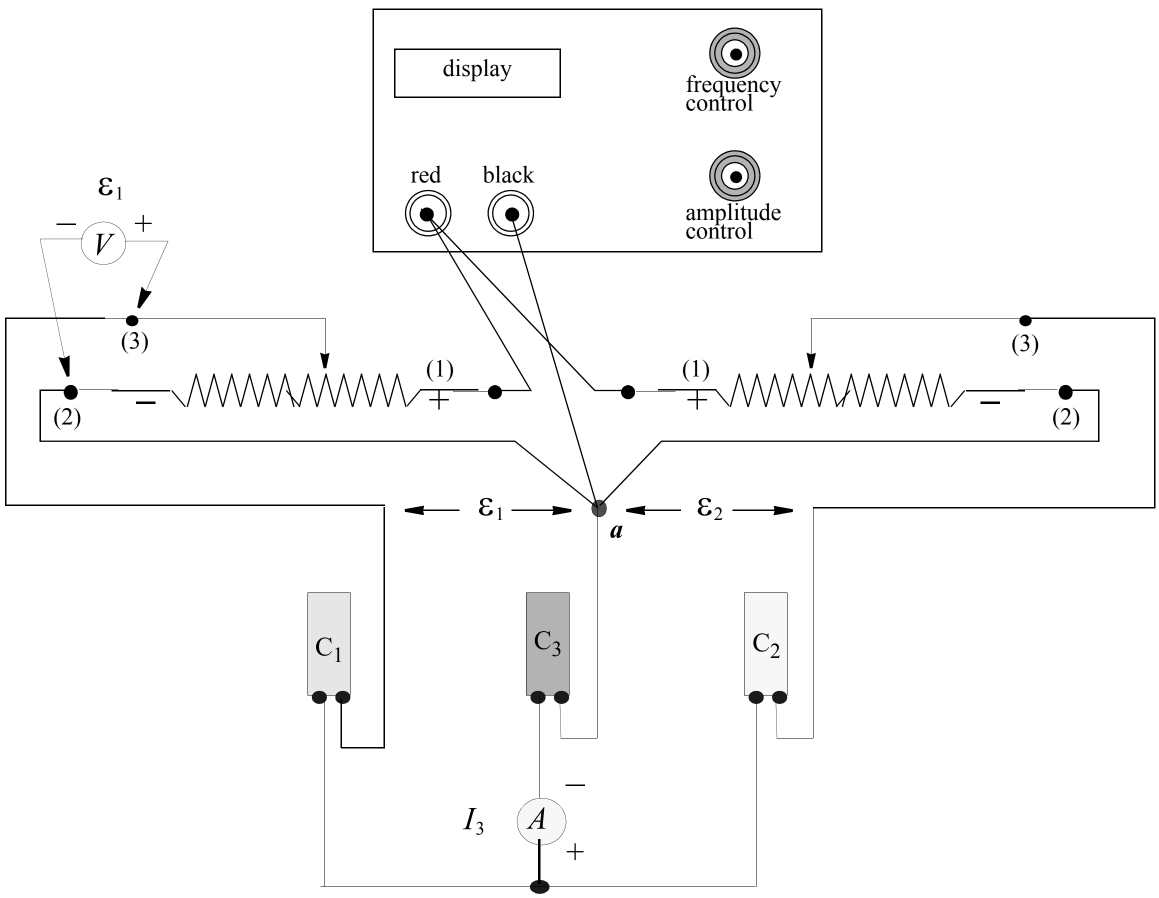

Fig (5) Circuit for Part (B) of the Experiment

Calculations And Graphs

(A) Verification of Eqns (14) through (20).

(1) Plug in the measured values of ε_1, ε_2, C_1, C_2 and C_3 in Eqn (14) and solve for I_3.

(2) Use Eqns (15) and (16) to calculate the values of I_1 and I_2.

(3) Calculate V_{C1}, V_{C2} and V_{C3} using Eqn (17).

(4) Calculate ε_1, ε_2 and $(\varepsilon_1 - \varepsilon_2)$ using Eqns (18), (19) and (20).

(5) Enter expected values (from step (1) through step (4) above) in the proper column in the first *Results* table. Then enter the experimental values from Data Sheet in the next column. Find percent errors enter in the last column of this table.

(B) The graph of I_3 *against* ε_1.

(1) Plot a graph of the 16 pairs of experimentally measured values of I_3 (on y-axis) against the 16 selected values of ε_1, (on the x-axis), using a computer.

(2) Calculate the intercept and slope using equations (22) and (23). These are *expected* values. Enter in proper columns in the second *Results* table.

(3) Write the values of the slope and the y-axis intercept, from the computer print-out. These are *experimental* values. Enter in proper columns in the second *Results* table. Compare each with its expected value. Find percent errors and record in the table.

Result Tables

A format for presenting the results of this experiment is given at the end.

Conclusions and Discussions

Write your conclusions from the experiment and discuss them.

What Did You Learn in this Experiment?.

A hearty and thoughtful account of what you learned in this experiment by way of the principle and the techniques of experimentation, should be given

Data & Data Tables

Name...................................   **Date**..............................

Instructor.............................   **Lab Section**...................

Partner.........................   **Table #**..........................

(A) Measurement of Currents and Voltages.

AC source voltage ε_1 :   volts

AC source voltage ε_2 volts

Table 1: Resistor Values

Resistor	Nominal value (as set on the Capacitance Box) (μF)	Actual Value (as found using an Ohmmeter) (μF)
C_1		
C_2		
C_3		

Table 2: Measured values of Parameters

	Parameters	Measured Values
1	V_{C1}	
2	V_{C2}	
3	V_{C3}	
4	I_1	
5	I_2	
6	I_3	

Table 2: Measured values of Parameters

	Parameters	Measured Values	
7	ε_1	a: step 7 b: step 9	
8	ε_2	a: step 7 b: step 9	
9	$\varepsilon_1 - \varepsilon_2$	a: step9	
10	$I_1 + I_2$	a: step 10	

(B) Plotting a graph of I_3 against ε_1.

Table 3:

Serial Number	ε_1	I_3	Serial Number	ε_1	I_3
1			9		
2			10		
3			11		
4			12		
5			13		
6			14		
7			15		
8			16		

Additional Data or Information (if any):

Table of Results **Name:** **Date:**

Table 4: (A) Direct Verification of Kirchhoff's Rules

#	Para-meters	Calculated Values,		Experimental Values,		Error Percentage
		Eqn. #	magnitude (units)	magnitude (units)	Source	
1	I_1	(14)			data table line 4	
2	I_2	(15)			data table line 5	
3	I_3	(16)			data table line 6	
4	V_{R1}	(17)			data table line 1	
5	V_{R2}	(17			data table line 2	
6	V_{R3}	(17)			data table line 3	
7	ε_1	(18)			data table line 7	
8	ε_2	(19)			data table line 8	
9	$\varepsilon_1 - \varepsilon_2$	(20)			data table line 9	
10	$I_1 + I_2$	(6)			data table line 10	

Table 5: Indirect Verification of Kirchhoff's Rules

Magnitude of I_3 when ε_1 is short-circuited (Expected)	Magnitude of I_3 when ε_1 is short-circuited (Experimental	% error	Conductance G_{eff} of C_3 when ε_2 is short-circuited (Expected)	Conductance G_{eff} of C_3 when ε_2 is short-circuited (Experimental)	% error

Appendix

We would like to prove a statement we made while interpreting the slope of the graph for part B of the experiment. It was stated that the slope of the graph is effective conductance G_{eff} of capacitor C_3 when ε_2 is shorted. We shall now prove this point by shorting ε_2 and calculating the current I_3. If we get the same value of for I_3 as given by Eqn (24), the above point will be proved.

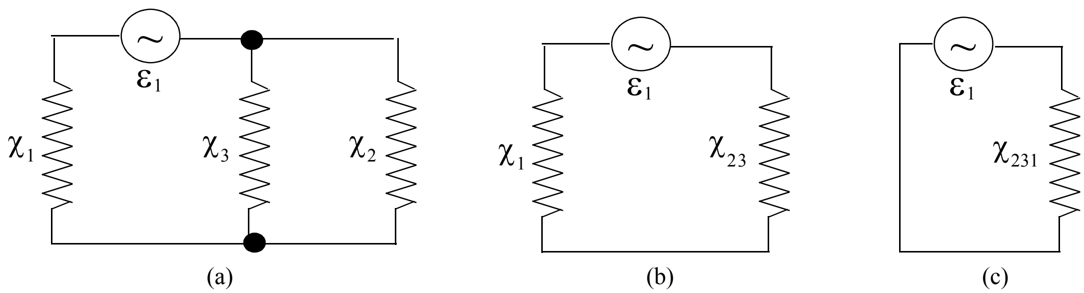

Fig (6) Crunching the Circuit of Fig (2)

(a) Combining χ_2 and χ_3 in parallel to get χ_{23}:

$$\chi_{2,3})_P = \frac{\chi_2\chi_3}{\chi_2 + \chi_3} = \chi_{23}$$

This is shown in Fig (6-b). Substituting the values of χ_2 and χ_3 from Eqn (3), we get:

$$\chi_{23} = \frac{\left(\frac{1}{\omega C_2}\right)\left(\frac{1}{\omega C_3}\right)}{\left(\frac{1}{\omega C_2}\right) + \left(\frac{1}{\omega C_3}\right)} = \frac{\frac{1}{\omega^2 C_2 C_3}}{\frac{C_2 + C_3}{\omega(C_2 C_3)}} = \frac{1}{\omega^2 C_2 C_3} \times \frac{\omega(C_2 C_3)}{C_2 + C_3} = \frac{1}{\omega(C_2 + C_3)}$$

Next we combine χ_{23} in series with χ_1 to get χ_{231}:

$$\chi_{23,1})_S = \chi_{23} + \chi_1 = \chi_{231}$$

This is shown in Fig (6-c). Substituting the values of χ_{23} and χ_1 we get:

$$\chi_{231} = \frac{1}{\omega(C_2 + C_3)} + \frac{1}{\omega C_1} = \frac{C_1 + C_2 + C_3}{\omega(C_2 + C_3)(C_1)}$$

Now we shall make a table of voltages and currents through each reactance.
The table appears on the next page.

Voltages and Currents for the Reactances of Fig (2)

S or P	Reactances		$V_C = \chi_C I_C$	$I_C = \dfrac{V_C}{\chi_C}$
	χ_C	values	voltages	currents
	χ_{231}	$\dfrac{C_1 + C_2 + C_3}{\omega(C_2 + C_3)(C_1)}$	ε_1	$\dfrac{\varepsilon_1}{\dfrac{C_1 + C_2 + C_3}{\omega(C_2 + C_3)(C_1)}} = \dfrac{\omega(C_2 + C_3)(C_1)}{C_1 + C_2 + C_3}(\varepsilon_1)$
S	χ_1	$\dfrac{1}{\omega C_1}$	$\dfrac{1}{\omega C_1} \times \dfrac{\omega(C_2 + C_3)(C_1)}{C_1 + C_2 + C_3}(\varepsilon_1) = V_{C1} = \left(\dfrac{C_2 + C_3}{C_1 + C_2 + C_3}\right)(\varepsilon_1)$	$I_1 = \dfrac{\omega(C_2 + C_3)(C_1)}{C_1 + C_2 + C_3}(\varepsilon_1)$
S	χ_{23}	$\dfrac{1}{\omega(C_2 + C_3)}$	$\dfrac{1}{\omega(C_2 + C_3)} \times \dfrac{\omega(C_2 + C_3)(C_1)}{C_1 + C_2 + C_3}(\varepsilon_1) = \left(\dfrac{C_1}{C_1 + C_2 + C_3}\right)(\varepsilon_1)$	$\dfrac{\omega(C_2 + C_3)(C_1)}{C_1 + C_2 + C_3}(\varepsilon_1)$
P	χ_2	$\left(\dfrac{1}{\omega C_2}\right)$	$V_{C2} = \left(\dfrac{C_1}{C_1 + C_2 + C_3}\right)(\varepsilon_1)$	$\dfrac{\left(\dfrac{C_1}{C_1 + C_2 + C_3}\right)(\varepsilon_1)}{\left(\dfrac{1}{\omega C_2}\right)} = I_2 = \left(\dfrac{\omega C_2 C_1}{C_1 + C_2 + C_3}\right)(\varepsilon_1)$
P	χ_3	$\left(\dfrac{1}{\omega C_3}\right)$	$V_{C3} = \left(\dfrac{C_1}{C_1 + C_2 + C_3}\right)(\varepsilon_1)$	$\dfrac{\left(\dfrac{C_1}{C_1 + C_2 + C_3}\right)(\varepsilon_1)}{\left(\dfrac{1}{\omega C_3}\right)} = I_3 = \left(\dfrac{\omega C_1 C_3}{C_1 + C_2 + C_3}\right)(\varepsilon_1)$

The value of I_3, as found for χ_3 in the ***last column of the last row*** in the above table, is exactly the same as that found in Eqn (22). Hence the above mentioned point is proved. It is good to know that two different techniques (crunching & Kirchhoff's rules) lead to identical results.