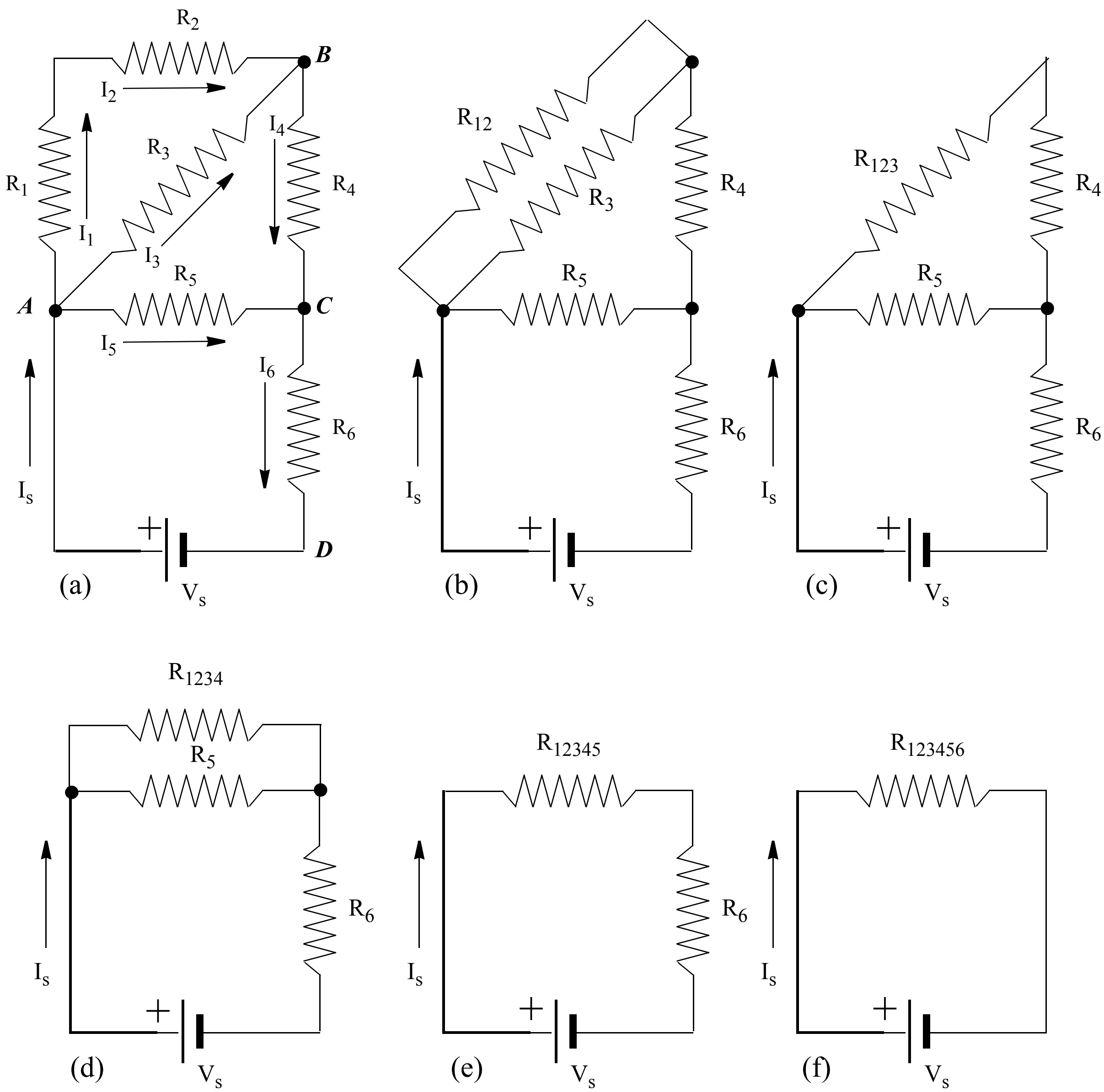
Circuit Analysis Using the Crunching Technique

(A) Crunching: Schematic

The circuit in Fig (1a)[*] is crunched in several steps, shown in Figs (1b), (1c), (1d) and (1e). The "crunched" circuit appears in Fig (1f).



Each resistor is 2.8 $k\Omega$ and the source is a 12 volt battery.

Find currents and voltages through each resistor,. Also find potential differences: (i) $V_{ab,}$ (ii) V_{ac}., and (iii) V_{bc}.

(*) Giancoli sixth edition

(B) Crunching: Analytic

$R_{1,2}$):S	$2.8 + 2.8 = 5.6\ k\Omega$	R_{12}	$R_{eq} = R_{123456} = 4.55000\ k\Omega$
$R_{12,3}$):P	$(2.8^{-1} + 5.6^{-1})^{-1} = 1.86667\ k\Omega$	R_{123}	$V_s = (R_{eq}) \times (I_s)$
			$12 = 4.55000\ I_s$
$R_{123,4}$):S	$2.8 + 1.86667 = 4.66667\ k\Omega$	R_{1234}	$I_s = 2.63736$ mA
			$I_{123456} = 2.63736$ mA
$R_{1234,5}$):P	$(2.8^{-1} + 4.66667^{-1})^{-1} = 1.75000\ k\Omega$	R_{12345}	
$R_{12345,6}$):S	$2, 8 + 1.75000 = 4.55000\ k\Omega$	R_{123456}	

(C) Calculating Currents through Each Resistor

Table 1: Calculating Currents through Each Resistor

Series or parallel	Resistors		Voltage (V = RI) (volts)	Current (I = V/R) (mA)
	symbol	value ($k\Omega$)		
	$R_{12345,6}$	**4.55000**	**12.0000**	12/4.55000 = **2.63736**
S	R_6	**2.8**	2.8 x 2.63736 = **738462**	**2.63736**
	$R_{1234,5}$	1.75000	1.75 x 2.63736 = 4.61538	2.63736
P	R_5	**2.8**	**4.61538**	4.61538/2.8 = **1.64835**
	$R_{123,4}$	4.66667	4.61538	4.61538/4.66667 = 0.98901
S	R_4	**2.8**	2.8 x 0.989013 = **2.76923**	**0.98901**
	$R_{12,3}$	1.86667	1.86667 x 0.98901 = 1.84616	0.98901
P	R_3	**2.8**	**1.84616**	1.84616/2.8 = **0.65934**
	$R_{1,2}$	5.6	1.84616	1.846165.6 = 0.32967
S	R_2	**2.8**	2.8 x 0.32967 = **0.92308**	**0.32967**
	R_1	**2.8**	2.8 x 0.32967 = **0.92308**	**0.32967**

(D) Calculating Potential Differences Between Given Points

For the routes between the given points, please see Fig (1a).

Potential difference between a pair of points in a circuit is independent of the route. This is shown by selecting three pairs of points (AB, AC and BC) in the circuit of Fig (1a) and calculating potential differences between them through all possible routes. The respective potential differences are $V_{AB,}$ V_{AC} and V_{BC}.

One follows the directions of currents through resistors. While using the formulae $V = \varepsilon - \Sigma I_n R_n$ or $V = \Sigma I_n R_n$, if one or more arrows are found to be oppositely directed to the direction of traversal through that route, then the contributions of such currents are deemed negative. Such contributions get subtracted if we are adding or get added if we are subtracting.

Of all possible routes, one will produce a negative potential difference. Any route with positive potential difference combined with the one with negative potential difference, makes a round trip or a “loop” to which Kirchhoff’s Loop Rule applies.

Please follow the calculations given below, very carefully.

Table 2: Calculating Potential Differences Between Given Points

	Route	Formula	Calculation (K¾ x mA = V)	Potential Difference (V)
V_{AB}	R_3	$V = \Sigma I_n R_n$	(2.8 x 0.0.65934) or, read directly from the table	1.84616
	R_1 & R_2	$V = \Sigma I_n R_n$	(2.8 x 0.32967) + (2.8 x 0.32967)	1.84615
	R_5 & R_4	$V = \Sigma I_n R_n$	(2.8 x 1.64835) – (2,8 x 0.98901)	1.84615
	battery, $R_{6\ \&}$ R_4	$V = \varepsilon - \Sigma I_n R_n$	–12 – (– 2.8 x 2.637364) – (– 2.8 x 0.98901)	– 1.84616
V_{AC}	R_5	$V = \Sigma I_n R_n$	(2.8 x 1.64835) or, read directly from the table	4.61538
	R_3 & R_4	$V = \Sigma I_n R_n$	(2.8 x 0.65934) + (2.8 x 0.98901)	4.61539
	R_1, R_2 & R_4	$V = \Sigma I_n R_n$	(2.8 x 0.32967) + (2.8 x 0.32967) + (2.8 x 0.98901)	4.61538
	battery & R_6	$V = \varepsilon - \Sigma I_n R_n$	– 12 – (– 2.8 x 2.63736)	– 4.61539
V_{AD}	battery	$V = \varepsilon - \Sigma I_n R_n$	– 12 – 0	– 12.00000
	R_5 & R_6	$V = \Sigma I_n R_n$	(2.8 x 1.64835) + (2.8 x 2.63736)	11.99999
	R_3, R_4 & R_6	$V = \Sigma I_n R_n$	(2.8 x 0.65934) + (2.8 x 0.98901) + (2.8 x 2.63736)	11.99999
	R_1, R_2, R_4 & R_6	$V = \Sigma I_n R_n$	(2.8 x 0.32967) + (2.8 x 0.32967) + (2.8 x 0.98901) + (2.8 x2.63736)	11.99999