A Resounding Proof of

Justin Timberlake's Theorem

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Introduction

Justin Timberlake's well known theorem, stated simply, reads:

"what goes around, comes back around."

The theorem, in the above form or in any of its many equivalent forms, is often used by authors, journalists and politicians. They all take it to be a true and well-established theorem. However, no formal or informal proof of the theorem has ever been reported in literature.

In what follows, we present a mathematically sound and scientifically valid proof of the theorem. It is based on the performance of a simple electrical circuit consisting of a resistor fed by a real-life battery. The equations are of a nature that permit us to *rotate* the variables. The final equation is an n^{th} order equation where n represents the number of rotations. No matter what value of n we choose, i.e. no matter how many times we rotate, we always get back to the same basic equation, thereby proving the theorem. This basic equation (Eqn 3, next page) is, of course, derived by applying Kirchhoff's loop rule to the circuit.

Consider a resistor R. When a potential difference V_{ab} is applied across its terminals, a current I flows through it. Such an arrangement is shown in Fig (1). According to the Ohm's law, we get:

$$V_a - V_b = V_{ab} = RI \qquad ....(1)$$

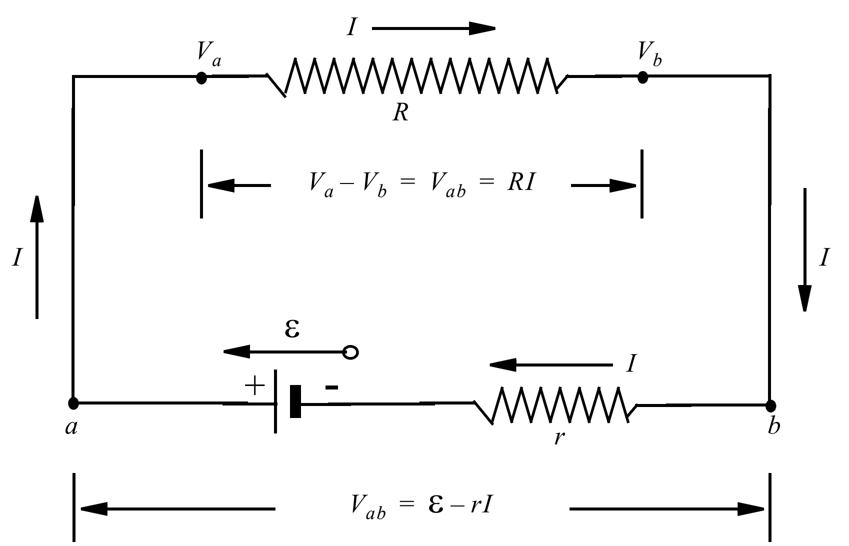

Fig (1) Resistor & Ohm's Law

The potential difference V_{ab} is also known as the "***Terminal Potential***".

The resistor is connected to a real-life battery of electro-motive force ε and internal resistance r. It is this battery that supplies the current I to the resistor. According to the battery-equation,

$$V_a - V_b = V_{ab} = \varepsilon - rI \qquad \text{..........(2)}$$

Applying the Kirchhoff's loop rule to the loop with the resistor and the real-life battery and rearranging, we get

$$\varepsilon = (R + r)I \qquad \text{..........(3)}$$

Eqn (3) is the basic equation of the circuit.

Solving Eqs (1) for I, we get:

$$I = \frac{V_{ab}}{R} \qquad \text{..........(4)}$$

From Eqn (3), we get another value of I:

$$I = \frac{\varepsilon}{R + r} \qquad \text{..........(5)}$$

Rotating The Terminal Potential V_{ab} and the Current I

Recall Eqn (2)

$$V_{ab} = \varepsilon - rI$$

Insert the value of I from Eqn (4) to get:

$$V_{ab} = \varepsilon - r\left(\frac{V_{ab}}{R}\right)$$

$$= \varepsilon - \left(\frac{r}{R}\right)(V_{ab})$$

Now we insert the value of V_{ab} from Eqn (2) in the last expression:

$$V_{ab} = \varepsilon - \left(\frac{r}{R}\right)(\varepsilon - rI)$$

$$= \varepsilon - \left(\frac{r}{R}\right)(\varepsilon) + \left(\frac{r}{R}\right)(rI)$$

$$= \varepsilon\left(1 - \frac{r}{R}\right) + \left(\frac{r^2}{R}\right)(I)$$

We insert the value of I from Eqn (4) to get:

$$V_{ab} = \varepsilon\left(1 - \frac{r}{R}\right) + \left(\frac{r^2}{R^2}\right)(V_{ab})$$

We repeat the above process by first inserting the value of V_{ab} from Eqn (2) and then inserting the value of I from Eqn (4). We first get:

$$= \varepsilon\left(1 - \frac{r}{R} + \frac{r^2}{R^2}\right) - \left(\frac{r^3}{R^2}\right)(I)$$

and then:

$$= \varepsilon\left(1 - \frac{r}{R} + \frac{r^2}{R^2}\right) - \left(\frac{r^3}{R^3}\right)(V_{ab})$$

Repeating the whole process all over again, we get:

$$= \varepsilon\left(1 - \frac{r}{R} + \frac{r^2}{R^2} - \frac{r^3}{R^3}\right) + \left(\frac{r^4}{R^3}\right)(I)$$

followed by:

$$= \varepsilon\left(1 - \frac{r}{R} + \frac{r^2}{R^2} - \frac{r^3}{R^3} + \frac{r^4}{R^4}\right) - \left(\frac{r^5}{R^5}\right)(V_{ab})$$

To be fully satisfied and totally confident, we repeat the process one more time. We get:

$$V_{ab} = \varepsilon\left(1 - \frac{r}{R} + \frac{r^2}{R^2} - \frac{r^3}{R^3} + \frac{r^4}{R^4} - \frac{r^5}{R^5}\right) + \left(\frac{r^6}{R^6}\right)(V_{ab}) \qquad \text{..........}(6)$$

We now have a pattern! It enables us to generalize Eqn (6) to an n^{th} order equation. Watch:

$$V_{ab} = \varepsilon\left[\sum_{n=0}^{n}(-1)^n\left(\frac{r}{R}\right)^n\right] + \left[(-1)^{n+1}\left(\frac{r}{R}\right)^{n+1}\right](V_{ab})$$

$$V_{ab} - \left[(-1)^{n+1}\left(\frac{r}{R}\right)^{n+1}\right](V_{ab}) = \varepsilon\left[\sum_{n=0}^{n}(-1)^n\left(\frac{r}{R}\right)^n\right]$$

$$V_{ab}\left[1 - (-1)^{n+1}\left(\frac{r}{R}\right)^{n+1}\right] = \varepsilon\left[\sum_{n=0}^{n}(-1)^n\left(\frac{r}{R}\right)^n\right]$$

Solving for V_{ab}, we get:

$$V_{ab} = \left\{\frac{\left[\sum_{n=0}^{n}(-1)^n\left(\frac{r}{R}\right)^n\right]}{\left[1 - (-1)^{n+1}\left(\frac{r}{R}\right)^{n+1}\right]}\right\}(\varepsilon) \qquad \text{..........}(7)$$

This is the terminal voltage applied to the resistor in Fig (1).

Rotating The emf ε and the Current I

Rearranging Eqn , (2) we get:

$$\varepsilon = V_{ab} + rI \qquad \text{..........}(8)$$

Recall Eqn (5)

$$I = \frac{\varepsilon}{R + r} \qquad \text{..........Eqn (5)}$$

Insert the value of ε from Eqn (8) into Eqn (5), we get:

$$I = \frac{V_{ab} + rI}{R + r}$$

$$= \frac{V_{ab}}{R + r} + \left(\frac{r}{R + r}\right)(I)$$

Insert the value of I from Eqn (5), to get:

$$= \frac{V_{ab}}{R + r} + \left(\frac{r}{R + r}\right)\left(\frac{\varepsilon}{R + r}\right)$$

$$I = \frac{V_{ab}}{R+r} + \left(\frac{r}{(R+r)^2}\right)(\varepsilon)$$

We replace ε by its value from Eqn (8), we get: (some rearrangement is carried out)

$$= \left(\frac{V_{ab}}{(R+r)}\right)\left(1 + \frac{r}{(R+r)}\right) + \left(\frac{r^2}{(R+r)^2}\right)(I)$$

We repeat the above process of replacing I by its equivalent from Eqn (5) and then replacing ε by its equivalent from Eqn (8), we first get;

$$= \left(\frac{V_{ab}}{(R+r)}\right)\left(1 + \frac{r}{(R+r)}\right) + \left(\frac{r^2}{(R+r)^3}\right)(\varepsilon)$$

and then: (some rearrangement is carried out)

$$I = \left(\frac{V_{ab}}{(R+r)}\right)\left(1 + \frac{r}{(R+r)} + \frac{r^2}{(R+r)^2}\right) + \left(\frac{r^3}{(R+r)^3}\right)(I)$$

For further satisfaction, we repeat the process once more. We first get:

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$$= \left(\frac{V_{ab}}{(R+r)}\right)\left(1 + \frac{r}{(R+r)} + \frac{r^2}{(R+r)^2}\right) + \left(\frac{r^3}{(R+r)^4}\right)(\varepsilon)$$

and then:

$$I = \left(\frac{V_{ab}}{(R+r)}\right)\left(1 + \frac{r}{(R+r)} + \frac{r^2}{(R+r)^2} + \frac{r^3}{(R+r)^3}\right) + \left(\frac{r^4}{(R+r)^4}\right)(I)$$

We have established a pattern. It is possible to find an n^{th} order expression for the terminal voltage V_{ab}, supplied by the real-life battery.

Watch carefully:

$$I = \left(\frac{V_{ab}}{(R+r)}\right)\left[\sum_{n=0}^{n}\left(\frac{r}{R+r}\right)^n\right] + \left[\left(\frac{r}{R+r}\right)^{n+1}\right](I)$$

$$I - \left[\left(\frac{r}{R+r}\right)^{n+1}\right](I) = \left(\frac{V_{ab}}{(R+r)}\right)\left[\sum_{n=0}^{n}\left(\frac{r}{R+r}\right)^n\right]$$

$$I\left[1 - \left(\frac{r}{R+r}\right)^{n+1}\right] = V_{ab}\left(\frac{1}{(R+r)}\right)\left[\sum_{n=0}^{n}\left(\frac{r}{R+r}\right)^n\right]$$

Solving for V_{ab}, we get:

$$V_{ab} = \left\{\frac{\left[1 - \left(\frac{r}{R+r}\right)^{n+1}\right]}{\left(\frac{1}{(R+r)}\right)\left[\sum_{n=0}^{n}\left(\frac{r}{R+r}\right)^n\right]}\right\}(I) \qquad \text{..........(9)}$$

The n^{th} order Rotation Equation

Equating the two values of V_{ab}, from Eqns (7) and (9), we get:

$$\left\{\frac{\left[\sum_{n=0}^{n}(-1)^{n}\left(\frac{r}{R}\right)^{n}\right]}{\left[1-(-1)^{n+1}\left(\frac{r}{R}\right)^{n+1}\right]}\right\}(\varepsilon) = \left\{\frac{\left[1-\left(\frac{r}{R+r}\right)^{n+1}\right]}{\left(\frac{1}{(R+r)}\right)\left[\sum_{n=0}^{n}\left(\frac{r}{R+r}\right)^{n}\right]}\right\}(I)$$

Rearranging:

$$\varepsilon = \left\{\frac{\left[1-\left(\frac{r}{R+r}\right)^{n+1}\right]\times\left[1-(-1)^{n+1}\left(\frac{r}{R}\right)^{n+1}\right]}{\left(\frac{1}{(R+r)}\right)\left[\sum_{n=0}^{n}\left(\frac{r}{R+r}\right)^{n}\right]\times\left[\sum_{n=0}^{n}(-1)^{n}\left(\frac{r}{R}\right)^{n}\right]}\right\}(I) \qquad \text{..........}(10)$$

Eqn (10) is the final equation that completes our proof. The parameter n represents the number of rotations. It can have any value. If you rotated 10 times, n will be 10. If you rotated only 3 times, n will be 3. No matter what value of n is chosen, when you solve the Eqn (10), the answer will invariably always be:

$$\varepsilon = (R+r)I$$

which is Eqn (3), the basic equation of the circuit!

This adequately proves Justin Timberlake’s Theorem:

“what goes around, comes back around”.