Linear Graphs

A graph is plotted for (one) dependent variable against (one) independent variable. These are respectively called the "y" and the "x" variables. The graph will be a straight line if the dependent variable y, is directly proportional to the independent variable x. There are two possibilities: (i) y increases as x increases, and (ii) y decreases as x increases. These graphs are shown in Fig (1).

$$y = b + mx$$

$$|y_f| > |y_i|$$

$$y = b - mx$$

$$|y_f| < |y_i|$$

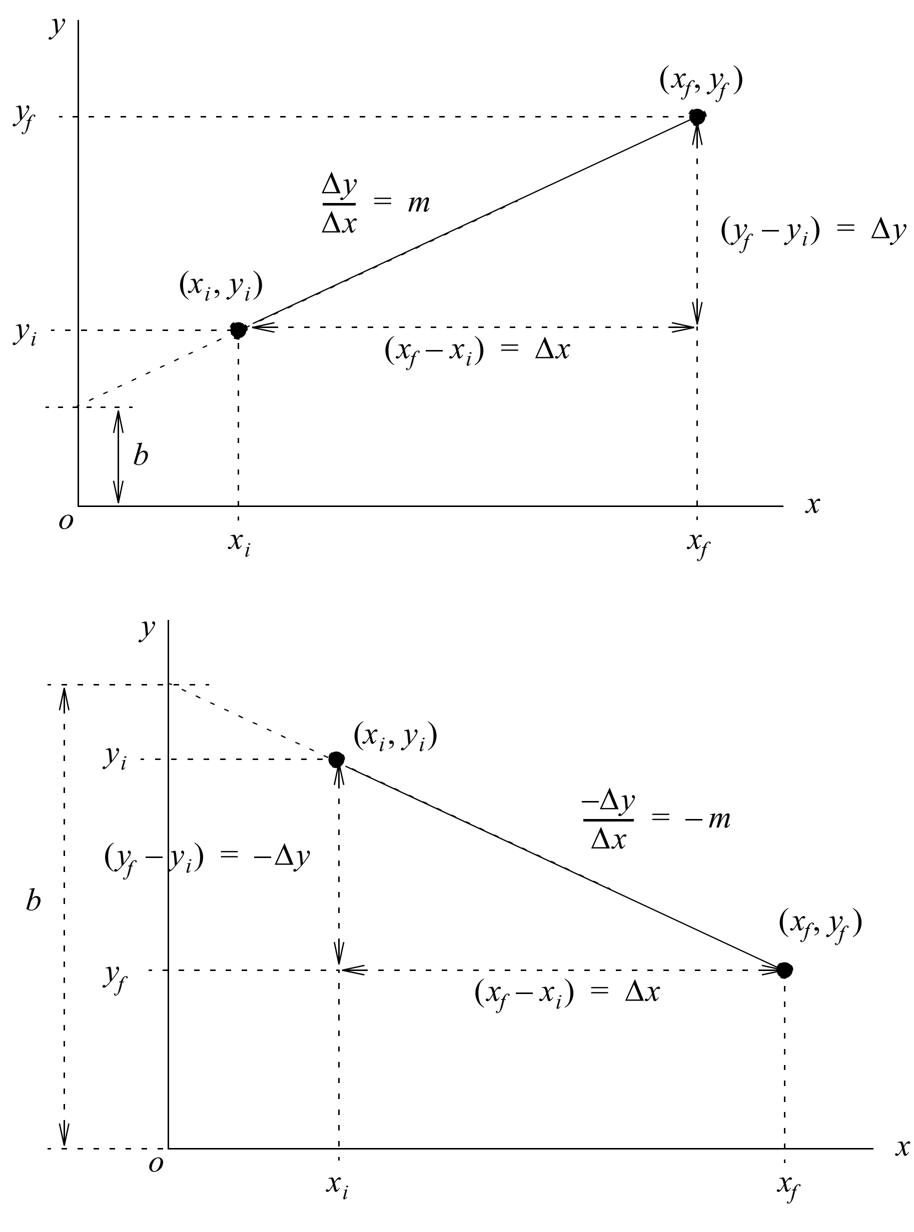


Fig (1) Straight Line Graphs with Positive and Negative Slope; Dependent Variable in the Positive Territory

Now, what if the dependent variable had only negative values while the dependent variable had only positive values? This is very important because the variable "time" is always positive while variables such as displacements and velocities can be positive or negative. With the x variable being positive and the y variable being negative, the graphs (straight lines) will now be in the fourth quadrant. These are shown in Fig (2).

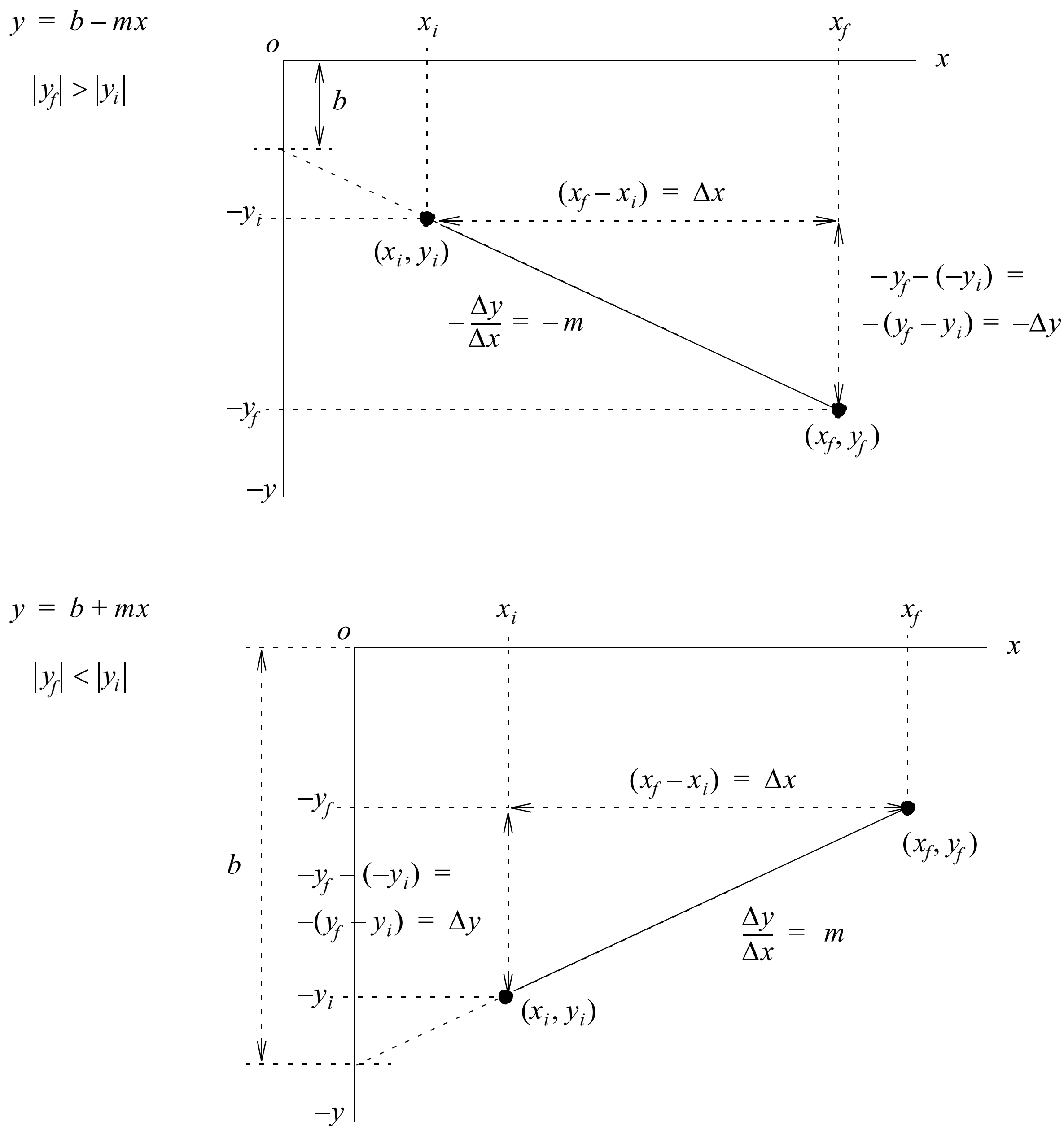


Fig (2) Straight Line Graphs with Positive and Negative Slope; Dependent Variable in the Negative Territory

To better understand the (hidden) conclusions, we have redrawn them, as shown in Figs (3) & (4). In Fig (3) Both straight lines have positive slopes even though the y variable is increasing in one case and decreasing in the other. The very orientation of the two lines is indicative of the positivity of the slopes and I am sure you could tell that even with your eyes closed!

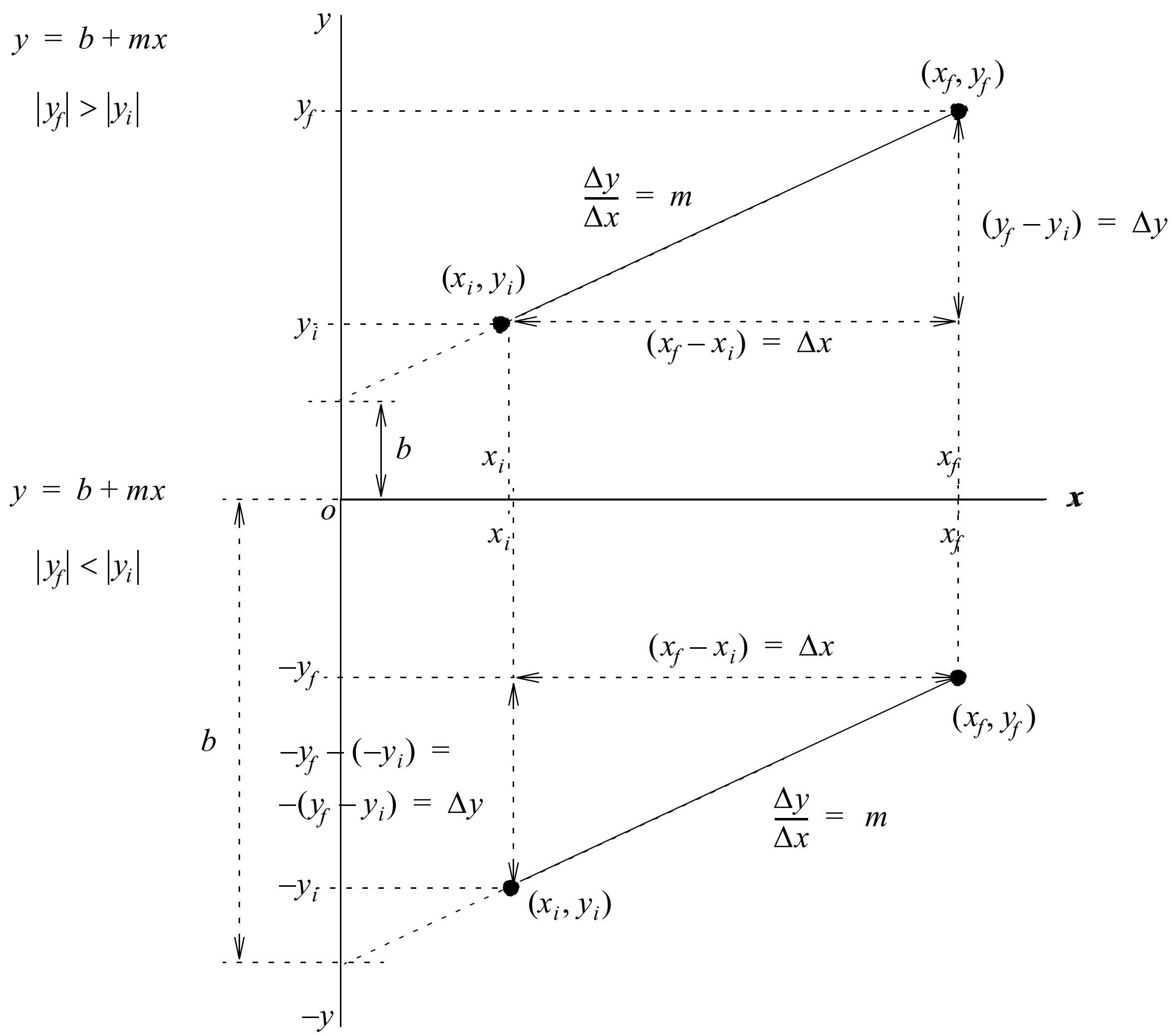
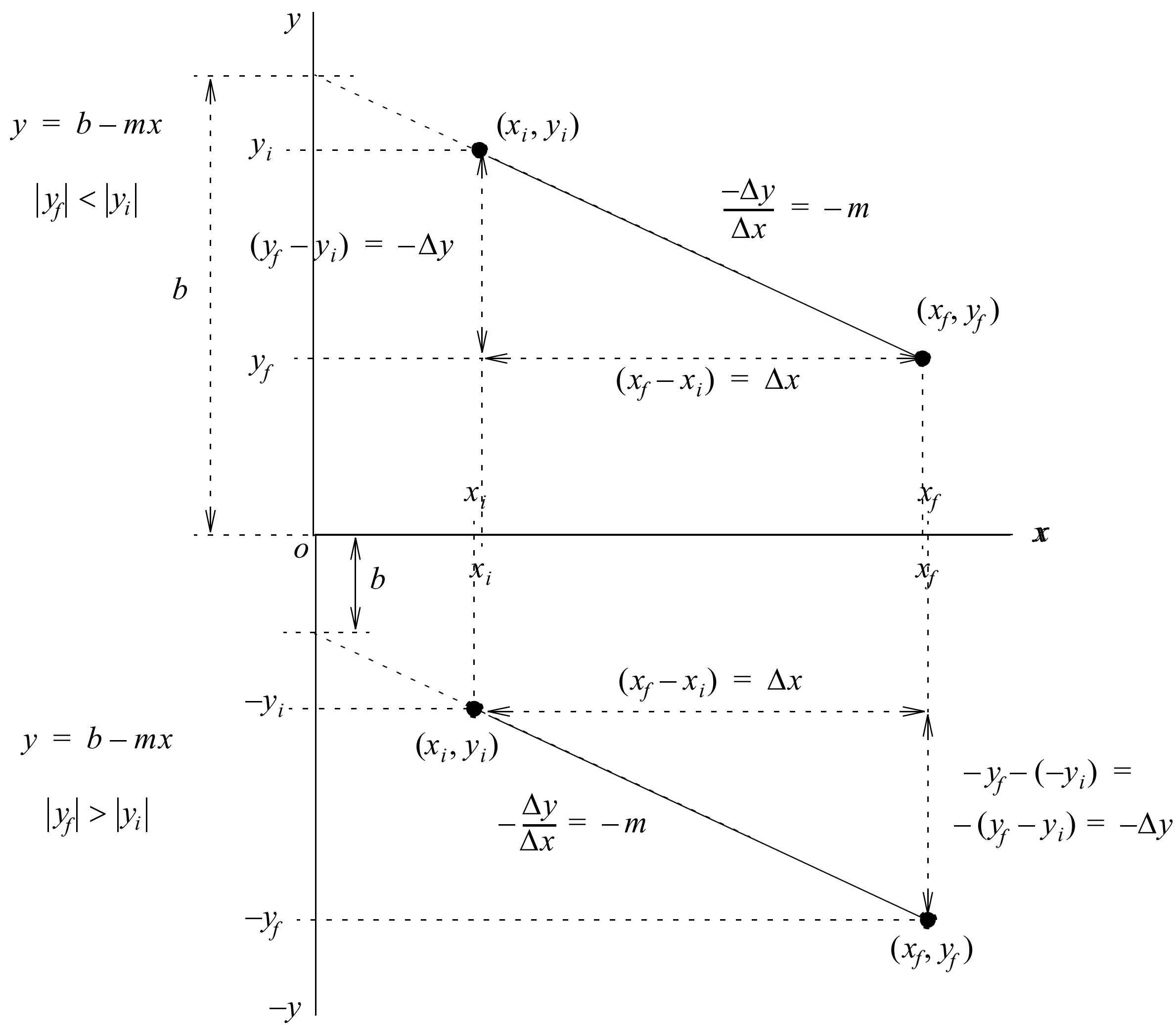


Fig (3) Positive Slopes for Both, the Increasing and the Decreasing Dependent Variable

In Fig (4) Both straight lines have negative slopes even though the y variable is increasing in one case and decreasing in the other. The very orientation of the two lines is indicative of the negativity of the slopes and I am sure you could tell that even with your eyes closed!



Negative Slopes for Both, The increasing and the Decreasing Dependent Variable

Now, let us suppose that the independent variable is "time t" and the dependent variable is "velocity v". The x-variable will always remain positive but velocity can be positive or negative. Consider a car. When we are driving it in a forward gear, the velocity is positive. When we are reversing it in the reverse gear, the velocity is negative. Alternatively, if an object is moving vertically up, (irrespective of its position in a reference frame), its velocity is positive. When it is moving vertically downward (again, irrespective of its position in a reference frame), its velocity is negative.

In Fig (5), we have replaced the variables x and y by t and v. The slope m is replaced by "acceleration, a". Many other elements from previous graphs have been eliminated to make the graphs less conjested.. In the top half of the graph, $|y_f| > |y_i|$, so you are accelerating. In the bottom half of the same graph, $|y_f| < |y_i|$. So you are braking. It is found that irrespective of whether you accelerate or brake,

The acceleration is positive in both cases!

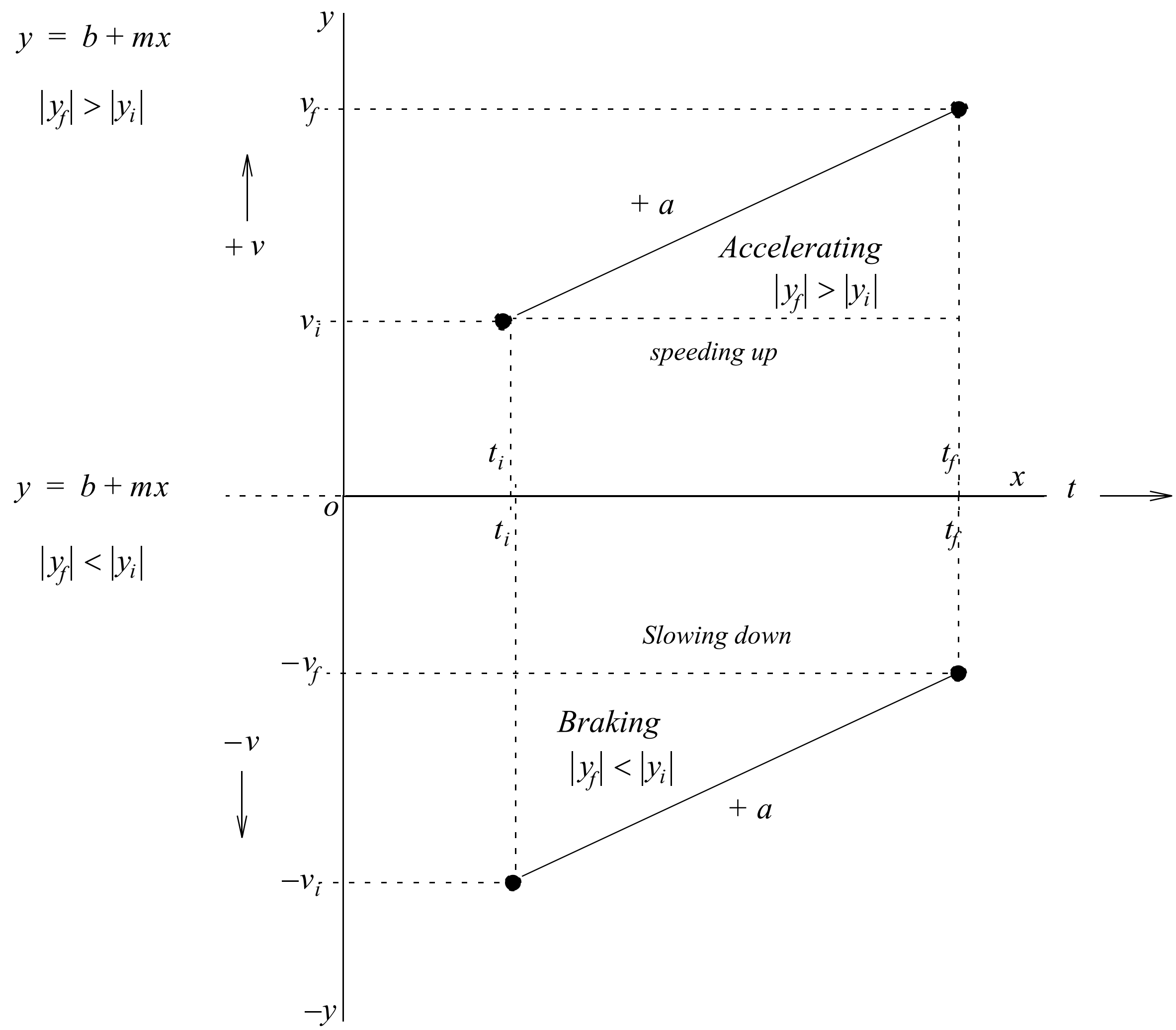


Fig (5) Positive Acceleration Both when (i) Speeding Up, and (ii) Slowing Down

If you tell your friend that while the dear friend is driving the car out of the garage, in reverse gear and is applying brakes, your friend is, in fact, applying a positive acceleration. Very soon we shall show that when your friend is accelerating the car (still in the reverse gear), your friend is, in fact, applying a negative acceleration (or is decelerating the car).

This is shown in Fig (6). Here we have drawn the two graphs each with a negative slope, together. It is seen that while braking (above the x-axis) the acceleration (the slope) is negative. This was expected. But we find that while accelerating, in the reverse gear, the acceleration is still negative.

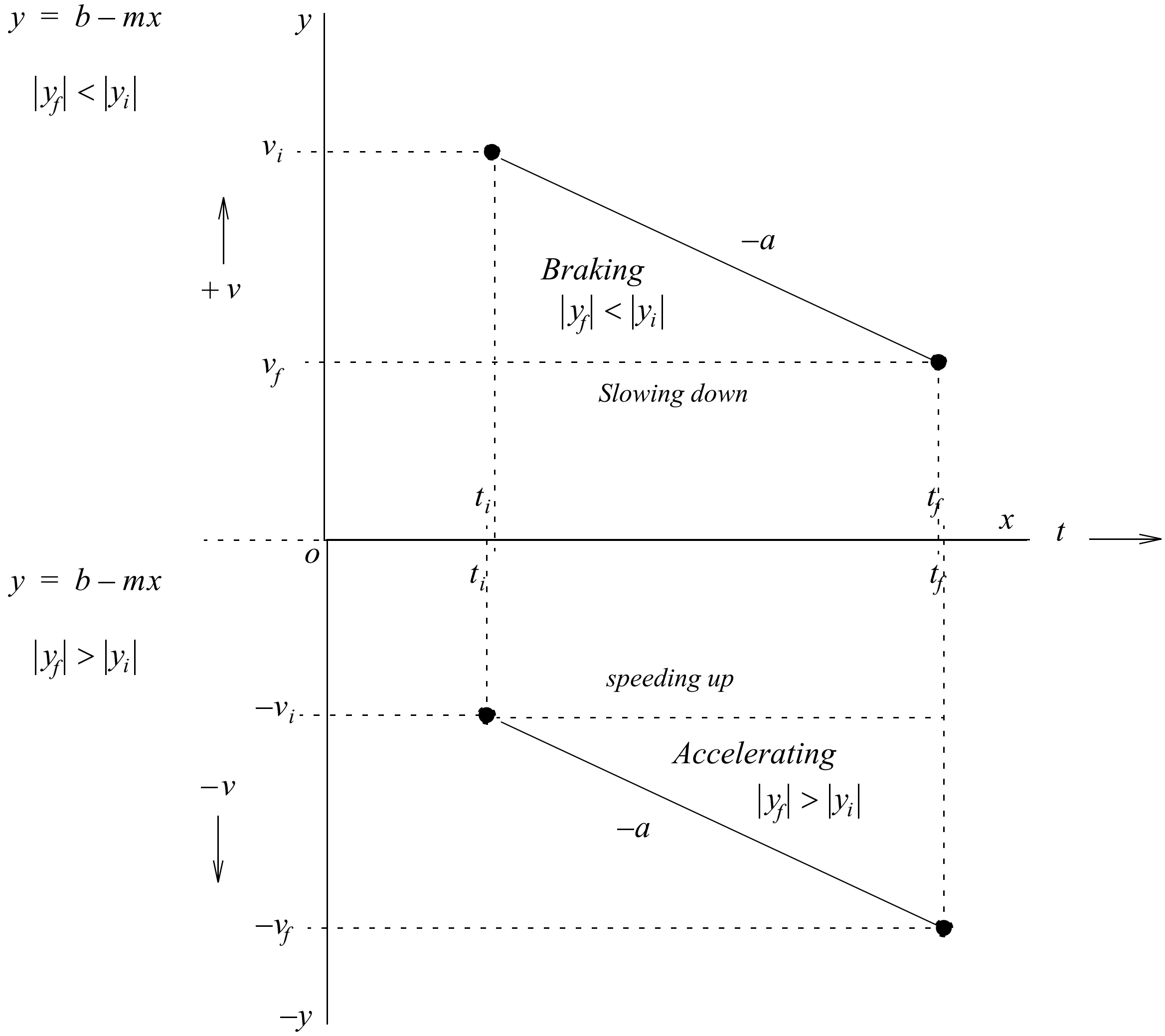


Fig (6) Negative Acceleration Both when (i) Speeding Up, and (ii) Slowing Down

The Rule

If the acceleration is causing the velocity to increase, then both (the velocity and the acceleration) must have the same algebraic sign: either both positive or both negative. The selections are:

(+v, +a) or (–v,– a)

If, on the other hand, the acceleration is causing the velocity to decrease, then both (the velocity and the acceleration) must have opposite algebraic sign: one positive and one negative. The selections are:

(+v, – a) or (–v,+ a)